Joint Data Detection for Punctured ARQ Diversity Systems

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***Abstract*— We investigate a bandwidth efficient ARQ scheme in which the Medium Access Control (MAC) only requests a partial retransmission after packet reception error. To reduce channel usage, the original packet is punctured prior to retransmission(s), thereby forming a modified ARQ scheme which we name Punctured Automatic Repeat Request (PARQ). The puncturing in the system can be viewed as a form of "diversity" to blindly estimate the channel. We focus on the detection of the original data sequence by presenting a Maximum Likelihood (ML) algorithm and a simpler multi-rate linear equalizer. We also investigate the BER performance based on estimated channel responses from both the blind and the semiblind channel estimation algorithms.**

I. Introduction

Cross-layer considerations are becoming important in bandwidth conservation of wireless networks. In future systems, more intelligent use of the network stack and transmission protocols will be used in combination with other traditional forms to achieve future network efficiency goals. This paper studies new retransmission protocols to allow for more flexible retransmissions and channel use. More specifically, we study the channel estimation and data detection for more flexible retransmissions involving puncturing.

In wireless systems, ARQ is an effective mechanism against frame errors after forward error code has failed to correct all the errors in a data frame. While traditional works on ARQ and hybrid ARQ have taken a FEC code design perspective, under frequency selective channel distortions, joint detection and channel estimation can critically impact the channel efficiency and system throughput. Thus, this work focuses on the joint channel estimation and joint data detection of ARQ systems. To conserve bandwidth, we proposed a *puncture* ARQ (PARQ) transmission in [1] with which packets are first punctured prior to retransmission. Since successive transmissions during PARQ still share common information, joint channel estimation and signal detection can benefit at the receiver. As a more general extension of simple ARQ repetition, we generalize the work in [2] by presenting new blind and semiblind channel estimators [1] [3] that utilizes the subspace method [4].

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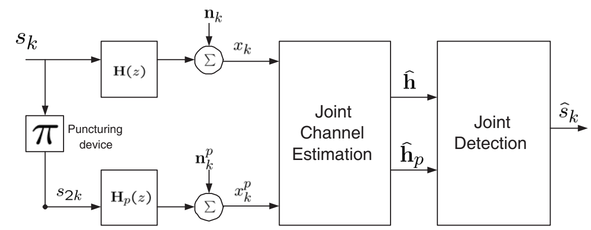

Fig. 1. PARQ System Model

It enables channel estimation of multiple (re)transmission channels without training (blind) or with training only during the first transmission (semiblind).

While recent works [1] [3] have demonstrated more relaxed channel identifiability conditions in a more robust algorithm for blind channel estimation as well as a simpler semiblind channel estimation method to remove training from punctured retransmissions, the actual detection of the transmitted data packets has not been studied. In this work, we will first investigate the optimal detection result by formulating a joint Maximum Likelihood (ML) algorithm for PARQ. Moreover, to reduce receiver complexity, we will also present a low complexity, vector linear equalizer. Based on a minimum mean square error design criterion, the vector equalizer parameters can be optimized for good error performance.

This paper is organized as follows. Section II presents the signal and system model for joint processing of PARQ. Section III summarizes blind and semiblind channel estimation algorithms and performance. Section IV introduces ML detection and presents a vector form linear equalizer design. Finally, Section V shows simulation BER performance results.

II. System Model

Consider a simple system diagram with only two transmissions shown in Fig. 1. The first and the punctured transmissions are modelled by two equations at baud rate, using superscript $.^p$ to represent punctured case:

$$x_k = \Sigma_{n=-\infty}^{\infty} s_n h_{k-n} + n_k \tag{1}$$

$$x_k^p = \Sigma_{n=-\infty}^{\infty} s_n^p h_{k-n}^p + n_k^p. \tag{2}$$

In the second transmission, we focus on signal $s^p[k]$ that is uniformly punctured without loss of generality. Thus, $s^p[k] = s[2k]$. As shown in [1] [3], the equivalent equations can be written with Toeplitz matrix $\mathbf{H}$ and block Toeplitz matrix $\mathbf{H}^p$, respectively,

