INTERFERENCE-FREE MULTI-USER MIMO-OFDM

Jonathan Duplicy[1], Jérôme Louveaux[2], and Luc Vandendorpe[1]

[1] Communications and Remote Sensing Laboratory
Université catholique de Louvain, Belgium
{duplicy, vandendorpe}@tele.ucl.ac.be

[2] Circuit &Systems, Dept. of Electrical Eng.
Delft University of Technology, The Netherlands
louveaux@cas.et.tudelft.nl

ABSTRACT

A multi-user downlink MIMO-OFDM scheme is considered. Perfect channel knowledge is assumed at the base station and linear processing is used at both the transmit and receive sides. The objective is to optimize the mean BER of the system while satisfying a transmit power constraint and fulfilling each user's rate. FDMA and SDMA are investigated. We show how the multi-user interference induced by SDMA can be annihilated and exhibit that this multiple access technique should be preferred to FDMA.

1. INTRODUCTION

Recently, MIMO (Multi-Input Multi-Output) systems arising from the use of antenna arrays at both the transmitter and the receiver have drawn considerable attention since they offer a large potential capacity increased in comparison with single antenna systems. Concurrently, Orthogonal Frequency Division Multiplexing (OFDM) has also encountered an increasing popularity since it provides a low complexity solution to the Inter-Symbol Interference (ISI) resulting from the frequency selectivity of the channels.

In this paper, we focus on a multi-user MIMO-OFDM system, which remains a largely unexplored topic. In [1], Doostnejad introduced space-frequency spreading codes for the downlink with no or second-order Channel State Information at the Transmitter (CSIT). [2] proposed a no-CSIT Multiple access scheme which allows to gradually vary the amount of user collision in signal space by assigning different subsets of the available OFDM tones to different users. Finally, assuming perfect CSIT, [3] presented an adaptive scheme for the uplink scenario. Assuming channel knowledge at the transmitter allows to improve performance significantly [4]. CSIT can be achieved either by means of a feedback channel or by estimating the received channel and exploiting reciprocity property when applicable.

In particular, we consider a downlink transmission with perfect CSI at the base station (BS) and linear processing at both the BS and the remote stations. The objective is to minimize the mean BER of the system while fulfilling each user's rate and satisfying a global transmit power constraint. Frequency Division Multiple Access (FDMA) and Space Division Multiple Access (SDMA) are investigated. Among others, we show how the Multi-User Interference (MUI) induced by SDMA can be annihilated by orthogonalizing the channels. We also exhibit that SDMA should be preferred to FDMA.

Jonathan Duplicy would like to thank the Belgian fund FRIA for its financial support. This work is also partly funded by the Federal Office for Scientific, Technical and Cultural Affairs, Belgium, through IAP contract No P5/11.

2. SYSTEM MODEL

2.1. Channel model

The frequency selective $N_r \times N_t$ MIMO channels are modeled as tap delay lines :

$$h_{ij}(t) = \sum_{l=0}^{L_{ij}-1} \beta_{ij}(l)\, \delta(t - \tau_{ij}(l)) \quad \begin{cases} 1 \leq i \leq N_r \\ 1 \leq j \leq N_t \end{cases} \tag{1}$$

where $\tau_{ij}(l)$ and $\beta_{ij}(l)$ are respectively the delay and the complex amplitude associated with the l^{th} path between the j^{th} transmit and i^{th} receive antennas. The $\beta_{ij}(l)$ are modeled as zero-mean, complex Gaussian random variables with variances σ^2_{ijl}. These variances are normalized so that $\sum_{l=0}^{L_{ij}-1} \sigma^2_{ijl}=1 \;\; \forall i, j$. Finally, channels are assumed quasi-static : invariant during each OFDM block, but variable from block to block. To deal with the frequency-selectivity of the channel, an N-tone OFDM modulation is adopted. Frequency channel response for tone n, $H_n \in \mathbb{C}^{N_r \times N_t}$, is given by :

$$(H_n)_{ij} = \sum_{l=0}^{L_{ij}-1} \beta_{ij}(l) e^{-j2\pi n \tau_{ij}(l)/N}. \tag{2}$$

