On Spatial Covariance Matrices for Downlink Eigen-Beamforming in Multi-Carrier CDMA

Thomas Sälzer [1], David Mottier [2]
[1] France Telecom, 38-40 rue du Général Leclerc, 92794 Issy-les-Moulineaux, France, thomas.salzer@francetelecom.com
[2] Mitsubishi Electric ITE-TCL, 1 allée de Beaulieu, CS 10806, 35708 Rennes, France, mottier@tcl.ite.mee.com

Abstract — **The present paper considers downlink eigen-beamforming for a Multi-Carrier CDMA mobile radio system. Three different versions of the spatial covariance matrix based on either short- or long-term averaging are investigated and their properties are assessed analytically and numerically. We highlight that, for environments with narrow angle spreads, the long-term spatial covariance matrix should be selected. In wide angle spreads, the short-term spatial covariance matrix averaged over the whole system bandwidth is an interesting trade-off between the beamforming gain and the robustness to Doppler variations.**

I. INTRODUCTION

Exploiting the spatial dimension of wireless channels is one of the key issues in the development of new radio interfaces for future mobile communication systems. In multi-user systems, beamforming and Space Division Multiple Access (SDMA) can efficiently use this dimension for separating the user's signals and increasing the user capacity of the system [1]. In this paper, we consider downlink beamforming for a Multi-Carrier Code Division Multiple Access (MC-CDMA) system. MC-CDMA combines Orthogonal Frequency Division Multiplex (OFDM) modulation with CDMA in the frequency domain and, thus, offers robustness to frequency selective fading and a very flexible multiple access scheme. Therefore, MC-CDMA is currently considered as a promising candidate for new air interfaces in wireless communications [2]-[5]. Nevertheless, the performance of MC-CDMA is limited by the Multiple Access Interference (MAI), which arises after propagation through a multi-path channel.

The combination of MC-CDMA and antenna arrays at the Base-Station (BS) for MAI mitigation was already considered by several authors. While Kim et al. propose beamforming in the uplink [6], we showed in [7] that active user separation with transmit antenna arrays is an interesting approach for the downlink as well, since it makes multi-user detection unnecessary and enables a light receiver design for the Mobile Terminal (MT). The crucial point for beamforming and more generally for pre-filtering approaches is the availability of reliable channel knowledge at the BS prior to transmission. In Time Division Duplex (TDD) systems, such knowledge can be obtained from channel estimation during the uplink time-slot and reused for transmission during the following downlink slot. However, depending on the mobility of the MT, Doppler variations may introduce a mismatch between the available estimate and the real channel state at the instant of transmission. Therefore in [7], we proposed space-frequency pre-filtering, which requires instantaneous knowledge of the channel fading, for indoor communications with low mobility and beamforming based only on long-term covariance knowledge for outdoor communications where higher velocities occur. For the latter case however, beamforming loses efficiency when the multi-paths cover a wide angle spread, which may occur especially in urban environments. To fill this gap, we present here spatial covariance matrices obtained by short-term averaging, which allow efficient transmit beamforming in MC-CDMA for wider angle spreads. We further show that these matrices are sufficiently robust w.r.t. Doppler variations arising with MT velocities in typical city traffic.

The remainder of this paper is organized as follows: We describe the proposed MC-CDMA downlink system with beamforming at the BS in section II. In section III, we introduce the different spatial covariance matrices, analytically assess their properties, and present the eigen-beamforming criterion. A performance analysis is given in section IV, where we compare the proposed MC-CDMA system with beamforming to the conventional single antenna system in a typical outdoor scenario. Finally, section V concludes the paper.

II. SYSTEM MODEL

The proposed downlink system model for MC-CDMA with beamforming at the BS is depicted in Fig. 1. The BS is equipped with M antenna branches and communicates with K active users who share the same set of L sub-carriers of the underlying OFDM transmission scheme. First, the data symbols, e.g. Quadrature Phase Shift Keying (QPSK) symbols, d_k of each user k are weighted with the beamforming vector $\mathbf{w}_k=[w_{k,0},..,w_{k,M-1}]^T$, where $(.)^T$ denotes vector transposition. The beamforming weights are calculated from the spatial covariance matrices $\mathbf{R}_k$. The M versions of each data symbol are then transmitted on the different antenna branches. On each branch the symbols are spread over L chips using Walsh-Hadamard codes denoted by $\mathbf{c}_k=[c_{k,0},\ldots,c_{k,L-1}]^T$, where $c_{k,\ell}\in\{-1/\sqrt{L},1/\sqrt{L}\}$. Since these chips are placed on adjacent sub-carriers of the OFDM system, ℓ denotes both the chip and the sub-carrier index. On each antenna branch, the signals of all active users are added chip-wise. Thus, the accumulated chips on sub-carrier ℓ and on the M antenna branches can be represented by the following vector:

$$\mathbf{s}_\ell = \sum_{k=1}^{K} \mathbf{w}_k^* \, c_{k,\ell} \, d_k \qquad (1)$$

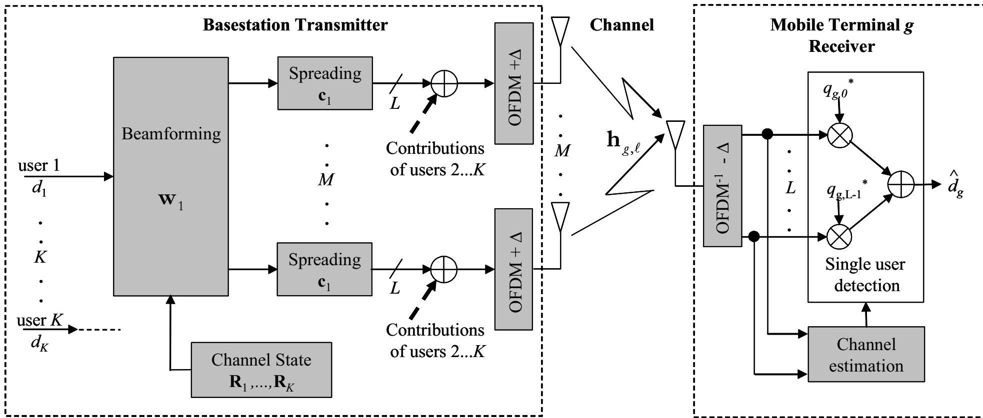

Figure 1: Proposed MC-CDMA downlink system with transmit beamforming.

Finally, OFDM modulation with cyclic prefix insertion is carried out on each antenna branch. Note that the OFDM system comprises a total number of N_C sub-carriers and that generally $N_C>>L$ [3][5]. Consequently, subsequent data symbols of the users are transmitted over different sub-carrier sets. For simplicity, we only describe here the transmission of one data symbol over a distinct subset of L sub-carriers.

We assume ideal OFDM transmission, which essentially means that the employed cyclic prefix Δ completely absorbs the multipath spread of the channel. Then, the effect of the channel can be represented by a single complex fading coefficient per sub-carrier. Thus, the channel between the M transmit antennas of the BS and the single receive antenna of a given MT g on sub-carrier ℓ can be represented by a vector $\mathbf{h}_{g,\ell} = [h_{g,\ell,0},\ldots,h_{g,\ell,M-1}]^T$. The combination of this channel vector and the beamforming vector of any MT k can be represented by an overall transfer function on the L considered sub-carriers, which is given by

$$\mathbf{u}_{k,g} = \left[\mathbf{w}_k^H \mathbf{h}_{g,0},\ldots,\mathbf{w}_k^H \mathbf{h}_{g,L-1}\right]^T \qquad (2)$$

where $(.)^H$ denotes hermitian transposition. After OFDM demodulation, the receiver of MT g combines the received signal versions from the L sub-carriers in order to get a decision variable for data symbol d_g. To meet the complexity limitations at the MT side, we focus on linear single-user detection techniques [2], where this combining can be represented by a vector $\mathbf{q}_g = [q_{g,0},\ldots,q_{g,L-1}]^T$. Then, the decision variable takes the following form:

$$\hat{d}_g = \underbrace{\mathbf{q}_g^H\left(\mathbf{u}_{g,g}\circ\mathbf{c}_g\right)d_g}_{\text{Desired Signal}} + \underbrace{\mathbf{q}_g^H\left(\sum_{k=1,k\neq g}^{K}\left(\mathbf{u}_{k,g}\circ\mathbf{c}_k\right)d_k\right)}_{\text{MAI}} + \underbrace{\mathbf{q}_g^H\mathbf{n}_g}_{\text{Noise}} \qquad (3)$$