$$\mathbf{x}_k = \mathbf{H}\mathbf{s}_k + \mathbf{n}_k, \tag{3}$$

$$\mathbf{x}_k^p = \mathbf{H}^p\mathbf{s}_k + \mathbf{n}_k^p. \tag{4}$$

By stacking two signals and noises together, we have

$$\tilde{\mathbf{x}}_k = \left[\begin{array}{ccc|ccc} x_k & \cdots & x_{k-m_1+1} & x_k^p & \cdots & x_{k-m_2+1}^p \end{array}\right]^T,$$

$$\tilde{\mathbf{n}}_k = \left[\begin{array}{ccc|ccc} n_k & \cdots & n_{k-m_1+1} & n_k^p & \cdots & n_{k-m_2+1}^p \end{array}\right]^T,$$

and a composite channel matrix,

$$\tilde{\mathbf{H}} = \begin{bmatrix} \mathbf{H} \\ \mathbf{H}^p \end{bmatrix}. \tag{5}$$

Therefore, the single matrix-vector description is given by

$$\tilde{\mathbf{x}}_k = \tilde{\mathbf{H}}\mathbf{s}_k + \tilde{\mathbf{n}}_k. \tag{6}$$

As shown in [3], with $\tilde{\mathbf{H}}$ full rank, the unknown channels (in two transmissions) can be uniquely identified by applying a punctured SSM on second order output statistics.

III. Joint Channel Estimation Algorithms

In essence, we now estimate

$$\tilde{\mathbf{h}} = \left[\begin{array}{c|c} \mathbf{h} & \mathbf{h}^p \end{array}\right]^T. \tag{7}$$

A main result of our algorithm is that $\tilde{\mathbf{H}}$ is not vector Toeplitz matrix as in SSM because of puncturing. It is in fact equivalently a block Toeplitz matrix as shown in [3]. As a result, the channel identifiability conditions are relaxed in SSM for $\tilde{\mathbf{h}}$.

Without any training, we solve

$$\mathbf{U}_n^H\tilde{\mathbf{H}} = 0. \tag{8}$$

Here $\mathbf{U}_n^H$ represents the noise subspace of the covariance matrix for signal vector $\tilde{\mathbf{x}}_k$. As in [4], which uses the standard SVD or EVD, this linear equation can be solved as a minimum eigenvector problem.

In practice, training is often present during the first transmission. By incorporate training and subspace orthogonality described in (8), we can form a *semiblind* channel estimation algorithm that simply combines the two costs. We present a simulation example that summarizes the channel estimation performance for all four different cases: blind estimation for full retransmission (full blind), semiblind estimation for full retransmission (full semiblind), blind estimation for PARQ (punc. blind), and semiblind estimation for PARQ (punc. semiblind). We use BPSK modulated symbols over a 1000 random 4-tap Rayleigh fading channels. 128 input symbols are used for channel estimation. The normalized mean square errors are compared against fully trained channel estimations. The results are very encouraging for both blind and semiblind PARQ channel estimations.

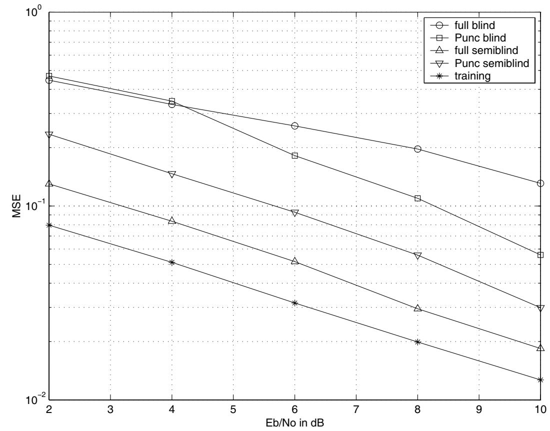

Fig. 2. Normalized Mean Square Error (NMSE) channel estimation performance.

Note that the training data length in fully trained cases is four times the channel length (which is 16), during both transmissions. While for semiblind PARQ, there is zero training during retransmission. The training sequence used is a CAZAC sequence described in [5].

