A MULTI USER BEAMFORMING SCHEME FOR DOWNLINK MIMO CHANNELS BASED ON MAXIMIZING SIGNAL-TO-LEAKAGE RATIOS

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ABSTRACT

Multi-user multiple-input multiple-output (MU-MIMO) wireless systems can provide a substantial gain in network downlink throughput by allowing multiple users to communicate in the same frequency and time slots. The challenge is to design transmit beamforming vectors for every user while limiting the co-channel interference (CCI) from other users. One approach is to perfectly cancel the CCI at every user, which requires a relatively large number of transmit antennas. In this paper, we consider an alternative approach based on maximizing the *signal-to-leakage ratio (SLR)* for designing transmit beamforming vectors in a multi-user system. One advantage of the proposed scheme is that it does not impose a restriction on the number of available transmit antennas; it also outperforms the conventional beamforming scheme.

1. INTRODUCTION

There has been considerable interest in MIMO wireless communications in view of their potential for dramatic improvement in channel capacity. In multi-user scenarios, several co-channel users with multiple antennas aim to communicate with a base station in the same frequency and time slots. In this case, it becomes necessary to design transmission schemes that are able to suppress the CCI at the end users.

One approach would be to deal with the CCI exclusively at the transmitter: the channel state information for all users would be required at the base station and the end users would not need the channel state information of co-channel users. This scheme would reduce overhead due to channel feedback. Several works have proposed such schemes for perfectly cancelling the CCI for each user (e.g., [1, 2, 3, 4]). While these methods result in superior performance by completely cancelling the CCI at every receiver, they tend to impose a restriction on the system configuration in terms of the number of antennas. Roughly, these methods require the number of transmit antennas at the base station to be larger than the sum of receive antennas at all users. This condition is necessary to provide enough degrees of freedom in order to make the CCI zero at each user.

In this paper, we consider an alternative approach for designing transmit beamforming vectors based on maximizing what we refer to as the signal-to-leakage ratio (SLR). The proposed criterion aims at maximizing the received desired signal power at every user, while minimizing the overall interference power caused by this user at all other co-channel users. The resulting solution does not impose a restriction on the number of available transmit antennas, and it determines the optimal procedure by solving a generalized eigenvalue problem.

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2. SYSTEM MODEL

Consider a downlink multi-user environment with a base station communicating with K users. The base station employs N transmit antennas and each user could be equipped with multiple antennas as well. Let M_i denote the number of receive antennas at the ith user. A block diagram of the system is shown in Figure 1, where $s_i(n)$ denotes the transmitted data intended for user i at time n. The scalar data $s_i(n)$ is multiplied by an $N \times 1$ beamforming vector $\mathbf{w}_i$ before being transmitted over the channel. In this way, the overall $N \times 1$ transmitted vector at time n is given by

$$\mathbf{x}(n) = \sum_{k=1}^{K} \mathbf{w}_k s_k(n) \tag{1}$$

The data $s_i(n)$ and the beamforming coefficients $\mathbf{w}_i$ are assumed to be normalized as follows:

$$\mathsf{E}|s_k(n)|^2 = 1, \quad \|\mathbf{w}_k\|^2 = 1$$

for $k = \{1, \ldots, K\}$.

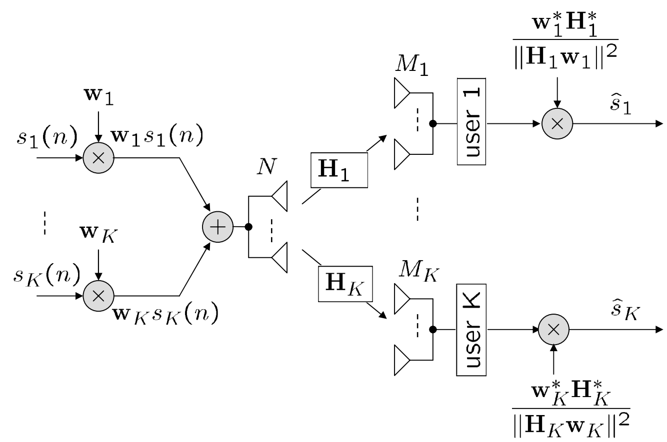

Fig. 1. Block diagram of the multi-user beamforming system.