Here, ∘ is the element-wise vector multiplication. The decision variable is a sum of three terms. The first term contains the desired signal of user g and the second one is the MAI introduced by the other active users. $\mathbf{n}_g$ is a vector containing the Gaussian noise samples on the L sub-carriers. Thus, the third term is the residual noise after sub-carrier combining. Since the overall transfer function in (2) is user-specific, single-user detection techniques based on orthogonality restoring for MAI cancellation cannot be applied anymore. So, we consider Equal Gain Combining (EGC), which just corrects the phase distortion on each sub-carrier and performs despreading with the code assigned to user g. The EGC coefficients are given by

$$q_{g,\ell} = \frac{c_{g,\ell}\, u_{g,g,\ell}}{\left|u_{g,g,\ell}\right|} = \frac{c_{g,\ell}\,\mathbf{w}_k^H \mathbf{h}_{g,\ell}}{\left|\mathbf{w}_k^H \mathbf{h}_{g,\ell}\right|} \qquad (4)$$

III. Spatial Covariance Matrices and Eigen-Beamforming Criterion

In the proposed system, we perform downlink beamforming based on the spatial covariance matrices of the different users' channels. In the sequel, we analyze three different versions of the spatial covariance matrix with respect to their robustness to Doppler variations due to the mobility of the terminals. The influence of the angle spread of the multi-path channel of each user is also considered. To simplify the analysis, we assume here that the channel coefficients are known, which results in perfectly estimated matrices. In practice, estimates of the channel coefficients may be available at the BS in TDD systems with low mobility, where uplink estimates can be reused in the downlink thanks to channel reciprocity. Note however, that the estimation of the different types of the covariance matrix does not require an explicit estimate of the channel coefficients, since they can be obtained by averaging over scattered pilot signals or even the data signal. Consequently, a low pilot overhead is generally sufficient for this purpose. The three presented matrices can then be distinguished by the way averaging is performed for their estimation.

A. Long-term averaged spatial covariance matrix

A long-term estimate of the spatial covariance matrix is obtained by averaging over time periods much longer than the coherence time of the channel. This means that a large number of consecutive OFDM symbols are involved in this average. The long-term spatial covariance matrix can be written as

$$\mathbf{R}_{g,\mathrm{long}} = \mathrm{E}_t\left\{\mathbf{h}_{g,\ell}(t)\,\mathbf{h}_{g,\ell}^H(t)\right\} \qquad (5)$$

where $\mathrm{E}_t\{.\}$ denotes the expectation in time. Here, we included the time variation of the channel coefficient vector on sub-carrier ℓ, which can be written as

$$\mathbf{h}_{g,\ell}(t) = \sum_{p=0}^{P-1} \mathbf{a}\left(\theta_{g,p}\right)\alpha_{g,p}(t)\, e^{-j2\pi\ell\Delta f\tau_{g,p}} \qquad (6)$$

In (6), we assume that the multi-path channel is composed of P paths, each of them characterized by a complex fast fading coefficient $\alpha_{g,p}(t)$, a delay $\tau_{g,p}$, and a direction $\theta_{g,p}$ w.r.t. the BS antenna array, which leads to an array response vector $\mathbf{a}(\theta_{g,p})$. Δf is the frequency spacing between adjacent sub-carriers. Note that within the considered time periods only the fast fading varies while the directions and the delays can be assumed constant. Since the fast fading processes of different paths are generally uncorrelated, the long-term spatial covariance matrix can be written as

$$\mathbf{R}_{g,long} = \sum_{p=0}^{P-1} \overline{\Omega}_{g,p} \mathbf{a}\left(\theta_{g,p}\right) \mathbf{a}^H\left(\theta_{g,p}\right) \tag{7}$$

where $\overline{\Omega}_{g,p}$ represents the average power of path p. This matrix is determined only by the directions and the average powers of the paths forming the radio channel and is consequently insensitive to Doppler variations. Hence, a reliable estimate of $\mathbf{R}_{g,long}$ can be obtained even for high MT velocities.

