SOME PROPERTIES OF THE CAPACITY OF MIMO SYSTEMS WITH CO-CHANNEL INTERFERENCE

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ABSTRACT

The mutual information of a multiple-input multiple-output (MIMO) system with co-channel interference is considered. Perfect channel information is assumed to be available to the receiver, but the transmitter has no channel information. It is theoretically proved that the worst interference condition is when the total power is equally distributed among all available interfering antennas. Moreover, equal-power interferers give worse performance than unequal-power interferers, and a smaller number of interferers each with larger power degrades performance less than a larger number of interferers each with lower power. Finally, it is shown that, for asymptotically large interference, when the number of interfering antennas N_I is smaller than the number of receive antennas N_R, the system is equivalent to a reduced MIMO system with $N_R - N_I$ receive antennas. When $N_I \geq N_R$, the mutual information approaches zero as interference becomes asymptotically large.

1. INTRODUCTION

In recent years multiple-input multiple-output (MIMO) systems have attracted great attention. MIMO systems have shown great potential for providing high spectral efficiency in isolated, single user, wireless links without interference [1]. There has also been some research on MIMO channels with co-channel interference [2]-[6]. In particular, [2] provided a closed-form solution for the capacity in the limit of a large number of antennas, including the case with interference. In [3], MIMO capacity with interference was analyzed in the low-power regime. In [4], the behavior of the capacity with varying number of interferers was studied through simulations. In this paper, we focus on the mutual information of MIMO systems with co-channel interference which use single-user detection. Our goal is to provide some insight on how interference affects a MIMO system and what kind of interference environment is more preferable (or undesirable) through theoretical analysis. In Section 2, the system model is introduced. Section 3 gives the analysis of the ergodic mutual information under different interference conditions which include considering a different number of interferers and different power allocation among the interferers. Then in Section 4, the asymptotical analysis is provided for systems with large interference. Finally, Section 5 gives the conclusion.

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2. SYSTEM MODEL

We consider a MIMO system with interference, where the desired user employs N_T transmit antennas and N_R receive antennas, and suffers from co-channel interference from other users. Initially we assume that there are totally K interferers, and the ith interferer has M_i transmit antennas. This is equivalent to a system with a single interferer which has totally $\sum_{i=1}^{K} M_i$ antennas and the appropriate covariance matrix for the interfering signal. Therefore, throughout the following discussion, a single interferer with totally N_I transmit antennas is assumed. Note that similar observations were also made in [2][3]. Under this assumption, the received complex baseband signal vector ($N_R \times 1$) of the desired user is given by

$$\mathbf{y} = \sqrt{\rho}\,\mathbf{H}\,\mathbf{x} + \sqrt{\eta}\mathbf{G}\mathbf{x}_I + \mathbf{n}, \tag{1}$$

where $\mathbf{H}$ ($N_R \times N_T$) denotes the channel matrix for the desired signal, $\mathbf{G}$ ($N_R \times N_I$) is the channel matrix for the interfering signal from the nominal N_I-antenna interferer, and $\mathbf{n}$ ($N_R \times 1$) is the noise vector. We assume $\mathbf{H}$, $\mathbf{G}$ and $\mathbf{n}$ all have independent and identically distributed (i.i.d.) complex Gaussian entries with zero mean and unit variance. Both the desired signal $\mathbf{x}$ ($N_T \times 1$) and the interfering signal $\mathbf{x}_I$ ($N_I \times 1$) are assumed to be complex Gaussian distributed with zero mean, with the covariance matrices $\mathbf{Q}$ and $\mathbf{P}$ satisfying tr$(\mathbf{Q}) = 1$ and tr$(\mathbf{P}) = 1$, respectively. Therefore, the signal-to-noise ratio (SNR) is ρ, and η is the total interference-to-noise ratio (INR).

The covariance matrix of the interference-plus-noise conditioned on $\mathbf{G}$ is $\mathbf{R} = \mathbf{I}_{N_R} + \eta\mathbf{G}\mathbf{P}\mathbf{G}^\dagger$. In this paper, we assume that the receiver knows $\mathbf{H}$ and $\mathbf{R}$, but the transmitter does not know $\mathbf{H}$ or $\mathbf{R}$. We further assume that the receiver always performs single-user detection.

