SPACE-TIME CODING OVER MIMO CHANNELS WITH IMPULSIVE NOISE

Ping Gao and Cihan Tepedelenlioğlu

Arizona State University Dept. of Electrical Engineering {gaoping,cihan}@asu.edu Tel: (480)965-6623

ABSTRACT

Impulsive, non-Gaussian noise is prevalent in wireless environments. We adopt an impulsive noise model over multi-input-multioutput (MIMO) channels and discuss receiver design and code design over MIMO channels with impulsive noise. We derive the code design criterion for the so-called genie-aided receiver and suboptimal receiver, both of which yield criteria identical to [11] when the noise is not impulsive. We propose a maximum a posteriori (MAP) receiver, whose performance can be tightly lower bounded by the genie-aided receiver. We simplify the optimal MAP receiver by an approximation and show that for all cases of practical interest, only 2 terms in the approximation is enough to get near-optimal performance.

1. INTRODUCTION

Impulsive, non-Gaussian noise is prevalent in many communication environments due to a variety of sources, such as man-made electromagnetic interference, atmospheric noise, or ignition noise [1, 2]. In such wireless environments, the performance is degraded both by fading, and impulsive noise. To combat fading, antenna arrays are often used, giving rise to MIMO systems. The statistical description of impulsive noise over MIMO systems are considered in [3], where a correlated impulsive noise model is derived for 2 closely spaced antennas. In [4], adaptive diversity receivers are proposed to adapt to the unknown parameters of the noise process, where the impulsive noise is modelled as independent over both space and time. The same model is used in [5], where the performance of DPSK with Equal Gain Combining (EGC) and Selection Combining (SC) over Ricean fading channels is considered. In [6], an adaptive receiver is proposed for correlated non-Gaussian noise. In [8] and [9], the performance of Maximum Ratio Combining (MRC), EGC, SC and Post Detection Combining (PDC) over Rayleigh fading channels are considered under two different impulsive noise models, which were originally proposed in [7].

In this paper, we consider the receiver design for MIMO channels with impulsive noise, for which we introduce a simplified impulsive noise model by adopting a specific noise correlation in space and in time for the Class A model of Middleton [1, 2]. Using this model, we derive code design criteria for space-time coded systems over impulsive noise channels for the first time in the literature and show that the code design criteria are the same as the Gaussian noise case. The paper is organized as follows. In Section 2, we propose the impulsive noise model for MIMO channels. In Section 3, we derive the code design criterion for both a genieaided receiver and suboptimal receiver, where the former is robust to impulsive noise while impossible to implement, and the latter is easy to implement but vulnerable to impulsive noise. Hence, in Section 4, we consider the MAP receiver, whose performance is very close to the genie-aided receiver while not requiring the knowledge of conditional noise variances at each time instance, as the genie-aided receiver does. To reduce its high computational complexity, we simplify the MAP receiver by an approximation, and we show that for all practical purposes, 2 terms in the approximation are enough to get a near-optimal performance.

2. SYSTEM MODEL

We consider a wireless communication system where the basestation is equipped with N_t transmitting antennas and the mobile is equipped with N_r receiving antennas. We consider the following MIMO flat-fading channel model:

$$\mathbf{Y} = \sqrt{\rho} \mathbf{H} \mathbf{S} + \mathbf{W},\tag{1}$$

where **Y** is the $N_r \times T_s$ matrix of received signals, and T_s is the length of the transmitted data block; **H** is an $N_r \times N_t$ matrix, with independent and identical distributed (iid) complex Gaussian entries with mean zero and variance 1; the average SNR per receiving antenna is denoted by ρ ; **S** is the $N_t \times T_s$ transmitted data block; **W** is the $N_r \times T_s$ additive impulsive noise matrix, with a distribution that will soon be specified.