The $N \times 1$ vector $\mathbf{x}(n)$ is then broadcast over the channel. Assuming a narrow-band (single-path) channel, the received vector of size $M_i \times 1$ at the ith user at time n is obtained as

$$\mathbf{y}_i(n) = \mathbf{H}_i \sum_{k=1}^{K} \mathbf{w}_k s_k(n) + \mathbf{v}_i(n) \tag{2}$$

where the entries of the $M_i \times N$ channel matrix $\mathbf{H}_i$ are denoted by

$$\mathbf{H}_i = \begin{bmatrix} h_i^{(1,1)} & \dots & h_i^{(1,N)} \\ \vdots & \ddots & \vdots \\ h_i^{(M_i,1)} & \dots & h_i^{(M_i,N)} \end{bmatrix} \tag{3}$$

with $h_i^{(k,l)}$ representing the channel coefficient from the lth antenna at the base station to the kth receiver antenna at user i. The elements of $\mathbf{H}_i$ are complex Gaussian variables with zero-mean and unit-variance. Furthermore, the additive noise $\mathbf{v}_i(n)$ has elements with distribution $\mathcal{CN}(0, \sigma_i^2)$ and is spatially and temporarily white. In other words,

$$\mathsf{E}\left[\mathbf{v}_i \mathbf{v}_i^*\right] = \sigma_i^2 \mathbf{I}_{M_i}$$

$$\mathsf{E}\ \mathsf{Trace}\left(\mathbf{H}_i \mathbf{H}_i^*\right) = M_i N$$

where $\mathbf{I}_{M_i}$ is the $M_i \times M_i$ identity matrix. Since the random quantities $\mathbf{H}_i$, s_i, and $\mathbf{v}_i$ are assumed independent, the received signal-to-noise ratio (SNR) at each receive antenna (per user) is $1/\sigma_i^2$, independently of N. We shall refer to $1/\sigma_i^2$ as the received SNR throughout the paper and the simulation results are plotted versus this quantity.

We assume that the channel matrices $\mathbf{H}_i, i = \{1, \dots, K\}$, are available at the base station (e.g., either through reverse channel estimation in time-division-duplex (TDD) or feedback in frequency-division-duplex (FDD)). We also assume that the channel matrix $\mathbf{H}_i$ is known at the corresponding receiver i, but is not required to be known by the other users. Furthermore, we assume a slow-fading wireless channel with packet-based transmission where the channel is quasi-static over a packet length, and changes independently between consecutive transmissions.

3. MULTI-USER BEAMFORMING

We reconsider the received signal in (2) by user i and drop the time index n for notational simplicity:

$$\mathbf{y}_i = \mathbf{H}_i \mathbf{w}_i s_i + \sum_{k=1, k\neq i}^{K} \mathbf{H}_i \mathbf{w}_k s_k + \mathbf{v}_i \tag{4}$$

where the second term is the co-channel interference (CCI) caused by the multi-user nature of the system. The goal is to estimate the transmitted scalar s_i from the received vector $\mathbf{y}_i$. In prior work on downlink multi-user MIMO systems, the major focus has been on cancelling the CCI term perfectly. For example in [3], the criterion for choosing the beamforming vectors $\mathbf{w}_i$, $i = \{1, \dots, K\}$, was to satisfy the conditions

$$\mathbf{H}_i \mathbf{w}_k = \mathbf{0} \tag{5}$$

for all $i, k = \{1, \dots, K\}, i \neq k$. This solution results in superior performance by completely cancelling the CCI at every receiver, but it imposes a strong condition on the system configuration in terms of the number of antennas. Specifically, in order for this problem to be well posed (i.e., for a solution $\mathbf{w}_i$ to exist), one needs to require

$$N \geq \sum_{k=1, k\neq i}^{K} M_k \tag{6}$$

for $i = \{1, \dots, K\}$. Thus the scheme requires an increase in the number of base station antennas as the number of users or the number of receive antennas by each user increase. Simulations indicate that this solution results in inferior performance when compared to conventional single user beamforming if condition (6) is not met.