IV. Joint Channel Equalizers for Punctured Retransmissions

A. Maximum Likelihood Receiver

By using a simple row permutation we can jointly rewrite two transmission output signals in the convolution expression of (6) into

$$\overline{\mathbf{x}}_k = \sum_i \mathbf{H}_i\mathbf{s}_{k-i} + \overline{\mathbf{n}}_k, \tag{9}$$

where the permuted vector-matrix elements are

$$\overline{\mathbf{x}}_k = \begin{bmatrix} x_{2k+1} \\ x_{2k} \\ x_k^p \end{bmatrix}, \mathbf{H}_i = \begin{bmatrix} h_{2i} & h_{2i+1} \\ h_{2i-1} & h_{2i} \\ 0 & h_i^p \end{bmatrix}, \overline{\mathbf{s}}_k = \begin{bmatrix} s_{2k+1} \\ s_{2k} \end{bmatrix}. \tag{10}$$

Using half rate puncturing, there is a non-trivial half rate puncturing matrix that "distorts" the shape of $\mathbf{H}^p$. Our vector Toeplitz matrix, $\tilde{\mathbf{H}}$, is now block Toeplitz with the form above. A block Toeplitz matrix can be viewed as a MIMO ISI system and it can be decoded in a straightforward, albeit expensive, manner by defining a trellis for BCJR and Viterbi algorithms. The input state-space changes because the block element has dimension 3×2. This means that instead of considering successive input symbols, one must consider input "pairs" of symbols to define the states of the trellis. This phenomenon effectively squares the amount of states in the trellis.

Although in general, a joint ML decoder will work for any puncturing rate, the complexity does vary and even

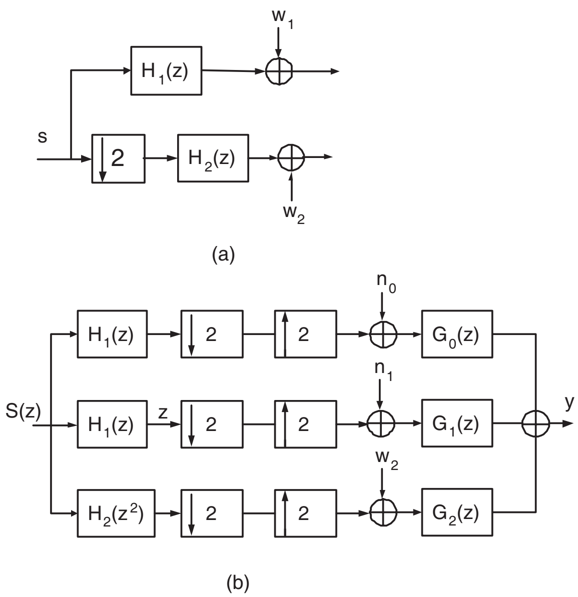

Fig. 3. (a) Single user multirate punctured ARQ system. (b) Linear equalization for PARQ system.

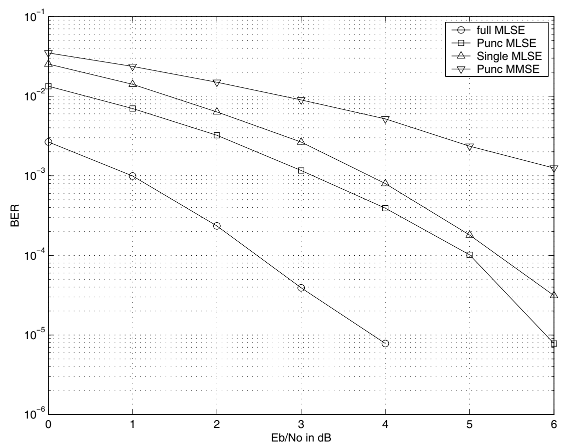

Fig. 4. BER performance of equalized PARQ system using perfect channel knowledge.

the general description becomes difficult to track. For this reason, we also would like to consider simpler receiver designs that are easier to implement for joint detection of multiple retransmissions.

B. Linear Minimum Mean Squared Error Receiver

For high rate wireless transmission, implementation of optimal ML equalizer may be challenging due to the complexity. Alternatively, linear equalizers with low complexity, low cost and low processing delays could be of interest for various applications in PARQ systems. In this section we investigate multirate linear equalization techniques based on minimum-mean-square-error (MMSE) criterion.