Based on these assumptions, the instantaneous mutual information conditioned on $\mathbf{H}$ and $\mathbf{G}$ is given by

$$C = \log_2 \det\left(\mathbf{I}_{N_R} + \rho\mathbf{H}\mathbf{Q}\mathbf{H}^\dagger\mathbf{R}^{-1}\right). \tag{2}$$

Therefore, the ergodic mutual information is given by

$$\begin{aligned} E\{C\} &= E_{\mathbf{H},\mathbf{G}}\left\{\log_2 \det\left(\mathbf{I}_{N_R} + \rho\mathbf{H}\mathbf{Q}\mathbf{H}^\dagger\mathbf{R}^{-1}\right)\right\} \\ &= E_{\mathbf{G}}\left\{S(\mathbf{G})\right\}, \end{aligned} \tag{3}$$

where $S(\mathbf{G}) \triangleq E_{\mathbf{H}}\left\{\log_2 \det\left(\mathbf{I} + \rho\mathbf{H}\mathbf{Q}\mathbf{H}^\dagger\mathbf{R}^{-1}\right)\right\}$.

Given the fact that $S(\mathbf{G})$ is a concave function with respect to $\mathbf{Q}$, we can prove the optimum $\mathbf{Q}$ to maximize $S(\mathbf{G})$ for a fixed $\mathbf{G}$ is given by $\mathbf{Q}_{opt} = \frac{1}{N_T}\mathbf{I}_{N_T}$, by following arguments similar as in [1]. This means that $\mathbf{Q}_{opt}$ is also optimum in the sense that it maximizes $E\{C\}$. However, all the discussions in this paper hold for any $\mathbf{Q}$.

3. DIFFERENT INTERFERING CONDITIONS

In this section, we study the ergodic mutual information $E\{C\}$ under different interfering conditions. Since different interfering conditions are fully characterized by the change in the covariance matrix $\mathbf{P}$, we explicitly express the instantaneous mutual information in (2) as a function of $\mathbf{P}$:

$$f(\mathbf{P}) = \log_2 \det\left(\mathbf{I} + \rho\mathbf{H}\mathbf{Q}\mathbf{H}^\dagger\left(\mathbf{I} + \eta\mathbf{G}\mathbf{P}\mathbf{G}^\dagger\right)^{-1}\right). \tag{4}$$

Then $E\{C\} = E\{f(\mathbf{P})\}$. Here $\mathbf{P} \in \Phi$, where Φ is the set of positive semi-definite matrices with unit trace.

Lemma 1 *The ergodic mutual information $E\{C\}$ is a convex function with respect to $\mathbf{P}$.*

This lemma follows from the fact that $f(\mathbf{P})$ is a convex function of $\mathbf{P}$. The detailed proof is omitted here.

Theorem 1 *The ergodic mutual information $E\{C\}$ is minimized by $\mathbf{P} = \frac{1}{N_I}\mathbf{I}_{N_I}$.*

Proof: Firstly, for any $\mathbf{P}$, the diagonal matrix consisting of the eigenvalues of $\mathbf{P}$ gives the same ergodic mutual information as $\mathbf{P}$. So we only need to consider diagonal matrices.

Then, given any non-negative diagonal $\mathbf{P}$, we consider $\mathbf{P}^\Pi = \Pi\mathbf{P}\Pi^\dagger$ for any possible permutation matrix Π. Due to the symmetry of our model with respect to the different antennas, we have $E\{f(\mathbf{P}^\Pi)\} = E\{f(\mathbf{P})\}$. Further we define $\hat{\mathbf{P}} = \frac{1}{N_I!}\sum_\Pi \mathbf{P}^\Pi = \frac{1}{N_I}\mathbf{I}_{N_I}$. According to Lemma 1, $E\{f(\mathbf{P})\}$ is a convex function of $\mathbf{P}$. Thus $E\{f(\hat{\mathbf{P}})\} \leq \frac{1}{N_I!}\sum_\Pi E\{f(\mathbf{P}^\Pi)\} = E\{f(\mathbf{P})\}$. This means that $\hat{\mathbf{P}} = \frac{1}{N_I}\mathbf{I}_{N_I}$ minimizes $E\{C\}$. □

Theorem 1 states that when the power is equally distributed among all the interfering antennas, this is the worst interfering condition. From the proof above, we know that only diagonal matrices need to be considered for $\mathbf{P}$. Therefore, $\mathbf{P}$ is assumed to be diagonal in the remainder of this paper. Based directly on the proof of Theorem 1, we have the following theorem on the effect of different power allocation among the interfering antennas.