Now let us assume the ith user estimates s_i from $\mathbf{y}_i$ in (4) according to a classical single-user maximum-likelihood detection scheme (without relying on knowledge of the other channels), i.e.,

$$\hat{s}_i \triangleq \frac{\mathbf{w}_i^* \mathbf{H}_i^* \mathbf{y}_i}{\|\mathbf{H}_i \mathbf{w}_i\|^2}$$

Then

$$\hat{s}_i = s_i + \frac{\mathbf{w}_i^* \mathbf{H}_i^* \sum_{k=1, k\neq i}^{K} \mathbf{H}_i \mathbf{w}_k s_k}{\|\mathbf{H}_i \mathbf{w}_i\|^2} + \frac{\mathbf{w}_i^* \mathbf{H}_i^* \mathbf{v}_i}{\|\mathbf{H}_i \mathbf{w}_i\|^2} \tag{7}$$

and the output signal-to-interference-noise ratio (SINR) for user i would be given by

$$\text{SINR}_i = \frac{\|\mathbf{H}_i \mathbf{w}_i\|^2}{\sigma_i^2 + \frac{\sum_{k=1,k\neq i}^{K} \|\mathbf{w}_i^* \mathbf{H}_i^* \mathbf{H}_i \mathbf{w}_k\|^2}{\|\mathbf{H}_i \mathbf{w}_i\|^2}} \tag{8}$$

Using this SINR expression for $i = \{1, \ldots, K\}$ as an optimization criterion for determining the $\{\mathbf{w}_i\}_{i=1}^K$ would generally result in a problem with K coupled variables $\{\mathbf{w}_i\}$ [5]. In the sequel, we propose an alternative criterion to design the beamforming coefficients $\{\mathbf{w}_i\}$, and which leads to a full characterization of the optimal solutions in terms of generalized eigenvalue problems.

Let us reconsider (4). The power of the desired signal $\mathbf{H}_i \mathbf{w}_i s_i$ in (4) is given by $\|\mathbf{H}_i \mathbf{w}_i\|^2$. At the same time, the power of the interference caused by this user i on the signal received by user k is given by $\|\mathbf{H}_k \mathbf{w}_i\|^2$. We define a quantity, called *leakage* for user i, as the total power leaked from this user to all other users:

$$\sum_{k=1,k\neq i}^{K} \|\mathbf{H}_k \mathbf{w}_i\|^2$$

Problem Statement. *Given a fixed transmit power for each user, design $\mathbf{w}_i$, $i = \{1, \ldots, K\}$, such that the signal-to-leakage ratio (SLR) is maximized for every user:*

$$\mathbf{w}_i^o = \arg \max_{\mathbf{w}_i \in \mathbb{C}^{N\times 1}} \underbrace{\frac{\|\mathbf{H}_i \mathbf{w}_i\|^2}{\sum_{k=1,k\neq i}^{K} \|\mathbf{H}_k \mathbf{w}_i\|^2}}_{\textit{SLR for user } i} \tag{9}$$

subject to $\|\mathbf{w}_i\|^2 = 1, \;\; i = \{1, \ldots, K\}$.

A key feature of the above criterion is that the design procedure for $\mathbf{w}_i$, $i = \{1, \ldots, K\}$, involves K decoupled optimization problems.

It can be verified that the SLR expression in (9) can be rewritten as

$$\text{SLR} = \frac{\|\mathbf{H}_i \mathbf{w}_i\|^2}{\|\tilde{\mathbf{H}}_i \mathbf{w}_i\|^2} \tag{10}$$

where

$$\tilde{\mathbf{H}}_i = \begin{bmatrix} \mathbf{H}_1^* & \ldots & \mathbf{H}_{i-1}^* & \mathbf{H}_{i+1}^* & \ldots & \mathbf{H}_K^* \end{bmatrix}^* \tag{11}$$

is an extended channel matrix that excludes $\mathbf{H}_i$ only. The SLR expression can be further rewritten as

$$\text{SLR} = \frac{\mathbf{w}_i^* \mathbf{H}_i^* \mathbf{H}_i \mathbf{w}_i}{\mathbf{w}_i^* \tilde{\mathbf{H}}_i^* \tilde{\mathbf{H}}_i \mathbf{w}_i} \tag{12}$$