2.2. Single-user case

We assume a convenient cyclic prefix and a perfect synchronization to avoid inter-symbol/carrier interference. Furthermore, linear pre/decoding is used at both sides of the link ; hence the system model for subcarrier n is written as :

$$\hat{s}_n = G_n(H_n F_n s_n + v_n) \qquad 1 \leq n \leq N\ , \tag{3}$$

where s_n and $\hat{s}_n \in \mathbb{C}$ are the transmitted and the estimated symbols for subcarrier n. $F_n \in \mathbb{C}^{N_t \times 1}$ and $G_n \in \mathbb{C}^{1 \times N_r}$

are the precoding and decoding vectors, respectively. $v_n \in \mathbb{C}^{N_r \times 1}$ is the additive white Gaussian noise with correlation matrix R_n. In addition, we assume $E\{v_n v_m^\dagger\} = 0_{(N_r \times N_r)}$ for $n \neq m$, where $(.)^\dagger$ means conjugate transpose.

Perfect CSI is assumed at both the transmit and receive sides and the following transmit power constraint is imposed ($\mathrm{tr}\{.\}$ is the trace operator) :

$$\sum_{n=1}^{N} \mathrm{tr}\left\{F_n {F_n}^\dagger\right\} \leq P_t \ . \tag{4}$$

Optimizing F_n and G_n subject to the power constraint has been well studied and it will not be discussed here. We refer the reader to [4] for a complete overview. Finally we restrict the problem to non-adaptive scenarios, i.e. the constellations (e.g. 4-QAM) are identical for the N subcarriers.

2.3. Multi-user case

This paper focuses on a multi-user downlink transmission. Perfect CSI of all the channels is assumed at the BS where transmit and receive weight optimization is conducted. Receive weights are transmitted to the U remote stations via control channels. The objective is to optimize the average BER (over all symbols of all users) while fulfilling each user's rate and satisfying a global transmit power, P_t.

3. FDMA

In this section, we focus on a (O)FDMA scheme : each OFDM tone is used by a *unique* user. The problem is to allocate the N subcarriers to the U users and to compute tx-rx weights to optimize the mean BER. Ideally, subcarrier allocation and weight computation should be done jointly. Unfortunately, such a joint optimization is too complex and practically untractable. Therefore, we propose the following 3-step suboptimal disjoint optimization :

1. Subcarrier allocation : For each subcarrier of each user, we compute the singular value decomposition (SVD) and retain the strongest singular value (λ_n^u) as a criterion of quality. Noting $\lambda_n^{u^*}$ the strongest singular value for the subcarrier n of the user to whom the n^{th} subcarrier has been allocated, the allocation is made so that :

$$\sum_{n=1}^{N} \lambda_n^{u^*} \quad \text{is maximized and users'rates filled.} \tag{5}$$

Problem (5) is a classical allocation problem whose optimal solution is given by the Hungarian algorithm [5]. Table 1 illustrates its application to a simple problem.

2. Power allocation : We have to answer the question : how to split the transmit power, P_t, between the U users? An easy answer is to allocate the power proportionally to the

	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
user 1	$\boxed{4}$	3	$\boxed{3}$	$\boxed{4}$	$\boxed{5}$	3
user 2	2	$\boxed{2}$	1	3	3	$\boxed{2}$

Table 1. Example of subcarrier allocation by the Hungarian algorithm for a system consisting of 6 subcarriers and 2 users (The rate of user 1 is twice the rate of user 2 and $\boxed{\text{x}}$ is the user selected for the subcarrier considered.)

number of subcarriers owned by the users : $P^u = P_t N^u / N$, where N^u is the number of subcarriers allocated to the u^{th} user. A better solution is to allocate the power regarding the quality of the channels. In this paper, we propose to use a simple allocation inversely proportional to the strongest singular values computed in step 1 :

$$P^u = \frac{P_t \cdot N}{\sum_{k=1}^{U} \left(\frac{1}{\sum_{\substack{n=1 \\ k=k^*}}^{N} \lambda_n^k} \right)} \cdot \frac{1}{\sum_{\substack{n=1 \\ u=u^*}}^{N} \lambda_n^u} \tag{6}$$

3. Weight computation : Once the subcarriers allocation and the power splitting between users are performed, the problem reverts to U single-user problems and classical solutions can be applied [4].

