COMBINING CIRCULANT SPACE-TIME CODING WITH IFFT/FFT AND SPREADING

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ABSTRACT

Space-time transmit structures for multi-antenna systems have received considerable interest. Circulant structures were among the first space-time coding techniques ever used for multiple-input multiple-output (MIMO) systems due to their simplicity and full rate. The fact that a circulant matrix is diagonalized by the discrete Fourier transformation matrix suggests that the circulant structure can be combined with an inverse fast Fourier transform (IFFT) at the transmitter and a fast Fourier transform (FFT) at the receiver. Using this method, the spatial mixing effect of the MIMO channel is decoupled but the diversity gain is lost. To recover the diversity advantage, we propose to spread the transmitted symbols over the diagonalized channel using the constellation rotation matrix for signal diversity designs. After spreading, every symbol experiences all the components of the frequency counterpart of the channel vector which makes our scheme provide full diversity. The proposed scheme is full rate and can be easily applied to any number of transmit antennas. Our simulation results show that the performance of our scheme is close to the performance of the ideal orthogonal space-time code and much better than the conventional circulant space-time code.

1. INTRODUCTION

Space-time transmit structures are very critical for multi-antenna systems and have attracted extensive research interest. The orthogonal space-time block code (OSTBC) is one of the most important space-time structures. The OSTBC was first introduced in [1] for two transmit antennas and was then extended to a general number of transmit antennas in [2]. The orthogonality in the code enables maximum likelihood (ML) detection based only on linear processing. However, it was shown that there is no complex orthogonal space-time design that provides full diversity and full rate (1 symbol per channel use) for more than two antennas. A rate loss of one-fourth or more is needed to keep the orthogonality and full diversity.

Space-time structures have also been proposed in a variety of other prior works (see for example the discussion and references in [3]). Circulant structures were among the first space-time coding techniques ever used for Multiple-Input Multiple-Output (MIMO) systems due to their simplicity and full rate [4, 5]. It is can be easily applied to any number of transmit antennas without any rate loss and design effort. Due to its special structure, a very important property of circulant matrix is that it can be diagonalized by the Fourier transformation matrix.

In this paper, we first combine the circulant structure with inverse fast Fourier Transform (IFFT) and fast Fourier transform (FFT) to utilize that special property of circulant matrix. By taking the IFFT at the transmitter, sending the circulant matrix through the multiple antenna channel, and taking the FFT at the receiver side, the spatial mixing effect of flat MIMO channel is eliminated. Similar to Orthogonal Frequency Division Multiplexing (OFDM), every symbol is affected by one element of the FFT vector of the channel. But there is a spatial diversity loss because of the same reason that OFDM loses the multipath diversity. To recover the diversity gain, we propose to spread the symbols before IFFT by a spreading matrix. The constellation rotation matrix in the signal diversity designs [6, 7] can be used here as spreading matrix to achieve full diversity. After spreading, every symbol experiences all the frequency coefficients of the channel vector. And the nonzero minimum produce distance property of the spreading matrix used here secures the full diversity property of our scheme. The scheme is always full rate and can be easily extended to any number of tansmit antennas without any design effort. Our simulation results on Quadrature Phase Shift Keying (QPSK) and 16 Quadrature Amplitude Modulation (16QAM) constellations show that the performance of our scheme is close to the performance of ideal orthogonal space-time coding and better than the conventional circulant matrix code scheme.

The outline of the paper is as follows. Section 2 describes the signal model and some space-time block code background. The proposed space-time transmision scheme is discussed in Section 3. The simulation results are presented in Section 4. Section 5 contains a concluding discussion.

2. PRELIMINARIES

2.1. Signal Model

For notational simplicity we consider a Multiple-Input Single-Output (MISO) channel with N_t transmit antennas and one receive antenna; extending the results in this paper to multiple receive antennas is straight forward.

A flat Rayleigh fading MISO channel can be described by a $N_t \times 1$ channel vector $\boldsymbol{h} = [h_1, \dots, h_{N_t}]^T$, where h_n is the fading coefficient between the nth transmit antenna and the receive antenna. We further assume that the channel is slowly time-varying so that $\boldsymbol{h}$ is constant during one code block. The elements of $\boldsymbol{h}$ are independent, identically distributed (i.i.d.) complex Gaussian random variables.