To solve (9) we then note that, in view of the Rayleigh-Ritz quotient result [6],

$$\frac{\mathbf{w}_i^* \mathbf{H}_i^* \mathbf{H}_i \mathbf{w}_i}{\mathbf{w}_i^* \tilde{\mathbf{H}}_i^* \tilde{\mathbf{H}}_i \mathbf{w}_i} \leq \lambda_{\text{max}} \left(\mathbf{H}_i^* \mathbf{H}_i, \tilde{\mathbf{H}}_i^* \tilde{\mathbf{H}}_i \right) \tag{13}$$

where λ_{max} is the largest generalized eigenvalue of the matrix pair $\mathbf{H}_i^* \mathbf{H}_i$ and $\tilde{\mathbf{H}}_i^* \tilde{\mathbf{H}}_i$. Equality occurs if $\mathbf{w}_i$ is proportional to a generalized eigenvector that corresponds to the largest generalized eigenvalue, written compactly as

$$\mathbf{w}_i^o \propto \max \;\; \text{gen. eigenvector} \left(\mathbf{H}_i^* \mathbf{H}_i, \tilde{\mathbf{H}}_i^* \tilde{\mathbf{H}}_i \right) \tag{14}$$

The proportionality constant is chosen to normalize the norm of $\mathbf{w}_i^o$ to unity. Compare this solution to the conventional single-user beamforming solution [7]:

$$\text{conventional } \mathbf{w}_i^o \propto \max \;\; \text{eigenvector} \left(\mathbf{H}_i^* \mathbf{H}_i \right) \tag{15}$$

where in contrast to (14), it only uses the channel information from user i. If $\tilde{\mathbf{H}}_i^* \tilde{\mathbf{H}}_i$ is invertible, then the generalized eigenvalue problem (14) reduces to

$$\lambda_{\text{max}} \left(\mathbf{H}_i^* \mathbf{H}_i, \tilde{\mathbf{H}}_i^* \tilde{\mathbf{H}}_i \right) = \lambda_{\text{max}} \left(\left(\tilde{\mathbf{H}}_i^* \tilde{\mathbf{H}}_i \right)^{-1} \mathbf{H}_i^* \mathbf{H}_i \right)$$

and

$$\mathbf{w}_i^o \propto \max \;\; \text{eigenvector} \left(\left(\tilde{\mathbf{H}}_i^* \tilde{\mathbf{H}}_i \right)^{-1} \mathbf{H}_i^* \mathbf{H}_i \right)$$

4. SIMULATION RESULTS

A typical multi-user MIMO system is simulated to evaluate the performance of the proposed multi-user beamforming scheme (14) in comparison to the conventional single-user beamforming scheme (15). A single-path quasi-static MIMO channel is used in the simulations with its elements generated as zero-mean and unit-variance independent and identically distributed (i.i.d) complex Gaussian random variables. All simulations are conducted using a QPSK transmit constellation and the results are averaged over several channel realizations. The noise variance per receive antenna is assumed the same for all users, $\sigma_1^2 = \ldots = \sigma_K^2 = \sigma^2$, and the BER curves are plotted versus $1/\sigma^2$, as the SNR:$1/\sigma^2$ functions as the SNR per receive antenna since the MIMO channel and beamforming coefficients are all normalized, as explained in Sec. 2. Figures 2 and 3 show the BER results for different system configurations.

While the conventional single-user beamforming fails in a multi-user environment in terms of BER (see Figures 2 and 3), the proposed scheme provides an acceptable BER in the presence of 2 and 3 active users. For instance in Figure 2, the proposed scheme maintains an acceptable 10^{-2} uncoded BER for 3 simultaneously active users at a received per antenna SNR of 3dB. To better understand the behavior of the proposed scheme, the SINR outage (or cumulative distribution function) is plotted in Figure 4 to show and compare the distribution of the achieved SINR given by (8). An outage value of 10% at SINR $=$ 10dB (for

