COMPARISON OF MULTIUSER DIVERSITY USING STBC AND TRANSMIT BEAMFORMING WITH OUTDATED FEEDBACK

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ABSTRACT

For multiuser diversity systems employing adaptive modulation, we compare the spectral efficiency for space-time block coding (STBC) and transmit beamforming (BF) with outdated channel feedback. Taking into account the channel feedback delay, we derive the optimal thresholds to maximize spectral efficiency subject to an average BER constraint, and illustrate the impact of feedback delay on the achievable multiuser diversity gain with either STBC or BF. We observe that more transmit antennas bring higher spectral efficiency for BF. But this becomes inverted using STBC, due to the effect of channel-hardening [1]. With a small feedback delay, the BF scheme outperforms the STBC scheme. However, when feedback delay becomes large enough, the STBC scheme achieves higher spectral efficiency due to the reduced diversity order of BF.

1. INTRODUCTION

Multi-antenna arrays are well known to offer spectral efficiency along with diversity benefits (e.g. STBC [2], and transmit BF [3]) over fading channels. Recent studies indicate that there is another form of diversity, called multiuser diversity, inherent in multiuser wireless systems [4] [5]. For multiuser diversity systems employing adaptive modulation, we compare the spectral efficiency for STBC and transmit BF with outdated channel feedback, and quantitatively determine whether STBC or BF is a better choice for exploiting the multiple transmit antennas in the presence of delay.

Good performance of adaptive modulation requires accurate channel estimation at the receiver and a reliable feedback path between the receiver and the transmitter. However, the channel feedback information will become outdated if the channel is changing rapidly. Performance comparison of adaptive STBC and BF with outdated feedback for single-user systems is performed in [6]. The impact of feedback delay on the achievable practical multiuser diversity is investigated in [5] for single-antenna systems. In this paper, we extend the results to multi-antenna systems employing either STBC or transmit BF. We consider the optimal channel assigning strategy where the channel is assigned to the user with the greatest instantaneous SNR [4] [5]. With the objective of maximizing spectral efficiency under an average BER constraint with respect to the switching thresholds, we consider a robust constant power, variable rate M-QAM scheme that is less sensitive to feedback delay. Based on the closed-form expressions for average BER and average rate, we compare the spectral efficiency for STBC and transmit BF with outdated channel feedback in adaptive modulation systems achieving multiuser diversity.

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2. SYSTEM MODEL

We address downlink transmission in a multiuser system with a single base station (BS) serving K users where the BS transmits in slots of some fixed duration. The BS has N_t transmit antennas while each user is equipped with N_r receive antennas. The BS transmits information on the MIMO link using either STBC or transmit BF. Dropping the time index for simplicity, the baseband input/output relationship is described by

$$\mathbf{y}_k = \sqrt{\bar{\gamma}_k}\mathbf{H}_k\mathbf{s} + \mathbf{w}_k, \tag{1}$$

where $\mathbf{y}_k$ denotes the $N_r \times 1$ received vector of the kth user, $\bar{\gamma}_k$ denotes the expected SNR at each receive antenna of the kth user, $\mathbf{s}$ is the $N_t \times 1$ vector broadcasted from the BS through N_t transmit antennas, $\mathbf{H}_k$ is the $N_r \times N_t$ flat fading channel matrix from the BS to the kth user, and $\mathbf{w}_k$ is the additive white Gaussian noise (AWGN) vector. We assume each user has the same expected SNR, i.e $\bar{\gamma}_k = \bar{\gamma}$, but this assumption can be relaxed as in [5], which considers the single-antenna case. The channel matrix $\mathbf{H}_k$ of each user is assumed independent identically distributed (iid), and each matrix entry $[\mathbf{H}_k]_{i,j} = h_{i,j}^{(k)} \sim \mathcal{CN}(0,1)$. The elements of the noise matrix $\mathbf{w}_k$ are also iid and $\mathcal{CN}(0,1)$. The transmit vector $\mathbf{s}$ contains the transmitted information bearing symbol in the chosen M-QAM constellation and the average energy is normalized so that $\mathrm{E}(\mathbf{s}^H\mathbf{s}) = 1$.