Denote $H_1(z)$ and $H_2(z)$ as channel responses for the first transmission and retransmission, respectively. The ISI channel output of the first transmission in Fig. 3(a) can be split into even and odd subsequences, respectively. Combined with the second retransmission, three equalizers can be designed jointly for the recovery of input symbols $S(z)$, as shown in Fig. 3(b). Using the multirate filterbank theory [6], we apply Type I polyphase decomposition to channel response $H_1(z)$ and input sequence $S(z)$,

$$H_1(z) = E_0(z^2) + E_1(z^2)z^{-1}, \tag{11}$$

$$S(z) = S_0(z^2) + S_1(z^2)z^{-1}. \tag{12}$$

It is clear that the equalized system output can be written [6] as

$$\begin{aligned} Y(z) = &\left[E_0(z^2)G_0(z) + E_1(z^2)G_1(z) + E_2(z^2)G_2(z)\right] \\ &S_0(z^2) + \left[z^{-2}E_1(z^2)G_0(z) + E_0(z^2)G_1(z)\right] S_1(z^2) \\ &+ G_0(z)n_0(z^2) + G_1(z)n_1(z^2) + G_2(z)w_2(z^2) \end{aligned} \tag{13}$$

where $n_0(z)$ and $n_1(z)$ are the polyphase components of the additive noise sequence w_1 of the first transmission. Also we change the notation for $H_2(z)$ to $E_2(z)$ for consistency.

Assume all the FIR equalizers $G_j(z),\ j = 0, 1, 2$ have length ρ. Define the $1 \times 3\rho$ equalizer vector $\mathbf{g}$ as

$$\mathbf{g} = \begin{pmatrix}\mathbf{g_0} & \mathbf{g_1} & \mathbf{g_2}\end{pmatrix} \tag{14}$$

with

$$\mathbf{g_j} = \begin{pmatrix}G_j(0) & G_j(1) & \cdots & G_j(\rho-1)\end{pmatrix}. \tag{15}$$

Let the maximum order of ISI channels $E_j(z^2)$ be d. Shorter channels can be zero-padded to reach identical length. Define the $\rho \times (\rho + d)$ Toeplitz filtering matrix associated with $E_j(z^2)$ as $\mathbf{E_0}$, $\mathbf{E_1}$ and $\mathbf{E_2}$, respectively. We can design a minimum MSE equalizer for a target system delay i. Therefore, the mean-square-error (MSE) of the equalized PARQ system is given by

$$\begin{aligned} J(\mathbf{g}) = &\sigma_s^2(\mathbf{g_0E_0} + \mathbf{g_1E_1} + \mathbf{g_2E_2} - \mathbf{e_0})(\mathbf{g_0E_0} + \mathbf{g_1E_1} + \\ &\mathbf{g_2E_2} - \mathbf{e_0})^H + \sigma_s^2(\mathbf{g_0E_1^d} + \mathbf{g_1E_0} - \mathbf{e_1})(\mathbf{g_0E_1^d} + \mathbf{g_1E_0} \\ &- \mathbf{e_1})^H + \sigma_{n_0}^2\mathbf{g_0}\mathbf{g_0}^H + \sigma_{n_1}^2\mathbf{g_1}\mathbf{g_1}^H + \sigma_{n_2}^2\mathbf{g_2}\mathbf{g_2}^H, \end{aligned} \tag{16}$$

where

$$\mathbf{e_0} = \begin{pmatrix}0 & 0 & \cdots & 0 & 1 & 0 & \cdots\end{pmatrix} \tag{17}$$

has only one non-zero element at the i-th position and $\mathbf{e_1}$ has only one non-zero element at the $(i+1)$-th position. $\mathbf{E_1^d}$ represents the filtering matrix associated with $z^{-2}E_2(z^2)$. The solution of the MMSE coefficients $\mathbf{g}$ can be obtained in closed-form by minimizing $J(\mathbf{g})$, given in (18) on the next page.

Using the same principle, zero-forcing equalizer minimizing the ISI can be found accordingly.