We next derive the impulsive noise model. Since the impulsive nature of the noise is due to the presence of interference from various sources, we first make the following assumptions on the interference using some ideas from [10]: 1) the distance between any pair of receiving antennas is small compared to the distance from the interfering source to the receiving antenna. This means that the distance from a specific interfering source to each antenna is approximately the same; 2) each receiving antenna receives interference from the same interfering sources. Based on these two assumptions, the instantaneous interference at each antenna is approximately the same. We further assume that the received interference plus a background Gaussian noise at the l^{th} antenna is given by $w_l(t) = w_{l,q}(t) + w_{l,i}(t)$, where $w_{l,q}(t)$ is the background white Gaussian noise component, and $w_{l,i}(t)$ is the received interference, which is the impulsive component and assumed independent of $w_{l,q}(t)$. The average power ratio of the Gaussian component to the impulsive component is denoted by T. The impulsive component $w_{l,i}(t)$ can be expressed as a series of impulses $w_{l,i}(t) := \sum_{k=-\infty}^{\infty} C_k \delta(t-t_k)$, where C_k are iid

The work in this paper was supported by NSF CAREER grant No. CCR-0133841.

Gaussian random variables with zero mean, and t_k is assumed to be generated by a Poisson point process with average rate λ .

The entries of **W** in (1) are the output of a matched filter (MF) with sampling frequency $1/T_b$. In order to get a simple and tractable model for the impulsive noise, we assume that the MF has a rectangular and time limited impulse response with a support of T_b . This means that the received signal is integrated over non-overlapped intervals, hence the noise at the output of the MF are independent over time [5]. Let $w_l[j] := [\mathbf{W}]_{l,j}$ be the j^{th} noise sample for the l^{th} antenna. The pdf of $w_l[j]$ can be written as the canonical Middleton's Class A model as,

$$p(w_l[j]) = \sum_{m=0}^{\infty} \frac{\alpha_m}{\pi \sigma_m^2} \exp\left(-\frac{|w_l[j]|^2}{2\sigma_m^2}\right),$$
(2)

where $A := \lambda T_b$ is the average number of impulses during a length T_b interval, and the number of impulses during the j^{th} length T_b interval, P_j , is a Poisson distributed random variable with parameter A, also called impulsive index; $\alpha_m := \exp(-A)A^m/m!$ is the probability that P_j takes the value of m; and $\sigma_m^2 := \sigma^2(m/A + T)/(1 + T)$. Note that even though (2) is not Gaussian, it can be viewed as conditional Gaussian, which means that $w_l[j]$ conditioned on the Poisson random variable P_j is Gaussian with mean zero, and conditional variance $v_{l,j} := \operatorname{var}(w_l[j]|P_j)$.

We have so far specified the pdf of each noise sample in (2). To set a full description of \mathbf{W} , we use the fact that all receiving antennas receive the same interference at any given time, but the interference is independent across time. This means that $v_{l,j} = v_{k,j} \forall l, k$, and also $v_{l,j}$ is independent of $v_{l,i}$ when $i \neq j$. We can then summarize our model as *dependent in space and independent in time*. Let $\mathbf{w}_j := [w_1[j], w_2[j], ..., w_{N_r}[j]]^T$ be the j^{th} column of \mathbf{W} , we can write their joint pdf as,

$$p(\mathbf{w}_j) = \sum_{m=0}^{\infty} \frac{\alpha_m}{\pi^{N_r} \sigma_m^{2N_r}} \exp\left(-\frac{\sum_{l=1}^{N_r} |w_l[j]|^2}{\sigma_m^2}\right).$$
 (3)

where $w_1[j], w_2[j], ..., w_{N_r}[j]$ are statistically dependent but uncorrelated [7]. Recall that during one data block, the noise samples $\mathbf{W} := [\mathbf{w}_0, \mathbf{w}_1, ..., \mathbf{w}_{T_s-1}]$, with pdf

$$p(\mathbf{W}) = \prod_{j=0}^{T_s-1} \sum_{m=0}^{+\infty} \frac{\alpha_m}{\pi^{N_r} \sigma_m^{2N_r}} \exp\left(-\frac{||\mathbf{w}_j||^2}{\sigma_m^2}\right).$$
 (4)

From the above analysis, we conclude that the conditional variances at any time j are not dependent on the antenna, $v_j := v_{l,j} = \sigma^2 (P_j/A + T)/(1 + T)$, $j = 0, ..., T_s - 1$, and for different times j, v_j are independent.

3. CODE DESIGN CRITERIA

Now that we have specified the impulsive noise model that we will adopt, we proceed to design the receiver and the space-time code to combat the fading and impulsive noise. We assume throughout that **H** is known at the receiver, and start with the genie-aided receiver for which the conditional noise variances are assumed known at the receiver. Since this assumption amounts to knowing how many impulsive interferers there are at a given interval, in practice, it is impossible to implement the genie-aided receiver. However, the performance of the genie-aided receiver is mathematically tractable and can serve as a benchmark for the performance of any realistic receiver that does not have the knowledge of conditional variances. In particular, as we will show in Section 4, the performance of MAP receiver is tightly lower bounded by the genie-aided receiver, which also motivates us to analyze the performance of genie-aided receiver.