For delay tolerant data applications, there has been recent interest in applying scheduling approaches to boost the data throughput in multiuser scenarios. One scheduling approach has the BS transmit to only one user who has the best instantaneous SNR, which is optimal in the sense of maximizing the throughput of a multiuser system [4]. We will adopt this scheduling strategy. Since the BS communicates with exactly one of the users in each slot, (1) is simplified to a single user model with $\mathbf{s}$ denoting the transmitted vector of the selected user.

For the adaptive STBC scheme, $\mathbf{s}$ is a column vector of the codeword matrix of the selected user, which contains the information symbol with an average energy of $1/N_t$. The total instantaneous received SNR of the kth user is given by [2] $\gamma_k^{STBC} := \frac{\bar{\gamma}}{N_t} \parallel \mathbf{H}_k \parallel^2 = \frac{\bar{\gamma}}{N_t}\sum_{i=1}^{N_r}\sum_{j=1}^{N_t} \mid h_{i,j}^{(k)} \mid^2$, where $\parallel \cdot \parallel$ denotes the Frobenius norm.

For the adaptive BF scheme, the transmitted vector is defined as $\mathbf{s} := \mathbf{b}_k s_k$, where $\mathbf{b}_k$ is a $N_t \times 1$ unit beamforming vector for the selected user k and s_k is the information bearing symbol with energy normalized to 1. The beamformer $\mathbf{b}_k$ is fed back to the BS by each mobile receiver. With perfect channel feedback, the BF scheme is optimal in view of SNR maximization at the receiver. For the kth user, the total received SNR can be written as [3] $\gamma_k^{BF} := \bar{\gamma} \parallel \mathbf{H}_k \mathbf{b}_k \parallel^2 = \bar{\gamma}\mathbf{b}_k^H \mathbf{H}_k^H \mathbf{H}_k \mathbf{b}_k$.

For both schemes, perfect channel estimation is assumed at the receiver and each mobile receiver feeds back its own instantaneous received SNR to the BS. As is commonly assumed (e.g. in [7]), we suppose an error-free feedback path from each user to the BS, which can be ensured by coding and ARQ protocols, and feedback has a time delay τ_k for user k. We use $\hat{\mathbf{H}}_k$ to denote the outdated version of $\mathbf{H}_k$.

For adaptive STBC, the constellation adaptation is performed based on the feedback estimate of the total received SNR $\hat{\gamma}_k^{STBC} := \frac{\bar{\gamma}}{N_t} \parallel \hat{\mathbf{H}}_k \parallel^2$. There is performance degradation due to the delayed channel feedback, since the actual channel SNR during transmission is $\gamma_k^{STBC} \neq \hat{\gamma}_k^{STBC}$. The probability density function (pdf) of the SNR estimate $\hat{\gamma}_k^{STBC}$ of user k is given by

$$p_{\hat{\gamma}_k^{STBC}}(\gamma_k) = \left(\frac{1}{\bar{\gamma}/N_t}\right)^{N_t N_r} \frac{\gamma_k^{N_t N_r - 1}}{\Gamma(N_t N_r)} \exp\left(-\frac{\gamma_k}{\bar{\gamma}/N_t}\right),$$

where $\gamma_k \geq 0$, and $\Gamma(\cdot)$ is the complete gamma function.

For BF, the beamformer is fed back and is given by the eigenvector $\mathbf{b}_k$ corresponding to the largest eigenvalue $\hat{\lambda}_k^{max}$ of the complex Wishart matrix $\hat{\mathbf{H}}_k^H \hat{\mathbf{H}}_k$. The constellation adaptation is performed based on the estimate of total received SNR $\hat{\gamma}_k^{BF} := \bar{\gamma} \parallel \hat{\mathbf{H}}_k \mathbf{b}_k \parallel^2 = \bar{\gamma}\mathbf{b}_k^H \hat{\mathbf{H}}_k^H \hat{\mathbf{H}}_k \mathbf{b}_k = \bar{\gamma}\hat{\lambda}_k^{max}$, while the total SNR during transmission is $\gamma_k^{BF} \neq \hat{\gamma}_k^{BF}$ when the delay $\tau_k \neq 0$. The pdf of $\hat{\lambda}_k^{max}$ is available in [8]. However, the evaluation of the cumulative distribution function (cdf) involves integration, which is numerically plausible but involved. Therefore we are especially interested in the simple multiple input and single output (MISO) systems with $N_r = 1$ receive antenna. In this case, the pdf of the SNR estimate $\hat{\gamma}_k^{BF}$ of user k is given by