4. MUI-FREE SDMA

In this section, we consider an SDMA scheme : each ODFM tone is used simultaneously by the U users. Hence, the estimated symbol for user u on tone n and the global transmit power constraint turn into :

$$\hat{s}_n^u = G_n^u \left(H_n \sum_{j=1}^{U} F_n^j s_n^j + v_n^j \right) \tag{7}$$

$$\sum_{u=1}^{U} \sum_{n=1}^{N} \mathrm{tr}\left\{ F_n^u {F_n^u}^\dagger \right\} \leq P_t \tag{8}$$

Such a multiple access system induces MUI, however we show how it can be easily annihilated by orthogonalizing the channels as proposed in [6] for the single-carrier case :

Ortho1 : For each subcarrier, the system model is rewritten in the following concatenated way (subcarrier index is left out for conciseness) : $\hat{s} = G\,(HFs + v)$, i.e.

$$\begin{pmatrix} \hat{s}^1 \\ \hat{s}^2 \\ \vdots \\ \hat{s}^U \end{pmatrix} = \mathrm{diag} \begin{pmatrix} G^1 \\ G^2 \\ \vdots \\ G^U \end{pmatrix} \left[\begin{pmatrix} H^1 \\ H^2 \\ \vdots \\ H^U \end{pmatrix} \times \right.$$
$$\left. \times \begin{pmatrix} F^1 \\ F^2 \\ \vdots \\ F^U \end{pmatrix}^T \begin{pmatrix} s^1 \\ s^2 \\ \vdots \\ s^U \end{pmatrix} + \begin{pmatrix} v^1 \\ v^2 \\ \vdots \\ v^U \end{pmatrix} \right], \tag{9}$$

where $diag$ denotes a block-diagonal matrix and $(.)^T$ is the transpose operator. As shown in Fig.1, we require that the symbol targeted to the u^{th} remote station generates a zero on each antenna of the other $U-1$ stations. Mathematically, it means that F^u lies in the null space of $\underline{H}^u$, the concatenated channel matrix where lines related to user u have been removed :

$$\underline{H}^u F^u = 0_{\left(\left(\sum_{j\neq u} N_{rj}\right)\times 1\right)} \;\equiv\; F^u \in \text{null}\{\underline{H}^u\} \quad \forall u \; . \tag{10}$$

To achieve this, the precoding matrix, F, is split in two matrices F_A and F_B :

$$F = F_A \cdot F_B = \left(F_A^1 \; F_A^2 \; \ldots \; F_A^U\right) \text{diag}\left(F_B^1 \; F_B^2 \; \ldots \; F_B^U\right) . \tag{11}$$

F_B^u is a $\left(\left(N_t - \sum_{j\neq u} N_r^j\right) \times 1\right)$ complex matrix while F_A^u is a $\left(N_t \times \left(N_t - \sum_{j\neq u} N_r^j\right)\right)$ complex matrix, basis of the null space of $\underline{H}^u$. Once F_A^u matrices have been computed, the multi-user system reverts to U single-user systems with enhanced channels ($H^u F_A^u$) and classical solutions can be applied to compute G^u and F_B^u matrices [4].

Naturally, this technique imposes a very restrictive feasibility condition (easily deduced from (10) and (11)) :

$$1 \leq N_t - \sum_{j\neq u} N_r^j \qquad \forall u \tag{12}$$

For example, a system with N_t=4 and N_r^u=3 $\forall u$ cannot accommodate more than 2 users. However, the capacity can be greatly improved by the method we propose hereafter.