3.1. Genie-aided Receiver

Since the receiver has the knowledge of the conditional variances $v_0, v_1,..., v_{T_s-1}$ at each time instance, we can normalize it with the conditional variances at both sides of the equation to make the normalized noise variance to be 1. This is equivalent to right multiplying (1) by a diagonal matrix $\mathbf{V} := \text{diag}[1/\sqrt{v_0}, 1/\sqrt{v_1}, ..., 1/\sqrt{v_{T_s-1}}]$. Hence we can rewrite (1) as

$$\mathbf{Y}\mathbf{V} = \sqrt{\rho}\mathbf{H}\mathbf{S}\mathbf{V} + \mathbf{W}\mathbf{V}.$$
 (5)

Since **WV** is now Gaussian, the optimal decision rule is to minimize the Euclidean distance:

$$\hat{\mathbf{S}} = \underset{\mathbf{S}}{\operatorname{argmin}} ||\mathbf{Y}\mathbf{V} - \sqrt{\rho}\mathbf{H}\mathbf{S}\mathbf{V}||^2.$$
(6)

Consider the PEP that the transmitted signal ${\bf S}$ is decoded as ${\bf S}',$

$$P(\mathbf{S} \to \mathbf{S}' | \mathbf{H}, \mathbf{V}) = Q\left(\rho \sqrt{\frac{||\mathbf{H}(\mathbf{S} - \mathbf{S}')\mathbf{V}||^2}{2}}\right), \quad (7)$$

which can be upper bounded using the Chernoff bound,

$$P(\mathbf{S} \to \mathbf{S}' | \mathbf{H}, \mathbf{V}) \le \exp\left(-\rho \frac{||\mathbf{H}(\mathbf{S} - \mathbf{S}')\mathbf{V}||^2}{4}\right).$$
 (8)

Since the elements of **H** are i.i.d Gaussian distributed random variables, taking expectation with respect to **H**, we get

$$E_{\mathbf{H}}P(\mathbf{S} \to \mathbf{S}' | \mathbf{H}, \mathbf{V}) \le \prod_{i=1}^{N_t} \left(\frac{1}{1 + \lambda_i \frac{\rho}{4}}\right)^{N_r},$$
 (9)

where λ_i is the *i*th eigenvalue of $\mathbf{B} := (\mathbf{S} - \mathbf{S}')\mathbf{V}\mathbf{V}^H(\mathbf{S} - \mathbf{S}')^H$. Ignoring the 1 term in the denominator, (9) can be further upper bounded by

$$E_{\mathbf{H}}P(\mathbf{S}\to\mathbf{S}'|\mathbf{H},\mathbf{V}) \le \prod_{i=1}^{r} \lambda_i^{-N_r} \left(\frac{\rho}{4}\right)^{-rN_r},$$
 (10)

where r is the rank of **B**. From (10), we can see that to obtain full diversity of $N_t N_r$, we need **B** to be a full rank matrix. Since **V** is diagonal with nonzero diagonal elements, if $\mathbf{S} - \mathbf{S}'$ is full row rank, then so is **B**. So the code design criterion turns out to be the same as [11]: to achieve the maximum diversity of $N_t N_r$, we require $\mathbf{S} - \mathbf{S}'$ to be full row rank for any pair of codewords **S** and \mathbf{S}' .

We note that if $\mathbf{S} - \mathbf{S}'$ is unitary, the eigenvalue λ_i of \mathbf{B} is given by the diagonal elements of $\mathbf{V}\mathbf{V}^H$, which is $1/v_i$ $(i = 0, ..., T_s - 1)$. In this case, (9) can be written as

$$E_{\mathbf{H}}P(\mathbf{S}\to\mathbf{S}'|\mathbf{H},\mathbf{V}) \le \prod_{i=1}^{N_t} \left(1+\frac{\rho}{4v_i}\right)^{-N_r}.$$
 (11)