$$p_{\hat{\gamma}_k^{BF}}(\gamma_k) = \left(\frac{1}{\bar{\gamma}}\right)^{N_t} \frac{\gamma_k^{N_t - 1}}{\Gamma(N_t)} \exp\left(-\frac{\gamma_k}{\bar{\gamma}}\right), \qquad \gamma_k \geq 0.$$

Since in each slot only the best user is scheduled for transmission, we focus on the best user and the user index is dropped in the sequel. We will refer to the feedback SNR for the selected best user as: $\hat{\gamma} := \max_k \hat{\gamma}_k$, for short, where $\hat{\gamma}_k$ represents the feedback SNR for either STBC or BF throughout the paper. Therefore the pdf of $\hat{\gamma}$ can be expressed using order statistics

$$p_{\hat{\gamma}}^{max}(y) = K p_{\hat{\gamma}_k}(y) F_{\hat{\gamma}_k}^{K-1}(y), \tag{2}$$

where $p_{\hat{\gamma}_k}(y)$ represents the pdf for $\hat{\gamma}_k^{STBC}$ or $\hat{\gamma}_k^{BF}$, and $F_{\hat{\gamma}_k}(y)$ represents their cdf.

For practical reasons, a discrete finite set of Gray-coded square M-QAM constellations with sizes in $\mathcal{M} = \{M_0, M_1, ..., M_{J-1}\}$ is adopted for our rate-adaptive schemes, where M_0 denotes no transmission. We compare two schemes where the BS transmits using either STBC or BF, and adapts the transmission rate for the selected best user by choosing a constellation from $\mathcal{M}$ in accordance with the feedback SNR $\hat{\gamma}$ of the selected user. More precisely, given a set of fixed switching thresholds $\mathbf{t} = [t_0, ..., t_J]$, the constellation size M_j is selected and used for transmission whenever $t_j \leq \hat{\gamma} < t_{j+1}$. We assume $t_0 = 0$, and $t_J = \infty$.

With no feedback delay, applying the well-known result $\lambda^{max}(\mathbf{H}^H\mathbf{H}) \geq \frac{1}{N_t}\mathrm{tr}(\mathbf{H}^H\mathbf{H})$, which holds for any matrix $\mathbf{H}$, we have $\gamma^{BF} \geq \gamma^{STBC}$. Therefore for a given set of switching thresholds, the BF scheme has superior performance to the STBC scheme with perfect feedback. However, when feedback delay is large, for BF, the beamformer $\mathbf{b}$ fed back to the transmitter would contain negligible channel information for the next transmission, and can be considered as a random vector independent of the actual channel $\mathbf{H}$. It is straightforward to show that the BF scheme has a maximum diversity order of N_r in this case. Thus for a given set of switching thresholds, the performance of the BF scheme gets worse in terms of feedback delay, due to the degraded diversity order. However, even though the STBC scheme suffers performance degradation with a large feedback delay, the diversity order remains $N_t N_r$. Therefore, we expect that the BF scheme outperforms the STBC scheme when feedback delay is small, and the reverse is true over the high SNR range as feedback delay gets larger, due to the degraded diversity order of BF.

3. BER AND AVERAGE RATE EXPRESSIONS

The figures of merit for our practical multiuser diversity scheme are BER and average rate. We now express these for STBC and BF, so that we can strike a balance between the conflicting requirements of increasing average rate and reducing BER by optimizing the switching thresholds in Section 4, for both schemes. Toward this goal, we will use the following BER approximation for Gray-coded square M-QAM over AWGN channels [7]

$$BER(M, \gamma) \approx 0.2\ e^{-\frac{3\gamma}{2(M-1)}}, \tag{3}$$

which is accurate to within $1dB$ for $M \geq 4$ and $0 \leq \gamma \leq 30dB$. As verified in [5], (3) yields a close approximation even for the exact average BER (under expectation).