Theorem 2 *Given the same number of interfering antennas, equal power distribution among these antennas gives the worst performance in terms of ergodic mutual information.*

Next we study how the number of interfering antennas affects the ergodic mutual information. Equal power distribution among all the interfering antennas employed is assumed here. In [4], simulation results showed that, for a fixed total interference power, a small number of large power interferers provides better performance than a large number of small power interferers in terms of outage capacity. The following theorem suggests a similar conclusion, but from the view of the ergodic mutual information.

Theorem 3 *Given a fixed total interference power, and assuming the power is equally distributed over all the interfering antennas employed, a smaller number of interfering antennas gives better performance in terms of ergodic mutual information.*

Proof: Without loss of generality, here we show that the case when N_I interfering antennas are present gives lower ergodic mutual information than the case when $M < N_I$ interfering antennas are present. The two cases of interest correspond to the covariance matrices of $\mathbf{P}_1 = \frac{1}{N_I}\mathbf{I}_{N_I}$ and $\mathbf{P}_2 = \frac{1}{M}\text{diag}\{1, \cdots, 1, 0, \cdots, 0\}$ (with M 1's), respectively. First we define a set Ψ, which includes all the diagonal matrices with M 1's scaled by $\frac{1}{M}$ (unit trace). The total number of elements in Ψ is $\binom{N_I}{M}$. Due the symmetry with respect to the different antennas, any matrix in Ψ gives the same performance as $\mathbf{P}_2$. Further, the average of all the matrices in Ψ equals $\mathbf{P}_1$. Therefore, by applying the convexity of the function $E\{f(\mathbf{P})\}$, we have $E\{f(\mathbf{P}_1)\} \leq E\{f(\mathbf{P}_2)\}$, that is, $\mathbf{P}_1$ gives worse performance than $\mathbf{P}_2$. □

Theorem 2 and 3 tell us that, given the total interference power, it is more favorable to have less number of interfering antennas and to have power unequally distributed among these antennas.

4. ASYMPTOTICALLY LARGE INTERFERENCE

In this section, we investigate how the ergodic mutual information $E\{C\}$ changes with the number of interfering antennas N_I for asymptotically large interference, i.e., as $\eta \to \infty$. Again we only consider diagonal $\mathbf{P}$. Moreover, since $\mathbf{P}$ with some zero diagonal entries actually means less interfering antennas, i.e., a smaller N_I, we restrict the discussions to $\mathbf{P}$ with positive diagonal entries.

We focus on the instantaneous mutual information C in (2). Note that $\mathbf{R}$ is a deterministic matrix when conditioned on $\mathbf{G}$. By eigenvalue decomposition, we have $\mathbf{R} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^\dagger$, where $\mathbf{U}$ is unitary and $\mathbf{\Lambda}$ is diagonal. Then

$$\begin{aligned} C &= \log_2 \det\left(\mathbf{I}_{N_R} + \rho \mathbf{H}\mathbf{Q}\mathbf{H}^\dagger \mathbf{U}^\dagger \mathbf{\Lambda}^{-1}\mathbf{U}\right) \quad (5) \\ &= \log_2 \det\left(\mathbf{I}_{N_R} + \rho \mathbf{\Lambda}^{-1/2}\tilde{\mathbf{H}}\mathbf{Q}\tilde{\mathbf{H}}^\dagger \mathbf{\Lambda}^{-1/2}\right), \quad (6) \end{aligned}$$

where $\tilde{\mathbf{H}} = \mathbf{U}\mathbf{H}$ has the same distribution as $\mathbf{H}$ because $\mathbf{U}$ is unitary. To obtain (6), we used the identity $\det(\mathbf{I} + \mathbf{B}\mathbf{C}) = \det(\mathbf{I} + \mathbf{C}\mathbf{B})$.

Let $\mathbf{A} = \mathbf{G}\mathbf{P}\mathbf{G}^\dagger$, then $\mathbf{R} = \mathbf{I}_{N_R} + \eta\mathbf{A}$. Since $\mathbf{G}$ has i.i.d. entries, $\mathbf{A}$ has $\min\{N_I, N_R\}$ nonzero eigenvalues with probability 1. We consider two cases separately, $N_I < N_R$ and $N_I \geq N_R$, and have the following theorems.