When $N_r = 1$, since $\left(1 + \frac{\rho}{4v_i}\right)^{-1}$ is a convex function of v_i , and v_i are independent for different *i*, taking expectation with respect

to v_i , we get,

$$E_{\mathbf{H},\mathbf{V}}P(\mathbf{S}\to\mathbf{S}'|\mathbf{H},\mathbf{V}) \leq E_{v_i}\prod_{i=1}^{N_t} \left(1+\frac{\rho}{4v_i}\right)^{-1}$$
 (12)

$$\leq \prod_{i=1}^{N_t} \left(1 + \frac{\rho}{4E[v_i]} \right)^{-1} = \prod_{i=1}^{N_t} \left(1 + \frac{\rho}{4\sigma^2} \right)^{-1}, \quad (13)$$

where we used Jensen's inequality and $E[v_i] = \sigma^2$. The righthand side of (13) is exactly the PEP upper bound for the Gaussian noise case. We see that the PEP upper bound for the impulsive noise case in (12) is smaller than that for the Gaussian noise case, which implies that the performance of the genie-aided receiver over the impulsive noise channel has a better coding gain than the Gaussian noise case. This is not surprising since the correlation of the impulsive noise in space is removed when only 1 receiving antenna is used. Moreover, the genie-aided receiver utilizes the impulsive structure to decode transmitted symbols, hence it has a better coding gain than Gaussian noise channel.

Considering now (10), and taking expectation with respect to \mathbf{V} , we get

$$E_{\mathbf{H},\mathbf{V}}P(\mathbf{S}\to\mathbf{S}'|\mathbf{H},\mathbf{V}) \le \left(\frac{\rho}{4\sigma^2}\right)^{-N_tN_r},$$
 (14)

where we use the properties that v_i are independent and $E[v_i] = \sigma^2$. Note that (14) is independent of A or T and is the same as the PEP upper bound at high SNR for the Gaussian noise case, which implies that for the genie-aided receiver, the PEP will approach to the Gaussian noise case at high SNR.

One special case which satisfies that $\mathbf{S} - \mathbf{S}'$ is unitary is Alamouti's code: the 2 × 2 orthogonal space-time block code. Given $N_r = 1$, $N_t = 2$, using Alamouti's code and BPSK constellation, we show in Fig. 1 the performance of the genie-aided receiver for two different channels. We use $(A, T) = (10^{-4}, 0.1)$ to represent highly impulsive channel and (A, T) = (1, 0.1) to represent near-Gaussian channel. We also show the upper bound in (12) for the highly impulsive channel, and the upper bound in (13) for the Gaussian noise channel. We see that (12) is smaller than (13), and at high SNR, (12) approaches to (13). This corroborates our analysis that the genie-aided receiver for impulsive noise channel outperforms that over Gaussian noise channel due to the better coding gain. Also at high SNR, they approach to each other, as expected.

Although the genie-aided receiver has a better performance over impulsive noise than Gaussian noise channels, it is not practical to estimate the conditional noise variances at each time instance, which motivates us to look for other receivers.

3.2. Suboptimal Receiver

We would now like to discuss the receiver that is optimal over Gaussian noise channels, which is given by,

$$\hat{\mathbf{S}} = \underset{\mathbf{S}}{\operatorname{argmin}} ||\mathbf{Y} - \sqrt{\rho} \mathbf{H} \mathbf{S}||^2 = \underset{\mathbf{s}_t}{\operatorname{argmin}} \sum_{t=0}^{T_s - 1} ||\mathbf{y}_t - \sqrt{\rho} \mathbf{H} \mathbf{s}_t||^2,$$
(15)

where \mathbf{y}_t and \mathbf{s}_t are the t^{th} columns of \mathbf{Y} and \mathbf{S} respectively. This receiver is suboptimal over impulsive noise channels, we will now derive PEP upper bound and the code design criterion for the suboptimal receiver. Since the conditional variances v_t , $(t = 0, ..., T_s - 1)$ are independent for different t, we get

$$P(\mathbf{S} \to \mathbf{S}' | \mathbf{H}, \mathbf{V}) = \prod_{t=0}^{T_s - 1} Q\left(\sqrt{\frac{\rho ||\mathbf{H}(\mathbf{s}_t - \mathbf{s}'_{t-1})||^2}{2v_t}}\right)$$