3.1. Average BER Conditioned on the Outdated Feedback

The BS has knowledge of the outdated $\hat{\mathbf{H}}$ while the channel is actually $\mathbf{H}$. We assume that $h_{i,j}$ is correlated with its τ delayed version, $\hat{h}_{i,j}$ by a correlation coefficient $\rho := J_0(2\pi f_d \tau)$, where $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind, and f_d is the Doppler spread of the selected best user. Investigating the impact of feedback delay requires the conditional pdf of the best user's SNR given its estimate from $\hat{\mathbf{H}}$. We should note that we are determining the conditional pdf of the current fading for a particular (the best) user based on the outdated channel SNR for that *same* user so that the conditional pdf involves two random variables always corresponding to the same user.

STBC: For STBC, the conditional pdf of the best user's instantaneous SNR given its estimate $p_{\gamma|\hat{\gamma}}^{STBC}(\gamma|\hat{\gamma})$ can be derived as

$$\frac{N_t}{\bar{\gamma}(1-\rho^2)}\left(\frac{\gamma}{\rho^2\hat{\gamma}}\right)^{\frac{L}{2}} \exp\left(-\frac{N_t(\rho^2\hat{\gamma}+\gamma)}{\bar{\gamma}(1-\rho^2)}\right) \mathrm{I}_L\left(\frac{2N_t\rho\sqrt{\hat{\gamma}\gamma}}{\bar{\gamma}(1-\rho^2)}\right), \tag{4}$$

where $\hat{\gamma}, \gamma \geq 0$, $L := N_t N_r - 1$, and $\mathrm{I}_a(\cdot)$ is the ath-order modified Bessel function of the first kind. The average BER, given the outdated SNR $\hat{\gamma}$ due to feedback delay can be expressed as

$$\widehat{BER}^{STBC}(M, \hat{\gamma}, \rho) := E_{\gamma|\hat{\gamma}}^{STBC}\left[BER(M, \gamma)\right]. \tag{5}$$

Inserting (3) and (4) into (5), we obtain:

$$\widehat{BER}^{STBC}(M, \hat{\gamma}, \rho) = c_1^{STBC}(M, \rho) e^{-c_2^{STBC}(M,\rho)\hat{\gamma}}, \tag{6}$$

where $c_1^{STBC}(M,\rho) = 0.2\left(\frac{2(M-1)}{3\bar{\gamma}(1-\rho^2)/N_t+2(M-1)}\right)^{N_tN_r}$, and $c_2^{STBC}(M,\rho) = \frac{3\rho^2}{3\bar{\gamma}(1-\rho^2)/N_t+2(M-1)}$. When $\rho = 1$, (6) becomes (3) for AWGN channels, which is the case with zero feedback delay.

BF: Now we take a look at adaptive BF. Based on the estimate $\hat{\mathbf{H}}$, the true channel matrix can be expressed as

$$\mathbf{H} = \rho\hat{\mathbf{H}} + \mathbf{\Xi}, \tag{7}$$

where each element of $\mathbf{\Xi}$ has zero mean and variance $\sigma_{\mathbf{\Xi}}^2 = 1-\rho^2$. The average BER, given the outdated SNR $\hat{\gamma}$ can be evaluated as

$$\widehat{BER}^{BF}(M,\hat{\gamma},\rho) := E_{\mathbf{H}|\hat{\mathbf{H}}}\left[BER(M,\mathbf{H},\hat{\mathbf{H}})\right], \tag{8}$$

where substituting γ^{BF} into (3), we can represent $BER(M,\mathbf{H},\hat{\mathbf{H}}) \approx 0.2\exp\left(-\frac{3\bar{\gamma}\mathbf{b}^H\mathbf{H}^H\mathbf{H}\mathbf{b}}{2(M-1)}\right)$. From (7), we can see that, conditioned on $\hat{\mathbf{H}}$, $\mathbf{H}\mathbf{b} \sim \mathcal{CN}(\rho\hat{\mathbf{H}}\mathbf{b}, \sigma_{\mathbf{\Xi}}^2\mathbf{I}_{\mathbf{N_r}})$. Then (8) can be evaluated by taking expectation of $BER(M,\mathbf{H},\hat{\mathbf{H}})$ in terms of the distribution of $\mathbf{H}\mathbf{b}$ [3]