Theorem 4 *When $N_I < N_R$, a MIMO system with N_T transmit antennas, N_R receive antennas, and an N_I-antenna interferer is statistically equivalent to a MIMO system with N_T transmit antennas and $(N_R - N_I)$ receive antennas for asymptotically large INR.*

Proof: When $N_I < N_R$, $\mathbf{A}$ has N_I nonzero eigenvalues with probability 1, denoted as $e_1, e_2, \cdots, e_{N_I}$ in non-increasing order. Let $(\lambda_1, \cdots, \lambda_{N_R})$ denote the eigenvalues, in non-increasing order, of $\mathbf{R} = \mathbf{I}_{N_R} + \eta\mathbf{A}$ so that

$$\lambda_i = \begin{cases} 1 + \eta\, e_i, & 1 \leq i \leq N_I \\ 1, & N_I < i \leq N_R \end{cases}. \quad (7)$$

As $\eta \to \infty$, $\lambda_i \to \infty$ for $1 \leq i \leq N_I$, which means that $\mathbf{\Lambda}^{-1/2} \to \mathrm{diag}(0, \cdots, 0, 1, \cdots, 1)$ (with N_I zeros). Further, we let

$$\tilde{\mathbf{H}} = \begin{bmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \\ \vdots \\ \mathbf{h}_{N_R} \end{bmatrix}, \quad (8)$$

where $\mathbf{h}_i$ is a $1 \times N_T$ row vector. Then for $\eta \to \infty$,

$$\mathbf{\Lambda}^{-1/2}\tilde{\mathbf{H}} = \begin{bmatrix} \mathbf{0} \\ \vdots \\ \mathbf{0} \\ \mathbf{h}_{N_I+1} \\ \vdots \\ \mathbf{h}_{N_R} \end{bmatrix}. \quad (9)$$

If we define

$$\hat{\mathbf{H}} = \begin{bmatrix} \mathbf{h}_{N_I+1} \\ \vdots \\ \mathbf{h}_{N_R} \end{bmatrix}, \quad (10)$$

then

$$\mathbf{\Lambda}^{-1/2}\tilde{\mathbf{H}}\mathbf{Q}\tilde{\mathbf{H}}^\dagger\mathbf{\Lambda}^{-1/2} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \hat{\mathbf{H}}\mathbf{Q}\hat{\mathbf{H}}^\dagger \end{bmatrix}, \quad (11)$$

It follows that

$$\begin{aligned} &\det\left(\mathbf{I}_{N_R} + \rho\mathbf{\Lambda}^{-1/2}\tilde{\mathbf{H}}\mathbf{Q}\tilde{\mathbf{H}}^\dagger\mathbf{\Lambda}^{-1/2}\right) \\ &= \det\begin{bmatrix} \mathbf{I}_{N_I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{(N_R-N_I)} + \rho\hat{\mathbf{H}}\mathbf{Q}\hat{\mathbf{H}}^\dagger \end{bmatrix} \quad (12) \\ &= \det\left(\mathbf{I}_{(N_R-N_I)} + \rho\hat{\mathbf{H}}\mathbf{Q}\hat{\mathbf{H}}^\dagger\right). \quad (13) \end{aligned}$$

By combining (6) and (13), we have

$$C = \log_2 \det\left(\mathbf{I}_{(N_R-N_I)} + \rho\hat{\mathbf{H}}\mathbf{Q}\hat{\mathbf{H}}^\dagger\right). \quad (14)$$

Since $\hat{\mathbf{H}}$ is an $(N_R - N_I) \times N_T$ matrix with i.i.d. components, (14) is basically the instantaneous mutual information of a MIMO channel with N_T transmit and $(N_R - N_I)$ receive antennas. Moreover, since the distribution of $\hat{\mathbf{H}}$ does not depend on $\mathbf{G}$, we conclude that the system of interest reduces to a MIMO channel with N_T transmit and $(N_R - N_I)$ receive antennas in the statistical sense. More specifically, this means that Theorem 4 holds in the sense of the distribution of the instantaneous mutual information, ergodic mutual information, and any related performance measures including the cumulative distribution functions and outage performance. This completes the proof. □

Theorem 5 *When $N_I \geq N_R$, the instantaneous mutual information of a MIMO system with N_T transmit antennas, N_R receive antennas, and an N_I-antenna interferer approaches zero for asymptotically large INR.*

Proof: When $N_I \geq N_R$, $\mathbf{A}$ is full rank and has N_R nonzero eigenvalues with probability 1. As $\eta \to \infty$, $\lambda_i \to \infty$ for all i, which means that $\mathbf{\Lambda}^{-1} \to \mathbf{0}$. It follows that the instantaneous mutual information $C \to 0$ with probability 1. □