Ortho2 : Conditions (10) are too stringent and not necessary. Fig.1 shows the principle of our new method. Instead of requiring zeros on each antenna of the other remote stations, we only require zeros on their *weighted symbols*. Hence the availability condition (12) turns into :

$$1 \leq N_t - (U-1) \qquad \forall u \; . \tag{13}$$

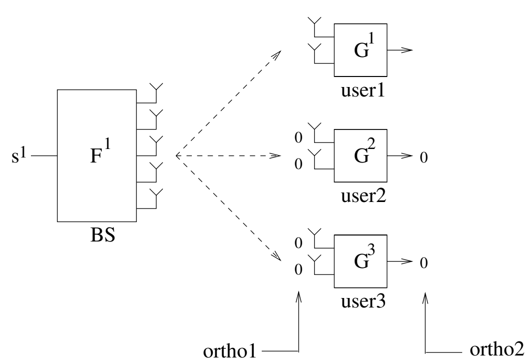

Fig. 1. Principles of *ortho1* and *ortho2*.

1. initialize G_n^u with the single-user case
2. compute ${F_A}_n^u$ for the enhanced channel $G_n^u H_n^u$
3. compute G_n^u and ${F_B}_n^u$ for the enhanced channel $H_n^u {F_A}_n^u$ while satisfying (8)
4. go to step 2 until convergence

Table 2. *ortho2* algorithm ($\forall$ n, u).

original chan.	chan. after *ortho1*	chan. after *ortho2*
$N_t \times N_r^u$	$\left(N_t - \sum_{j\neq u} N_r^j\right) \times N_r^u$	$(N_t-(U-1))\times N_r^u$

Table 3. Channel dimensions for user u.

The application of this improved scheme to our example (N_t=4 and N_r^u=3 $\forall u$) doubles the capacity of the system : 4 users can now be accommodated simultaneously. Unfortunately, F_A is needed to compute F_B and G and vice versa. Therefore, we have to resort to the iterative processing given by table 2. Table 3 summarizes the dimensions of the channels before and after orthogonalization.

Power allocation : As for the FDMA scheme, a proportional and an adaptive power allocation are considered. For *ortho1* the proportional scheme allocates the same power to each user whereas the adaptive allocation is given by (14). For *ortho2*, the proportional scheme allocates the same power to each subcarrier of each user, i.e. $P_n^u = P_t/(N \cdot U)$ while the adaptive allocation is described by (15). We emphasize that power allocation is made on a user basis for *ortho1* while it is made on a subcarrier basis for ortho2.

$$P^u = \frac{P_t \cdot N}{\sum_{k=1}^{U}\left(\frac{1}{\sum_{n=1}^{N}\lambda_n^k}\right)} \cdot \frac{1}{\sum_{n=1}^{N}\lambda_n^u} \tag{14}$$

$$P_n^u = \frac{P_t \cdot N}{\sum_{k=1}^{U}\left(\frac{1}{\sum_{n=1}^{N}\lambda_n^k}\right)} \cdot \frac{1}{\lambda_n^u} \tag{15}$$

5. SIMULATION RESULTS

A two-equal-rate-user system with N_t=4 and N_r^1=N_r^2=2, 3 is investigated. The channel model is the HiperLan model A [7] and uncorrelated channels and noise are assumed. The 20MHz channel is turned into N=64 subchannels. Moreover, to have a fair comparison, 4-QAM (Gray-mapped) constellations are used for SDMA whereas 16-QAM constellations are used for FDMA. Once the multi-user system is reverted to single-user systems, the *max-MSE* method developed in [4] is used to compute the tx-rx weights.

FDMA : We first consider the FDMA scheme. The set of N=64 subcarriers is equally shared between the two users. Figure 2 compares various subcarriers/power allocation methods. It emphasizes the benefit from the subcarrier allocation made by the Hungarian algorithm with respect to a random allocation. We can also notice a small performance improvement by using the power allocation given by (6) rather than assigning the same power to each user.

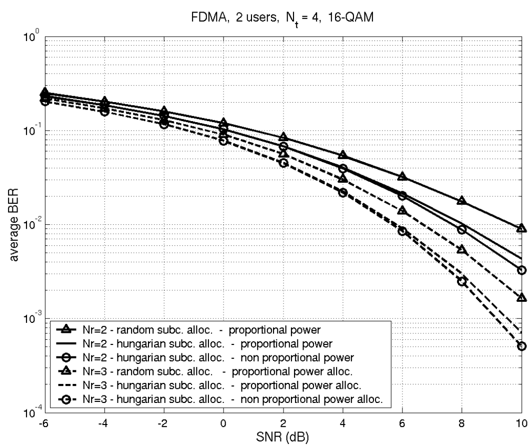

Fig. 2. FDMA for various numbers of receive antennas and power allocation schemes.