$$\widehat{BER}^{BF}(M,\hat{\gamma},\rho) = c_1^{BF}(M,\rho)e^{-c_2^{BF}(M,\rho)\hat{\gamma}}, \tag{9}$$

where $c_1^{BF}(M,\rho) = 0.2\left(\frac{2(M-1)}{3\bar{\gamma}(1-\rho^2)+2(M-1)}\right)^{N_r}$, and $c_2^{BF}(M,\rho) = \frac{3\rho^2}{3\bar{\gamma}(1-\rho^2)+2(M-1)}$.

Comparing (6) and (9), we observe that BF is more sensitive to delay as it becomes large enough.

3.2. Average Rate and BER

In what follows we derive the average sum rate and BER expressions for our scheme. Recall that given a set of switching thresholds $\mathbf{t} = [t_0,...,t_J]$, the constellation size M_j is selected and used for transmission whenever $t_j \le \hat{\gamma} < t_{j+1}$. A *generic* set of switching thresholds $\mathbf{t}^g$ is defined as the value of γ at which the instantaneous BER achieves the target BER, $\overline{BER}_0$, for any realization of channel when there is no feedback delay. More precisely, using (3), t_j^g satisfies $t_j^g = -2(M_j-1)\ln\left(5\overline{BER}_0\right)/3$, where $j = 1,...,J-1$, and $t_0 = 0$, $t_J = \infty$. When the Doppler spread and feedback delay are not known at the BS, generic switching thresholds can be used.

The average Bit Per Symbol (BPS) sum rate $\bar{R}$ is formulated as the sum rate of the individual M-QAM constellations weighted by their probability, which depends on the set of thresholds $\mathbf{t}$:

$$\bar{R}(\mathbf{t}) = \sum_{j=0}^{J-1}\int_{t_j}^{t_{j+1}} R_j p_{\hat{\gamma}}^{max}(y)dy,$$

where $R_j := \log_2 M_j$. For adaptive STBC, the average BPS sum rate is denoted by $\bar{R}^{STBC}(\mathbf{t})$ and given by

$$\sum_{j=0}^{J-1}R_j\left[\left[1-\frac{\Gamma\left(N_tN_r,t_{j+1}N_t/\bar{\gamma}\right)}{\Gamma(N_tN_r)}\right]^K-\left[1-\frac{\Gamma\left(N_tN_r,t_jN_t/\bar{\gamma}\right)}{\Gamma(N_tN_r)}\right]^K\right],$$

while for BF with $N_r = 1$ receive antenna, the average sum rate $\bar{R}^{BF}(\mathbf{t})$ is given by

$$\sum_{j=0}^{J-1}R_j\left[\left[1-\frac{\Gamma\left(N_t,t_{j+1}/\bar{\gamma}\right)}{\Gamma(N_t)}\right]^K-\left[1-\frac{\Gamma\left(N_t,t_j/\bar{\gamma}\right)}{\Gamma(N_t)}\right]^K\right],$$

where $\Gamma(a,x) := \int_x^\infty t^{a-1}e^{-t}dt$ is the upper incomplete gamma function. The same procedure can be extended to cases with more than one receive antenna, although the final expression is more complicated.

The average BER can be expressed as the sum of the BER of individual constellations divided by the average rate, which is

$$\overline{BER}(\mathbf{t}) = \frac{1}{\bar{R}(\mathbf{t})}\sum_{j=0}^{J-1}R_j\int_{t_j}^{t_{j+1}}\widehat{BER}(M_j,y,\rho)p_{\hat{\gamma}}^{max}(y)dy. \tag{10}$$