$$\mathbf{g} = \begin{pmatrix}\mathbf{g_0} & \mathbf{g_1} & \mathbf{g_2}\end{pmatrix} = \begin{pmatrix} \mathbf{E_0}\mathbf{E_0}^H + \mathbf{E_1^d}\mathbf{E_1^d}^H + \frac{\sigma_{n_0}^2}{\sigma_s^2}\mathbf{I}_\rho & \mathbf{E_0}\mathbf{E_1}^H + \mathbf{E_1^d}\mathbf{E_0}^H & \mathbf{E_0}\mathbf{E_2}^H \\ \mathbf{E_1}\mathbf{E_0}^H + \mathbf{E_0}\mathbf{E_1^d}^H & \mathbf{E_1}\mathbf{E_1}^H + \mathbf{E_0}\mathbf{E_0}^H + \frac{\sigma_{n_1}^2}{\sigma_s^2}\mathbf{I}_\rho & \mathbf{E_1}\mathbf{E_2}^H \\ \mathbf{E_2}\mathbf{E_0}^H & \mathbf{E_2}\mathbf{E_1}^H & \mathbf{E_2}\mathbf{E_2}^H + \frac{\sigma_{w_2}^2}{\sigma_s^2}\mathbf{I}_\rho \end{pmatrix}^{-1}$$

$$\begin{pmatrix}\mathbf{e_0}\mathbf{E_0}^H + \mathbf{e_1}\mathbf{E_1^d}^H & \mathbf{e_0}\mathbf{E_1}^H + \mathbf{e_1}\mathbf{E_0}^H & \mathbf{e_0}\mathbf{E_2}^H\end{pmatrix}. \quad (18)$$

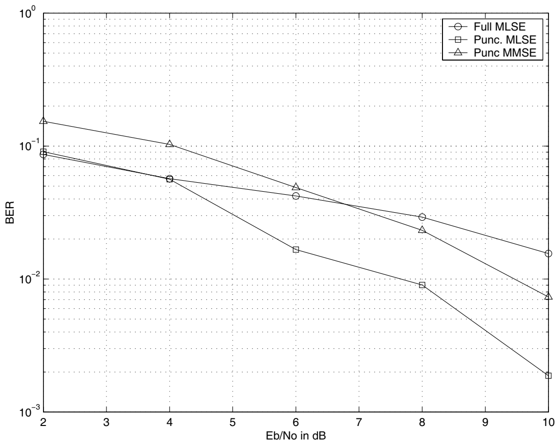

Fig. 5. BER performance using blind estimation and MLSE equalization algorithms.

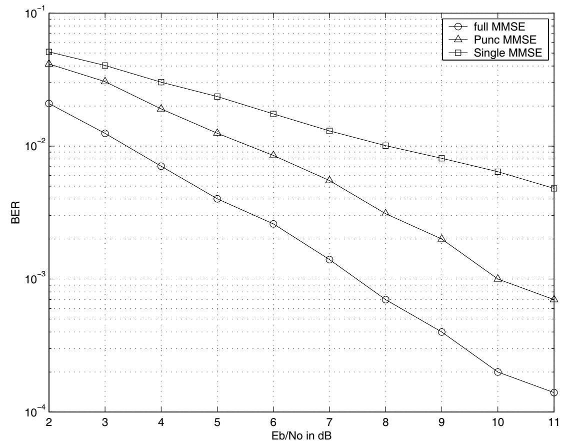

Fig. 6. BER performance using semiblind estimation and MMSE equalization algorithms.

V. System Error Rate Performance

We now consider the joint detection performance of the PARQ retransmission. We assume that the channel is quasi-static over one packet of 128 symbols and is independent from packet to packet. Fig. 4 shows the relative performance of 4 different equalizers: (a) The full retransmission MLSE, (b) the punctured retransmission MLSE, (c) the single transmission MLSE, (d) and the punctured retransmission linear equalizer. Fig. 5 compares the performance of a full retransmission system employing blind SSM for channel estimation with the half retransmission system employing the blind PSSM algorithm. Because we average over 100 random channels, a few bad channels may be ill-conditioned for the SSM algorithm and result in almost 50 percent errors. These errors dominate the average and while the PARQ blind algorithm has a more relaxed channel identifiability condition and yields better results.

In Fig. 6 we investigate the performance of the linear equalizer when the channel is estimated using a more stable semiblind approach. Here we compare the full retransmission, the punctured retransmission, and the single transmission case. The results are satisfactory and as expected.

VI. Conclusion

We investigate the problem of data detection in PARQ systems. First, channel estimation in PARQ with blind and semiblind algorithms is described. A joint ML sequence estimation algorithm is formulated and tested. For low complexity receivers, a linear vector MMSE equalizer is designed and shown to provide robust detection performance. Simulations also show encouraging results for PARQ which may be advantageous in packet transmission systems whose packet error rate may be satisfied with BER of partial retransmission without consuming excessive channel bandwidth.

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