Fig. 1. Genie-aided receiver vs. suboptimal receiver over a highly impulsive channel and a near-Gaussian channel

$$\leq \exp\left(-\sum_{t=0}^{T_s-1} \frac{\rho ||\mathbf{H}(\mathbf{s}_t - \mathbf{s}'_t)||^2}{4v_t}\right) \leq \exp\left(-\frac{\rho ||\mathbf{H}(\mathbf{S} - \mathbf{S}')||^2}{4v_{max}}\right)$$
(16)

where v_{max} is the maximum conditional variance among $v_0, v_1, ..., v_{T_s-1}$. Taking expectation with respect to v_{max} , we get

$$E_{\mathbf{V}}P(\mathbf{S} \to \mathbf{S}' | \mathbf{H}, \mathbf{V}) \leq \sum_{m=0}^{\infty} P(v_{max} = \sigma_m^2)$$
$$\cdot \exp\left(-\frac{\rho ||\mathbf{H}(\mathbf{S} - \mathbf{S}')||^2}{4\sigma_m^2}\right), \quad (17)$$

where $P(v_{max} = \sigma_m^2) = \left(\sum_{k=0}^m e^{-A} \frac{A^k}{k!}\right)^{T_s} - \left(\sum_{k=0}^{m-1} e^{-A} \frac{A^k}{k!}\right)^{T_s}$. Using Binomial Theorem, it can be shown that $P(v_{max} = \sigma_m^2) \leq T_s \alpha_m$. Taking expectation with respect to **H**, we get the PEP upper bound as

$$E_{\mathbf{H},\mathbf{V}}P(\mathbf{S}\to\mathbf{S}'|\mathbf{H},\mathbf{V}) \leq \sum_{m=0}^{\infty} T_s \alpha_m \prod_{i=1}^{N_t} \left(\frac{1}{1+\psi_i\left(\frac{\rho}{4\sigma_m^2}\right)}\right)^{N_r}$$
$$\leq \left[\left(\prod_{i=1}^r \psi_i\right)^{-N_r} \left(\frac{\rho}{4\sigma^2}\right)^{-rN_r}\right] \left[T_s \sum_{m=0}^{\infty} \left(\frac{\alpha_m}{\frac{m/A+T}{1+T}}\right)^{-rN_r}\right],(18)$$

where ψ_i and r are the i^{th} eigenvalue and rank of $(\mathbf{S} - \mathbf{S}')(\mathbf{S} - \mathbf{S}')^H$ respectively. To get the highest diversity order, $\mathbf{S} - \mathbf{S}'$ needs to be full row rank, which is the same criterion as before.

From (18), we see that the diversity order of the suboptimal receiver is the same as that of the genie-aided receiver, and the first term is equal to (14) if $\mathbf{S} - \mathbf{S}'$ is unitary. However, the second term is always larger than 1, which can be easily shown using Jensen's inequality. Hence the second term leads to a coding gain loss, which is expected since the suboptimal receiver does not utilize any information about the impulsive noise. In Fig. 1, we show the simulated BER for Alamouti's code using suboptimal receiver over a highly impulsive channel and a near-Gaussian channel. We compare them with the genie-aided receiver. There is a large gap between the genie-aided receiver and the suboptimal receiver. In particular, a flattening of the BER curve between 10 - 30 dB for the suboptimal receiver over highly impulsive channel can be observed. This coding gain loss can also be observed from



Fig. 2. Comparison of MAP receiver using different M over a highly impulsive channel and a near-Gaussian channel

the upper bound for the genie-aided receiver in (12) and the upper bound for the suboptimal receiver in (18). When the channel is near-Gaussian, since the suboptimal receiver is optimal for Gaussian noise, the difference between the genie-aided receiver and the suboptimal receiver is small. This phenomenon indicates that the suboptimal receiver is in fact ineffective in combating highly impulsive noise due to the large coding gain loss, which motivates us to find the receiver that is implementable while combating the impulsive noise effectively.

4. MAP RECEIVER

We now consider the optimal MAP receiver, which is given by

$$\hat{\mathbf{S}} = \underset{\mathbf{S}}{\operatorname{argmax}} \prod_{i=0}^{T_s-1} \sum_{m=0}^{+\infty} \frac{\alpha_m}{\pi^{N_r} \sigma^{2N_r}} \exp\left(-\frac{|\mathbf{y}_i - \mathbf{Hs}_i|^2}{\sigma_m^2}\right). (19)$$

Since (19) involves infinitely many terms, we can approximate it by including only finite many terms. The amount of terms that is enough to approximate (19) needs to be examined. Assume Mfinite terms are added together to approximate the sum in (19), we simulate different values for M = 1, 2, 25, over different channels to see how large M needs to be.