B. Short-term averaged spatial covariance matrices

In contrast to the long-term covariance matrix, short-term covariance matrices can be obtained from a single OFDM symbol by averaging in frequency. Time-averaging over a low number of consecutive OFDM symbols is still possible, e.g. for noise reduction, but the corresponding time period has to be much shorter than the coherence time of the channel. In this case, the time variation of the channel vector defined in (6) can be dropped.

Then, a first version of the short-term covariance matrix is obtained by averaging over the L sub-carriers employed for transmitting the considered data symbol. It can be written as

$$\mathbf{R}_{g,L} = \frac{1}{L} \sum_{\ell=0}^{L-1} \mathbf{h}_{g,\ell} \mathbf{h}_{g,\ell}^H \tag{8}$$

$\mathbf{R}_{g,L}$ clearly depends on the fast fading coefficients of the different paths and cannot be related to their directions of arrival for a considerable angle spread and a low number of sub-carriers L. As a consequence, $\mathbf{R}_{g,L}$ is very sensitive to Doppler variations. To overcome this drawback, we propose to average the covariance matrix over the whole number of sub-carriers N_C of the OFDM system, i.e.

$$\mathbf{R}_{g,Nc} = \frac{1}{N_C} \sum_{\ell=0}^{Nc-1} \mathbf{h}_{g,\ell} \mathbf{h}_{g,\ell}^H \tag{9}$$

In practice, the average in (9) spans a larger number of sub-carriers than in (8) (Typical values are N_C=736 and L=16 [4]) and a wider frequency range. In this case, we may approximate $\mathbf{R}_{g,Nc}$ as

$$\mathbf{R}_{g,Nc} = \frac{1}{N_C} \sum_{\ell=0}^{Nc-1} \mathbf{h}_{g,\ell} \mathbf{h}_{g,\ell}^H \approx \sum_{p=0}^{P-1} \left|\alpha_{g,p}\right|^2 \mathbf{a}\left(\theta_{g,p}\right) \mathbf{a}^H\left(\theta_{g,p}\right) \tag{10}$$

Indeed, the cross terms in the covariance matrix involving e.g. two paths $p1$ and $p2$ vanish all the more as their respective delays differ, which induces different phase rotations on the subcarriers (cf. (6)). Hence, approximation (10) particularly holds for large values of N_C and $\Delta f(\tau_{p1}-\tau_{p2})$. As an important result, $\mathbf{R}_{g,Nc}$ can again be related to the directions of the paths similar to (7), but it is now a function of the instantaneous power of each path. Therefore, $\mathbf{R}_{g,Nc}$ is expected to be less sensitive to Doppler variations than $\mathbf{R}_{g,L}$.

Note that all presented versions of the spatial covariance matrix are equivalent, up to a scalar factor, if all paths have the same directions, i.e. $\theta_{g,p}=\theta_g \; \forall p=0 \ldots P\text{-}1$. Their differences, however, increase with increasing angle spread.

C. Eigen-beamforming criterion

Following a preliminary study in [6], we consider downlink Eigen-Beamforming (Eig-BF) [9] in order to maximize the Signal-to-Noise Ratio (SNR) in the decision value given in (3). In principle, Eigen-Beamforming for MC-CDMA could also be designed to combat MAI by maximizing the Signal-to-Interference-plus-Noise Ratio (SINR). However, we showed the limitations of such an approach in [7]. In contrast, the robust SNR-based approach presented here can be efficiently combined with directional code allocation for additional MAI mitigation [7]. From (3) and (4), assuming unit power for the data symbols, the desired signal power for EGC detection can be written as

$$P_{D,g} = \left|\mathbf{q}_g^H\left(\mathbf{u}_{\mathbf{g},\mathbf{g}} \circ \mathbf{c}_\mathbf{g}\right)\right|^2 = \left(\frac{1}{L}\sum_{\ell=0}^{L-1}\left|\mathbf{w}_g^H \mathbf{h}_{g,\ell}\right|\right)^2 \tag{11}$$

Then, it is easily seen that the SNR is maximized by maximizing the following term:

$$\frac{1}{L}\sum_{\ell=0}^{L-1}\left|\mathbf{w}_g^H \mathbf{h}_{g,\ell}\right|^2 = \mathbf{w}_g^H\left(\frac{1}{L}\sum_{\ell=0}^{L-1}\mathbf{h}_{g,\ell}\mathbf{h}_{g,\ell}^H\right)\mathbf{w}_g = \mathbf{w}_g^H \mathbf{R}_{g,L} \mathbf{w}_g \tag{12}$$