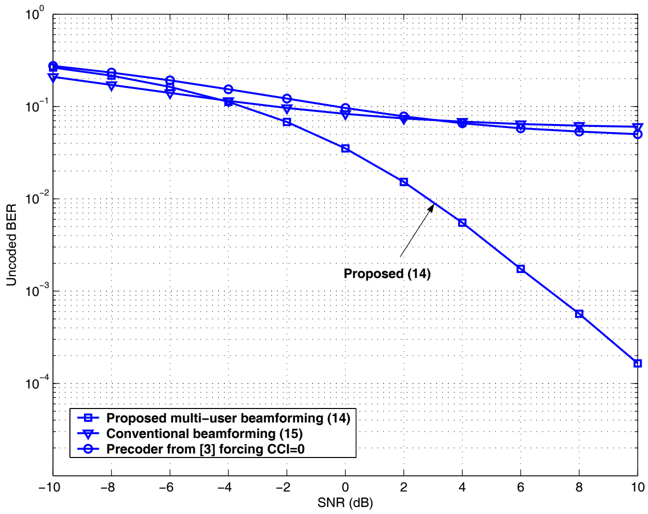

Fig. 2. A multiuser MIMO system with $N = 5$ transmit antennas and $K = 3$ users, each equipped with $M_i = 3$ receive antennas.

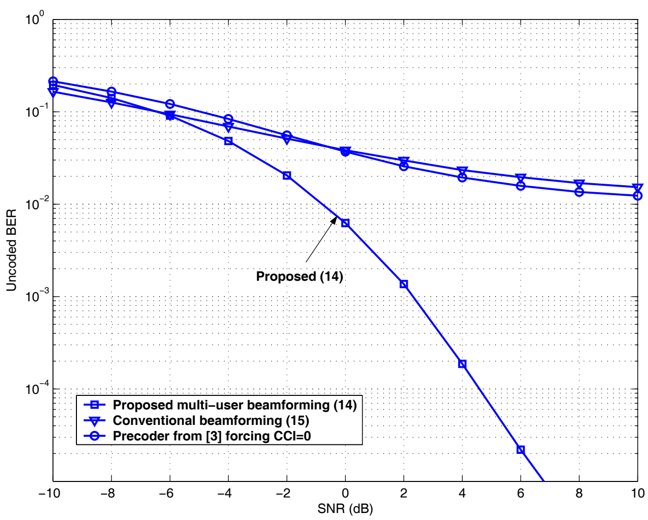

Fig. 3. A multiuser MIMO system with $N = 4$ transmit antennas and $K = 2$ users, each equipped with $M_i = 6$ receive antennas.

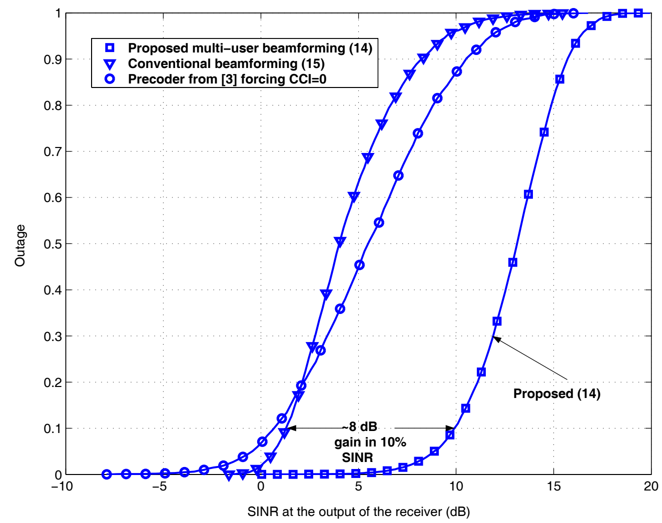

Fig. 4. A system with $N = 5$ transmit antennas and $K = 3$ users, each equipped with $M_i = 3$ receive antennas. The outage values are plotted for SNR $= 1/\sigma^2 = 8$dB.

the proposed scheme) means for 90% of the channel realizations, the achieved SINR is larger than 10dB. As shown in the figure, using the proposed scheme results in an 8dB improvement in 10% outage value compared to the conventional scheme.

5. CONCLUSIONS

In this paper, we considered a performance criterion based on optimizing the signal-to-leakage ratio (SLR) for designing multi-user transmit beamforming vectors in a MIMO system. The decoupled nature of this criterion allows for a characterization of the solution to the multi-user beamforming problem in terms of generalized eigenvalue problems. Moreover, the proposed scheme does not impose a restriction on the number of available transmit antennas.

6. REFERENCES

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