Assume Alamouti's code and BPSK signaling are used over a highly impulsive channel and a near-Gaussian channel. The simulation results are shown in Fig. 2. It is not surprising that M = 1 leads to a poor performance since it is equivalent to the suboptimal receiver given by (15). However, when M = 2, the performance is improved significantly, and very close to M = 25 with only 0.02 dB difference. We also note that the performance of MAP receiver is very close to the genie-aided receiver, hence the genie-aided receiver though unrealistic provides a tight lower bound as the MAP receiver, and can act as a good benchmark for the performance of MAP receiver. For near-Gaussian channel, there is a small difference between M = 1 and M = 25, because the suboptimal receiver (M = 1) is optimal for Gaussian noise case.

To implement the MAP receiver, we need to know A, T and σ^2 , which are deterministic noise parameters, while the genieaided receiver requires v_j ($j = 0, 1, ..., T_s - 1$) at each time instance, which are random variables related to the Poisson points. As for the suboptimal receiver, although it does not require any knowledge of the impulsive noise parameters, it is ineffective in combating highly impulsive noise.

5. CONCLUSION

In this paper, we adopt an impulsive noise model for MIMO channels, and discuss the receiver and code design issues. We derive the coding criterion for the genie-aided receiver and suboptimal receiver, and analyze their performance using the PEP upper bound. The genie-aided receiver turns out to have a much better coding gain than the suboptimal receiver, however, it is difficult to implement. While MAP receiver has a close performance to the genieaided receiver, and it can be simplified by an approximation which only needs 2 terms to get near-optimal performance.

6. REFERENCES

- A. D. Spaulding and D. Middleton, "Optimum reception in an impulsive interference environment - Part I: Coherent Detection," *IEEE Trans. on Commun.* vol. COM-25, no. 9, Sept. 1977, pp. 910-923.
- [2] D. Middleton, "Non-Gaussian noise models in signal processing for telecommunications: new methods and results for class A and class B noise models", *IEEE Trans. on Info. Theory*, vol. 45, no. 4, May 1999, pp. 1129-1149.
- [3] K. F. McDonald and R. S. Blum, "A physically-based noise model for array observations," *Proceedings of the 31st Asilomar conference on signals, systems and computers*, pp. 448-452, Nov. 2-5, 1997.
- [4] R. S. Blum, R. J. Kozick, B. M. Sadler, "An adaptive spatial diversity receiver for non-Gaussian interference and noise," *IEEE Trans. on Signal Proc.* vol. 47, no. 8, Aug. 1999, pp. 2100-2111.
- [5] J. F. Weng and S. H. Leung, "On the performance of DPSK in Rician fading with class A noise," *IEEE Trans. on Vehicular Technology*, vol. 49, no. 5, Sept. 2000, pp. 1934-1949.
- [6] Y. Zhang and R. S. Blum, "An adaptive spatial diversity receiver for correlated non-Gaussian interference and noise," *Conference record of the thirty-second Asilomar conference on signals, systems and computers*, vol. 2: 1-4, November 1998.
- [7] P. A. Delaney, "Signal detection in multivariate class-A interference," *IEEE Trans. on Commun.* vol. 43, no. 2, Feb. 1995, pp. 365-373.
- [8] C. Tepedelenlioglu and P. Gao, "Performance of diversity reception over fading channels with impulsive noise," *Intl. Conf. on Acoustics, Speech, and Signal Processing 2004* (ICASSP '04), vol. 4, 17-21 May 2004 pp. 389 - 392.
- [9] C. Tepedelenlioglu and P. Gao, "Performance of diversity reception over fading channels with impulsive noise," *Globecom 2004*, (to appear).
- [10] Leslie A Berry, "Understanding Middleton's canonical formula for class A noise," *IEEE Trans. on Electromagnetic Compatibility*, vol. EMC-23, no. 4, November 1981.
- [11] V. Tarokh, N. Seshadri and A. R. Calderbank, "Space-time codes for high data rate wireless communication: performance criterion and code construction," *IEEE Trans. on Information Theory*, vol. 44, no. 2, March 1998.