To obtain a simple accurate closed-form expression for the average BER, through asymptotic analysis [9], it can be shown that in the limit of large number of users K, $p_{\hat{\gamma}}^{max}(y)$ will converge to the Gumbel distribution and the asymptotic average BER for both schemes can be approximated as

$$\frac{1}{\bar{R}(\mathbf{t})}\sum_{j=0}^{J-1}R_jc_1(M_j,\rho)\exp(-c_2(M_j,\rho)a_K)\left[\Gamma\left(c_2(M_j,\rho)b_K+1,\exp\left(-\frac{t_{j+1}-a_K}{b_K}\right)\right)-\Gamma\left(c_2(M_j,\rho)b_K+1,\exp\left(-\frac{t_j-a_K}{b_K}\right)\right)\right],$$

where $a_K = F_{\hat{\gamma}_k}^{-1}\left(1-\frac{1}{K}\right)$, and $b_K = F_{\hat{\gamma}_k}^{-1}\left(1-\frac{1}{Ke}\right) - a_K$.

To fairly evaluate the STBC scheme with different antennas, we are interested in the spectral efficiency, which is defined as $\mathcal{R}_{N_t}^{STBC} = \bar{R}^{STBC}\cdot\eta_{N_t}$, where η_{N_t} denotes the data rate of STBC with N_t transmit antennas, e.g. $\eta_{N_t} = 1$ for $N_t = 1,2$, $\eta_{N_t} = 3/4$ for $N_t = 3,4$ [2]. For BF, the spectral efficiency is given by $\mathcal{R}_{N_t}^{BF} = \bar{R}^{BF}$.

4. OPTIMAL THRESHOLD DESIGN

The generic thresholds defined in the beginning Section 3.2, are designed to maintain the instantaneous BER, $BER(M,\gamma)$, below the target BER for all channel realizations when there is no feedback delay. On the other hand, when there is feedback delay, the BER might go beyond the target BER. Since there is a tradeoff between the average BER and average rate for adaptive modulation systems, it is possible to optimize $\mathbf{t}$ to maximize the average rate subject to an average BER constraint. Therefore, assuming the Doppler spread is known at the BS, we can find the optimal set of thresholds, $\mathbf{t}^{opt}$, that maximizes the average rate $\bar{R}(\mathbf{t})$ subject to:

$$\overline{BER}(\mathbf{t}) \le \overline{BER}_0 \tag{11}$$

using a standard Lagrange multiplier approach. Denoting $P_{BER}(\mathbf{t}) := \bar{R}(\mathbf{t})\overline{BER}(\mathbf{t})$, the average BER constraint can be represented equivalently as $P_{BER}(\mathbf{t}) \le \bar{R}(\mathbf{t})\overline{BER}_0$.

Since we assume that the boundary thresholds are fixed to $t_0 = 0$ and $t_J = \infty$, we are concerned with a $J-1$ dimensional optimization problem. Using a Lagrange multiplier λ, the $J-1$ dimensional optimization will now be shown to be converted into a one dimensional optimization problem. The cost function is given by

$$\Phi_{ave}(\mathbf{t}) := \bar{R}(\mathbf{t}) + \lambda(P_{BER}(\mathbf{t}) - \bar{R}(\mathbf{t})\overline{BER}_0). \tag{12}$$

Taking the derivatives of (12) to t_j and equating to zero, we obtain the following relationship, which relates all thresholds $t_j, j = 2,...,J-1$ to t_1:

$$\frac{R_j\widehat{BER}(M_j,t_j,\rho) - R_{j-1}\widehat{BER}(M_{j-1},t_j,\rho)}{R_j - R_{j-1}} = \widehat{BER}(M_1,t_1,\rho).$$

Therefore the optimum vector $\mathbf{t}^{opt}$ is completely determined by t_1, which can be found numerically to have the maximum average rate while satisfying the constraint in (11). The computation of optimal switching thresholds is all done off-line and stored in a look-up table.