When a $T \times N_t$ space-time block signal $\boldsymbol{X}$ is transmitted, we receive

$$\boldsymbol{y} = \boldsymbol{X}\boldsymbol{h} + \boldsymbol{w}, \tag{1}$$

where T is the number of time slots, $\boldsymbol{y}$ is the $T \times 1$ received signal, and $\boldsymbol{w}$ is zero-mean, white, complex Gaussian noise with variance $N_0/2$ per real and imaginary dimension. The total average transmitted power over a time slot is defined as $E_t = \operatorname{tr}\left(\boldsymbol{X}^H \boldsymbol{X}\right)/T$, where $\operatorname{tr}(\cdot)$ denotes the trace.

2.2. Space-Time Block Code

Most of the space-time code designs can be seen as matrix coding. A block of p symbols $\boldsymbol{s}$ is coded into a $T \times N_t$ space time matrix $\boldsymbol{X}$. The rate of this code is $R = p/T$ symbols per channel use.

Generally a linear complex space-time block code is a linear function of the symbols $s_k, k = 1, \dots, p$, and can be written as

$$\boldsymbol{X} = \sum_{k=1}^{p} \boldsymbol{X}_k(s_k^{(r)}, s_k^{(i)}) = \sum_{k=1}^{p} \boldsymbol{A}_k s_k^{(r)} + \sum_{k=1}^{p} \boldsymbol{B}_k s_k^{(i)} \tag{2}$$

where both $\boldsymbol{A}_k$ and $\boldsymbol{B}_k$ are matrices with complex entries, $s_k^{(r)}$ and $s_k^{(i)}$ are the real and imaginary part of s_k which is from the complex constellation $\mathcal{A}$.

Transmit diversity gain D_t is one of the most important features of a space time code. Under the assumptions given in Section **??**, D_t of a given STBC with maximum likelihood decoding is given as [8]:

$$D_t = \min_{\boldsymbol{s}\neq\boldsymbol{v};\boldsymbol{s},\boldsymbol{v}\in\mathcal{A}^p} \operatorname{rank}(\boldsymbol{X}(\boldsymbol{s}) - \boldsymbol{X}(\boldsymbol{v})). \tag{3}$$

The maximum D_t achievable for a MIMO system with N_t transmit antennas is N_t.

Also the coding gain G of the code is given by the determinant criterion [8]:

$$G = \min_{\boldsymbol{s}\neq\boldsymbol{v};\boldsymbol{s},\boldsymbol{v}\in\mathcal{A}^p} \det(\boldsymbol{X}(\boldsymbol{s}) - \boldsymbol{X}(\boldsymbol{s}))^H(\boldsymbol{X}(\boldsymbol{s}) - \boldsymbol{X}(\boldsymbol{v})) \tag{4}$$

where $\det(\cdot)$ means determinatant.

3. THE SPACE-TIME TRANSMISION SCHEME BASED ON CIRCULANT MATRIX

3.1. Combining Circulant Structure with IFFT/FFT

A $N \times N$ circulant matrix is one having the form

$$\mathcal{C} = \begin{bmatrix} c_0 & c_{N-1} & \dots & c_1 \\ c_1 & c_0 & \dots & c_2 \\ \vdots & \ddots & \ddots & \vdots \\ c_{N-1} & \dots & c_1 & c_0 \end{bmatrix}, \tag{5}$$

where each column is a cyclic shift of the previous column. It is obvious that $\mathcal{C}$ is completely specified by its first column. So by $\mathcal{C}(\boldsymbol{c})$ we denote the circulant matrix whose first column is $\boldsymbol{c}$.