Now we provide some Monte Carlo simulation results to support Theorem 4 and 5. We consider a MIMO system with 4 transmit and 4 receive antennas. Fig. 1 and 2 give the ergodic mutual information for the cases $N_I < N_R$ and $N_I \geq N_R$, respectively, when SNR = 10 dB. As seen from the curves, when $N_I = 2$ and $N_I = 3$, the ergodic mutual information approaches that of a 4×2 and 4×1 MIMO system, respectively, as INR increases. When $N_I \geq N_R$,

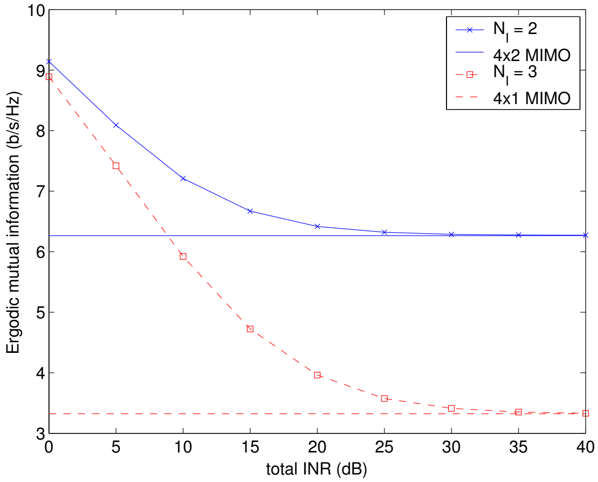

Fig. 1. Ergodic mutual information for $N_T = N_R = 4$ and SNR $= 10$ dB when $N_I < N_R$.

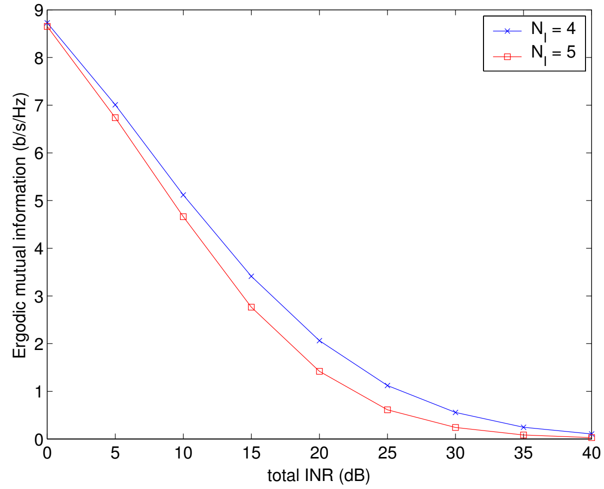

Fig. 2. Ergodic mutual information for $N_T = N_R = 4$ and SNR $= 10$ dB when $N_I \geq N_R$.

the ergodic mutual information approaches zero. These observations agree perfectly with the theorems.

By following similar arguments, the conclusions in Theorem 4 and 5 can also be extended to the case when the transmitter exploits knowledge of $\mathbf{H}$ and $\mathbf{R}$.

Theorem 4 and 5 also agree with some observations in [2]. Based on the analysis of systems with an asymptotically large number of antennas, one observation in [2] was, once the number of the interfering antennas exceeds the number of receive antennas, the high-SNR performance is determined mostly by SIR. Both our results and the observations in [2] can be explained using the concept of the degrees of freedom. In general, the receiver needs to distribute the total degrees of freedom N_R between interference suppression and signal detection. In our case, since the interference becomes asymptotically large, it is preferable to use enough degrees of freedom to suppress the interference first, which results in a reduced system with less receive antennas when $N_I < N_R$. When $N_I \geq N_R$, the receiver no longer has extra degrees of freedom for signal detection after interference suppression. Therefore, the capacity approaches zero.

5. CONCLUSION

In this paper, we study the impact of co-channel interference on a MIMO system with theoretical analysis. We proved that, in terms of ergodic mutual information, it is preferable to have fewer interferers with power unevenly distributed among the interferers. This can also be interpreted spatially: it is more favorable to have the interference dominate only a few dimensions, rather than to have the interference evenly spread over all dimensions. The asymptotic analysis showed that as INR approaches infinity, a MIMO system with interference can be viewed as a reduced MIMO system with fewer receive antennas, and this reduced system deteriorates when $N_I \geq N_R$. This suggests that one extra receive antenna is capable of suppressing one infinitely strong single-antenna interfering signal.

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