5. NUMERICAL RESULTS

We now present the numerical results for both adaptive STBC and BF schemes for a multiuser system over Rayleigh flat fading channels. For practical reasons, we assume $\mathcal{M} = \{0, 4, 16, 64, 256\}$. The target BER is set to $\overline{BER}_0 = 10^{-4}$. We consider a multiuser system of $K = 50$ users, adopting STBC or BF, with $N_t = 1, 2, 4$ transmit antennas and $N_r = 1$ receive antenna. The spectral efficiency using optimal switching thresholds with a normalized feedback delay $f_d\tau = 0.01$ is depicted in Fig. 1. Since with perfect channel feedback, BF is the optimal, it outperforms STBC, when feedback delay is small and multiple transmit antennas are employed. We also observe that more transmit antennas yield higher spectral efficiency for BF. However, this becomes inverted using STBC, due to the effect of channel-hardening [1], which is different from the result for single-user systems, where no channel-hardening effect occurs. Similar phenomenon is observed using generic thresholds (not shown), although higher spectral efficiency can be obtained using the optimal thresholds.

We now present the effects of outdated feedback on the achievable performance gains for both STBC and BF schemes. Using the optimal switching thresholds, the spectral efficiency vs the normalized feedback delay is plotted in Fig. 2. With a small feedback delay, the BF scheme outperforms the STBC scheme. However, due to the reduced diversity order of BF, the STBC scheme achieves higher spectral efficiency as feedback delay becomes large enough ($f_d\tau \gg 0$). The same phenomenon is also observed for single-user systems [6]. Further, it is important to point out, as the feedback delay becomes larger, the adaptive M-QAM system using the optimal thresholds converges to a non-adaptive M-QAM system with constellation M_j, which is most often selected to satisfy the BER requirement (e.g. for STBC with $N_t = 4$ transmit antennas, the system converges to a non-adaptive M-QAM system with constellation size 4). We also observed that multiuser diversity brings less sensitivity to feedback delay, although we do not include the figures here due to the space limit. Hence more users result in higher spectral efficiency for either STBC or BF.

6. CONCLUSIONS

Taking into account the channel feedback delay, we have compared the spectral efficiency for STBC and transmit BF in adaptive modulation systems achieving multiuser diversity. We note that the BF scheme is more complex since the beamformer as well as the SNR estimate are fed back while only the SNR estimate is required for STBC. We optimize the thresholds to maximize spectral efficiency subject to an average BER constraint. We observe that multiuser diversity brings less sensitivity to feedback delay. In a practical multiuser diversity system, more transmit antennas bring higher spectral efficiency for BF. But this becomes inverted using STBC, due to the effect of channel-hardening. With a small feedback delay, the BF scheme outperforms the STBC scheme, which is reversed at large delays.

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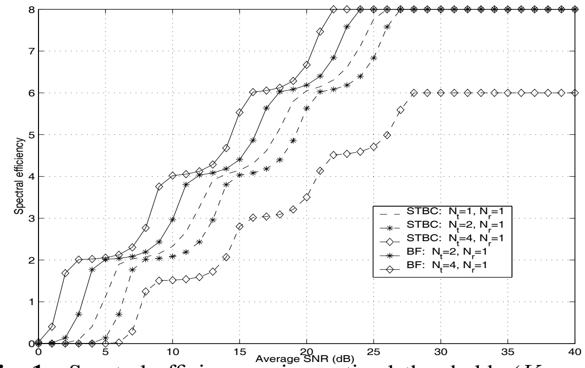

Fig. 1. Spectral efficiency using optimal thresholds ($K = 50$ users, $\overline{BER}_0 = 10^{-4}$, $f_d\tau = 0.01$)

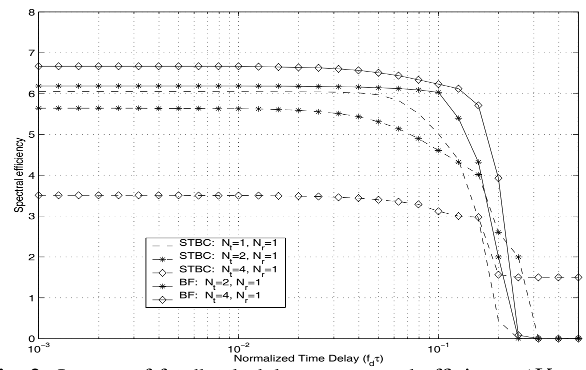

Fig. 2. Impact of feedback delay on spectral efficiency ($K = 50$ users, $\overline{BER}_0 = 10^{-4}$, $\bar{\gamma} = 20dB$, optimal thresholds)

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