It is known that a circulant matrix can be diagonalized by the Fourier transformation matrix. Therefore, matrix-vector multiplication can be written as

$$\mathcal{C}(\boldsymbol{c})\boldsymbol{v} = \operatorname{ifft}\left(\operatorname{fft}(\boldsymbol{c}) \odot \operatorname{fft}(\boldsymbol{v})\right), \tag{6}$$

where $\operatorname{ifft}(\cdot)$ and $\operatorname{fft}(\cdot)$ denote the IFFT and FFT transform, $\odot$ is the componentwise product of two vectors. Based on that property, we can decouple the spatial mixing effect of multiple transmit antennas channel by combining circulant structure with IFFT/FFT.

First the symbol vector $\boldsymbol{s}$ of length N_t is transformed into $\boldsymbol{f} = \operatorname{ifft}(\boldsymbol{s})$ by IFFT. Then a circulant matrix $X = \mathcal{C}(\boldsymbol{f})$ is constructed from $\boldsymbol{f}$ and transmitted through the channel $\boldsymbol{h}$.

$$\boldsymbol{y} = \boldsymbol{X}\boldsymbol{h} + \boldsymbol{w} \tag{7}$$

Due to the circulant structure of $\boldsymbol{X}$, we have

$$\begin{aligned} \boldsymbol{y} &= \operatorname{ifft}(\operatorname{fft}(\boldsymbol{f}) \odot \operatorname{fft}(\boldsymbol{h})) + \boldsymbol{w} && (8) \\ &= \operatorname{ifft}\left(\boldsymbol{s} \odot \operatorname{fft}(\boldsymbol{h})\right) + \boldsymbol{w}. && (9) \end{aligned}$$

So after the FFT operation on $\boldsymbol{y}$, we get

$$\boldsymbol{z} = \operatorname{fft}(\boldsymbol{y}) = \boldsymbol{s} \odot \operatorname{fft}(\boldsymbol{h}) + \operatorname{fft}(\boldsymbol{w}). \tag{10}$$

Similar to OFDM, every symbol of $\boldsymbol{s}$ fades according to one corresponding element of the FFT vector of the channel.

3.2. The Spreading Matrix

Unfortunately, the transmitting model in (10) suffers a diversity loss for the same reason that OFDM loses multipath diversity. Since every symbol now only experiences one component of the FFT vector of the channel. To recover the diversity gain, we propose to spread the symbols first before the IFFT transform, that is to multiply the symbol vector by a spreading matrix $\boldsymbol{Q}$ which gives

$$\boldsymbol{d} = \rho\boldsymbol{Q}\boldsymbol{s}, \tag{11}$$

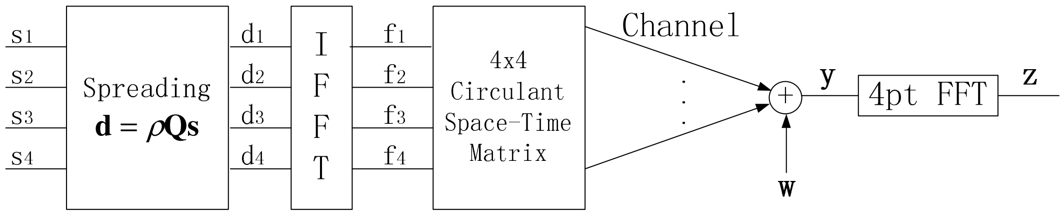

Fig. 1. Block diagram of the proposed space-time transmision scheme for $N_t = 4$

where ρ is the normalization coeffcient that makes the total tranmitting power over a time slot equal to the symbol energy of the constellation E_s i.e., $E_t = E_s$. In Fig. 1 we show a diagram of the proposed space-time transmission scheme for $N_t = 4$.

The transmission relation between $\boldsymbol{s}$ and the received signal $\boldsymbol{z}$ after the FFT changes into following:

$$\begin{aligned} \boldsymbol{z} &= \rho\,\mathrm{diag}(\tilde{\boldsymbol{h}})\boldsymbol{Q}\boldsymbol{s} + \tilde{\boldsymbol{w}} & (12)\\ &= \rho\,\mathrm{diag}(\boldsymbol{Q}\boldsymbol{s})\tilde{\boldsymbol{h}} + \tilde{\boldsymbol{w}} & (13)\end{aligned}$$

where $\tilde{\boldsymbol{h}} = \mathrm{fft}(\boldsymbol{h})$, $\tilde{\boldsymbol{w}} = \mathrm{fft}(\boldsymbol{w})$, and $\mathrm{diag}(\boldsymbol{a})$ denotes the diagonal matrix with vector $\boldsymbol{a}$ on its diagonal line.