Since power control is beyond the scope of this paper, we introduce the power constraint $\mathbf{w}_g^H \mathbf{w}_g = 1$. The Eig-BF vector maximizing (12) is then given by

$$\mathbf{w}_g = \text{m\_eig}\left(\mathbf{R}_{g,L}\right) \tag{13}$$

where m_eig(.) denotes the normalized principal eigenvector of a matrix. Thus, to be optimal, Eig-BF should be based on the short-term averaged matrix $\mathbf{R}_{g,L}$. However, since a reliable estimation of $\mathbf{R}_{g,L}$ is not always available at the BS, we alternatively employ $\mathbf{R}_{g,Nc}$ and $\mathbf{R}_{g,\text{long}}$.

IV. Numerical Results

The proposed system employing Eig-BF with 4 transmit antennas has been compared to the conventional system employing a single antenna by computer simulations using realistic system parameters [4], which are summarized in Tab.1.

Tab.1: System parameters.

Carrier frequency	5 GHz
Bandwidth	57.6 MHz
FFT-size / Used sub-carriers, N_C	1024 / 736
Carrier spacing, Δf	56.3 kHz
Symbol time / Cyclic prefix length	21.5 µs / 3.75 µs
Channel Model	Spatially extended BRAN E, max. delay 1.72 µs
Angle spread	10°, 30°
Symbol alphabet	QPSK
Coding	UMTS convol., rate 2/3
Spreading factor, L	16
Uniform Linear Array	half-wave spaced, M=1,4

We assume an outdoor scenario and MT velocities corresponding to city traffic. MT positions are randomly chosen within a 120° sector. Channel estimation at the MT receiver is assumed perfect. In contrast, we assume that the covariance matrices defined in (7), (8) and (9) are estimated from the uplink and used for downlink Eig-BF after a typical delay of 1 ms. Hence, the short-term spatial covariance matrices may suffer from a mismatch due to Doppler variations. The given results represent the Bit Error Rate (BER) averaged over all users and a large number of channel realizations as a function of the SNR. Since channel coding is included, we compare the results at an operation point of BER=10^{-4}.

Fig.2 concerns a typical sub-urban environment with an Angle Spread (AS) of 10° for the multi-path channel of each user. For a single user (K=1), the proposed system using the short-term covariance matrix $\mathbf{R}_{g,L}$ achieves a beamforming gain of 6 dB compared to the conventional system. At full system load (K=L=16), the conventional system suffers from MAI, while the proposed one still achieves the same performance. Since the different spatial covariance matrices are almost identical for low angle spreads, only a slight loss of 0.5 dB is encountered by employing the long-term covariance matrix $\mathbf{R}_{g,\text{long}}$. Therefore, $\mathbf{R}_{g,\text{long}}$ should be favoured here for its robustness.

In Fig.3, the AS is increased to 30° (typically an outdoor urban environment), where the differences between the three versions of the covariance matrix are clearly visible. For negligible Doppler (0 km/h), beamforming using $\mathbf{R}_{g,L}$ obviously performs best and a loss of around 2 dB is experienced by using $\mathbf{R}_{g,Nc}$. Note that beamforming based on $\mathbf{R}_{g,\text{long}}$ is not advantageous for this wide angle spread. At 45 km/h, $\mathbf{R}_{g,L}$ loses its reliability due to Doppler variations. In contrast, the proposed robust short-term version $\mathbf{R}_{g,Nc}$ only shows a 0.5 dB degradation at BER=10^{-4}. Hence, the use of $\mathbf{R}_{g,Nc}$ represents a good trade-off between the beamforming gain in wide angle spread and robustness to Doppler variations.