SDMA : Fig.3 illustrates the average BER for the *ortho1* and *ortho2* schemes. Focusing on *ortho1*, we observe that the system with three receive antennas exhibits worse performances than the system with two receive antennas. The explanation of this paradox is to be found in the dimensions of the enhanced channel resulting of the orthogonalization process. As summarized by table 4, *ortho1* applied to the present scenario turns the 4×2 and 4×3 channels into 2×2 and 1×3 channels, respectively. This explains why a 2-receive antenna system should be preferred to a 3-receive antenna system. On the contrary, *ortho2* preserves the benefit of using more receive antennas. It is also worth noticing the gain resulting from the use of an adaptive power allocation rather than a proportional power allocation. Observe that the gain for *ortho1* is almost negligible whereas the gain for *ortho2* is significant. It comes from the fact that for *ortho1*, the allocation is made for the whole set of subcarriers whereas it is made on a subcarrier basis for *ortho2*.

original chan.	chan. after *ortho1*	chan. after *ortho2*
4×2	2×2	3×2
4×3	1×3	3×3

Table 4. Dimensions of the simulated channels.

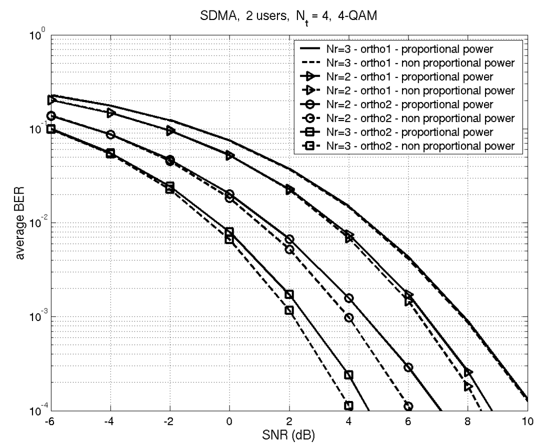

Fig. 3. *ortho1* and *ortho2* for various numbers of receive antennas and power allocation schemes.

6. DISCUSSION AND CONCLUSION

We have considered the downlink of a MIMO-OFDM system with prefect CSIT and linear processing at both the transmit and receive sides. The goal was to minimize the average BER of the system subject to power and rate constraints. Among others, we showed how MUI induced by SDMA could be annihilated by orthogonalizing the channels. Finally, by comparing figures 3 and 2, one can also affirm that zero-MUI SDMA performs significantly better than FDMA.

7. REFERENCES

[1] R. Doostnejad, T.J. Lim, and E.S. Sousa, “On spreading codes for the downlink in a multiuser MIMO/OFDM system,” *Proc. VTC 2003-Fall*, vol. 1, pp. 498-502, Oct. 2003

[2] S. Visuri and H. Bolcskei, “MIMO-OFDM Multiple access with variable amount of collision,” *Proc. ICC 2004*, Paris, France, June 2004.

[3] Y.J. Zhang, K.B. Letaief, “Optimizing power and resource management for multiuser MIMO/OFDM systems,” *Proc. GLOBECOM 2003*, no. 1, pp. 179-183, San Francisco, Ca., Dec. 2003.

[4] D.P. Palomar, “A Unified Framework for Communications through MIMO Channels”, thesis, Universitat Politcnica de Catalunya, May 2003.

[5] H. Kuhn, “The Hungarian method for the assignment problem, ” *Naval Research Logistics Quaterly*, pp. 83-97, 1955.

[6] A. Bourdoux and N.Khaled, “Joint TX-RX optimisation for MIMO-SDMA based on a null-space constraint,” *Proc. of IEEE Vehicular Technology Conference (VTC-02 Fall)*, pp. 171-174, Vancouver, Canada, Sept. 2002.

[7] ETSI, “Channel models for HIPERLAN/2 in different indoor scenarios,” ETSI EP BRAN 3ERI085B, March 1998.