The spreading matrix needs to be designed to recover the diversity loss. Due to the fact that i.) $\tilde{\boldsymbol{h}} = \mathrm{fft}(\boldsymbol{h})$ is still an i.i.d. complex Gaussian random vector when h is i.i.d. complex Gaussian, and ii.) the Parseval's Theorem, the rank criterion in [8] can be applied to transmission model (12). The proposed scheme will achieve full diversity when

$$\min_{\boldsymbol{s}\neq\boldsymbol{v}} \mathrm{rank}\{\mathrm{diag}(\boldsymbol{Q}(\boldsymbol{s}-\boldsymbol{v}))\} = N_t. \qquad (14)$$

Since the rank of a diagonal matrix is nothing but the number of nonzero entries along the diagonal, the full diversity requirement for the spreading matrix can be expressed as

$$|\Delta_i^{(\boldsymbol{s},\boldsymbol{v})}| \neq 0, \forall i \in [1,\ldots,N_t], \forall \boldsymbol{s} \neq \boldsymbol{v}; \boldsymbol{s},\boldsymbol{v} \in \mathcal{A}^{N_t}, \quad (15)$$

where $\Delta_i^{(\boldsymbol{s},\boldsymbol{v})}$ is the ith element of $\boldsymbol{\Delta}^{(\boldsymbol{s},\boldsymbol{v})} = [\Delta_1^{(\boldsymbol{s},\boldsymbol{v})},\ldots,\Delta_{N_t}^{(\boldsymbol{s},\boldsymbol{v})}]^T = \boldsymbol{Q}(\boldsymbol{s}-\boldsymbol{v})$.

In fact, this criterion is equivalent to the *nonzero minimum product distance criterion* in signal space diversity designs [6, 7] due to statistical and energy equivalence of $\tilde{\boldsymbol{h}}$ and $\boldsymbol{h}$. Therefore precoding or constellation rotation matrices proposed for those problems can be used as the spreading matrix achieving full diversity in our scheme.

Here, we use the unitary precoding matrix proposed in [7] as the spreading matrix

$$\boldsymbol{Q} = \boldsymbol{F}_{N_t}^T \,\mathrm{diag}\left(1, \alpha, \ldots, \alpha^{N_t-1}\right), \qquad (16)$$

where $\boldsymbol{F}_{N_t}$ is the FFT matrix with (m,n)st entry given by $N_t^{-1/2}\exp(j2\pi(m-1)(n-1)/N_t)$. When N_t is a power of 2, α is selected as $\alpha = \exp(j\pi/(2N_t))$. For any N_t which is not a power of 2, α is selected such that the minimal polynomial of α over the algebraic field $\mathbb{Q}(j)$ has degree greater than or equal to N_t. For examples, when $N_t = 3$, α is selected to be $\exp(j\pi/9)$; when $N_t = 5$, α is selected to be $\exp(j\pi/25)$. This design meets the full diversity criterion regardless of the constellation and can be constructed for any value of N_t without design effort [7].

It can be seen that the resulting scheme can be easily applied to any number of transmit antennas. The proposed scheme also achieves full diversity without rate loss.

3.3. Symbol Detection at the Receiver

The transmission model between $\boldsymbol{s}$ and the received signal after the FFT can be expressed as follows:

$$\boldsymbol{z} = \mathrm{diag}(\tilde{\boldsymbol{h}})\boldsymbol{Q}\boldsymbol{s} + \tilde{\boldsymbol{w}}. \qquad (17)$$

According to this model and assuming perfect channel knowledge, there are several methods for symbol detection at the receiver side. Maximum likelihood decoding can be employed to detect $\boldsymbol{s}$ optimally, but the detection complexity increases exponentially with N_t.

Alternatively, the sphere decoding algorithm [9] can be applied to achieve near-optimium performance. Due to the spreading nature of our scheme, the parallel interference cancellation in CDMA can also be used to do the detection, but has worse performance because of error propagation.