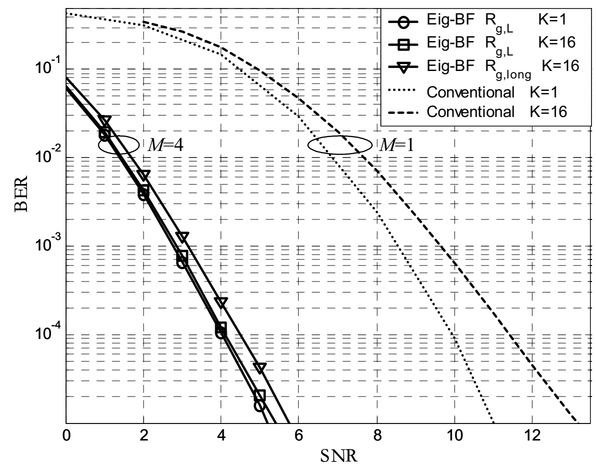

Fig.2: Comparison in sub-urban environment (AS=10°)

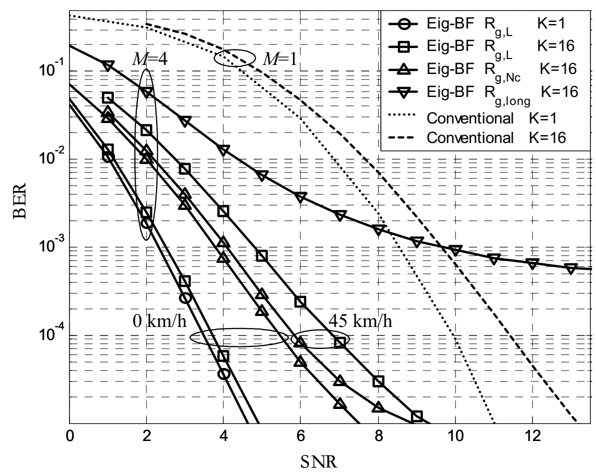

Fig.3: Comparison in urban environment (AS=30°)

V. Conclusion

We investigated different versions of the spatial covariance matrix for downlink eigen-beamforming in MC-CDMA. At small angle spreads, the short- and long-term based matrices achieve almost the same performance independently of the MT's velocity. Therefore, the long-term covariance matrix should be preferred for its robustness. For wider angle spreads, we showed that the short-term covariance matrix averaged over the whole system bandwidth yields a good trade-off between beamforming gain and robustness to Doppler variations.

Acknowledgement

The presented work is carried out in the framework of the 4MORE (4G MC-CDMA multiple antenna system On chip for Radio Enhancements) project supported by the European Commission under the contract number IST-2002-507039 [5].

References

[1] A.F. Naguib, A.J. Paulraj, T. Kailath, ‘Capacity improvement with base-station antenna arrays in cellular CDMA,’ *IEEE Trans. on Vehic. Tech.*, vol. 43, no. 3, pp. 691-698, Aug. 1994.

[2] S. Hara, R. Prasad, ‘Overview of multicarrier CDMA,’ *IEEE Comm. Magazine*, vol. 35, pp. 126-133, Dec. 1997.

[3] N. Maeda, Y. Kishiyama, K. Higuchi, H. Atarashi, M. Sawahashi, ‘Experimental evaluation of throughput performance in broadband packet wireless access based on VSF-OFCDM and VSF-CDMA,’ *IEEE Proc. of PIMRC'03*, vol. 1, pp. 6-11, Sept. 2003.

[4] European IST MATRICE project, web site http://www.ist-matrice.org.

[5] European IST 4MORE project, web site http://www.ist-4more.org.

[6] C. K. Kim, Y. S. Cho, ‘Performance of a wireless MC-CDMA system with an Antenna Array in a fading channel: reverse link,’ *IEEE Trans. on Communications*, vol. 48, no. 8, pp. 1257-1261, Aug. 2000.

[7] T. Sälzer, D. Mottier, 'Downlink Strategies Using Antenna Arrays for Interference Mitigation in Multi-Carrier CDMA,' *Proc. of int. Workshop on Multi-Carrier Spread Spectrum & Rel. Topics (MC-SS’03)*, pp. 315-325, Sept. 2003.

[8] D. Mottier, T. Sälzer, ‘Spreading Sequence Assignment in the Downlink of OFCDM Systems Using Multiple Transmit Antennas,’ *IEEE Proc. of VTC’04 Spring*, May 2004.

[9] S.A. Jafar, S. Vishwanath, A. Goldsmith, ‘Channel capacity and beamforming for multiple transmit and receive antennas with covariance feedback,’ *IEEE Proc. of ICC’01*, vol.7, pp. 2266-2270, June 2001.