4. SIMULATION RESULTS

In this section, we provide simulation results for the proposed scheme and compare it with the ideal OSTBC, the DAST code [10], and the circulant code [5]. In all simulations, we assume the channel to be quasistatic as previously mentioned and i.i.d. complex Gaussian with variance one.

In the first simulation, we consider the case with $N_t = 3$ transmit antennas, one receive antenna, and QPSK constellation. The symbol error rate (SER) performances of the proposed scheme using ML decoding is shown in Fig. 2. We also plot the performance of ideal OSTBC which can be seen as a performance bound because no full-rate complex orthogonal design exists for $N_t > 2$. Our performance is much better than the conventional circulant code and very close to the ideal OSTBC.

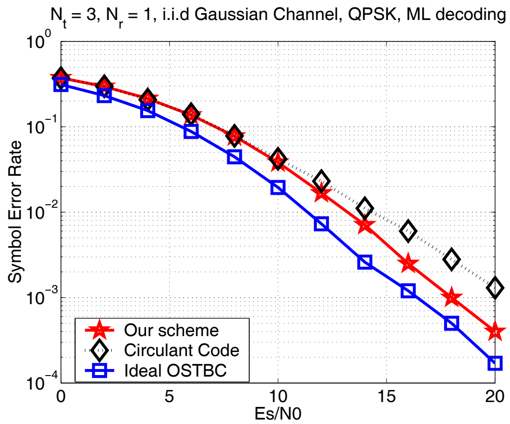

Fig. 2. Symbol error rate vs E_s/N_0 for $N_t = 3$, QPSK

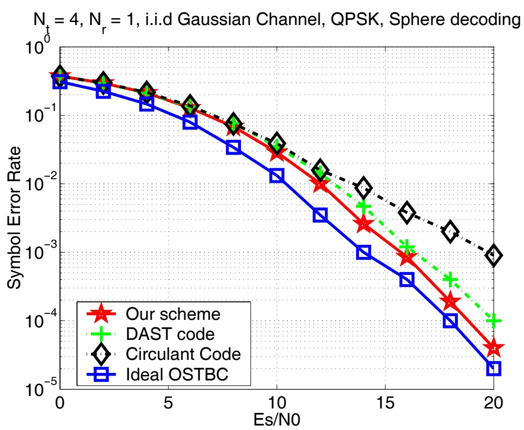

Fig. 3. Symbol error rate vs E_s/N_0 for $N_t = 4$, QPSK

Fig. 3 provides simulation results for $N_t = 4$ and QPSK constellation. The results for $N_t = 4$ and 16QAM constellation is shown in Fig. 4. It is also compared with another full rate DAST code. In all these simulations our scheme has very close performance to ideal OSTBC.

5. CONCLUSION

In this paper, we proposed to combine the circulant matrix structure with IFFT/FFT and spreading for space-time transmission in MIMO. The proposed scheme is rate one with full transmit diversity and can be easily applied to any number of transmit antenna. Simulation results on QPSK and 16QAM show that the performance of our scheme is close to the performance of ideal OSTBC and outperforms the conventional circulant space-time code.

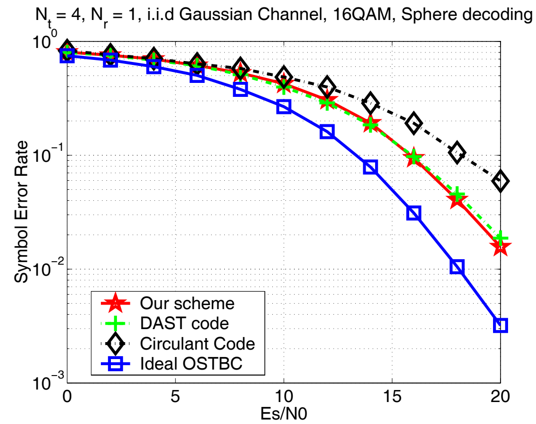

Fig. 4. Symbol error rate vs E_s/N_0 for $N_t = 4$, QAM

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