ON THE PERFORMANCE OF SUPER-ORTHOGONAL SPACE-TIME CODES FOR CORRELATED MIMO CHANNELS

Sau-Hsuan Wu and C.-C. Jay Kuo

Department of Electrical Engineering and Integrated Media Systems Center
University of Southern California, Los Angeles, California 90089-2564, USA
E-mail: sauhsuaw@costard.usc.edu, cckuo@sipi.usc.edu

ABSTRACT

The exact expression of pairwise error probabilities (PEPs) for space-time orthogonal codes is derived for spatially and temporally correlated Rayleigh fading channels in this research. We adopt a moment generating function approach in developing a unified framework to facilitate the evaluation of PEPs for various types of channel correlations. Under this new framework, the complexity for evaluating PEPs is greatly simplified. Moreover, the simple expression provides valuable insights into the performance of space-time codes in the presence of correlated fading channels.

1. INTRODUCTION

Space-Time coding (STC) has attracted a significant amount of attention since the pioneering work by Tarokh *et al.* [1]. The notion of super-orthogonal space-time trellis codes (STTCs) was proposed in [2] to provide a wilder set of orthogonal space-time codes originally introduced in [3]. The STC design is primarily based on the rank and the coding gain distance criteria, which are usually obtained from an upper bound of the pairwise error probability (PEP). The reason of using the upper bound is that the resultant simple expression can provide more insights into code designs.

Approaches to evaluate PEPs exactly based on the moment generating function (MGF) have recently been proposed for some STCs, *e.g.* the super-orthogonal (SO)-STTCs in [4]. However, these exact formulas are still too complicated to be useful in code design, and they are only valid for uncorrelated channels. Due to some mathematical difficulties, STC is primarily designed for two extreme cases under fast and slow fading channel assumptions, respectively. For fast fading, channel coefficients are assumed to change from symbol to symbol independently, while, for slow fading, the coefficients remain steady over the entire coding block and change from block to block also independently. These two extreme cases often serve as the upper and the lower performance bounds for STCs in realistic fading channels.

It was shown in [5], among others, that STCs designed for uncorrelated fading channels have severe performance degradation when applied to correlated channels. To provide a higher performance gain, robust STC design for correlated fading channel is necessary. This motivates us to look for an exact expression of PEP for spatially and temporally correlated channels. To the best of our knowledge, no similar formula has been reported, even thought simpler forms were obtained for simplified channel conditions. Here, we investigate this problem using the MGF approach and obtain exact PEPs for spatially and/or temporally correlated Rayleigh fading channels when both transmit and receive antennas employ uniform linear arrays (ULA). The derived expressions of PEPs are even simpler than those given in [4] for uncorrelated channels, thus providing useful insights into the performance of STC over correlated fading channels.

This research has been funded in part by the Integrated Media Systems Center, a National Science Foundation Engineering Research Center, Cooperative Agreement No. EEC-9529152.

2. SPACE-TIME BLOCK CODES AND SYSTEM MODEL

We consider signal constellations for SO-STTC in spatially and temporally (ST) correlated Rayleigh fading channels. In this system, there are L_t complex symbols being transmitted for each time index n, denoted by $\mathbf{x}_n = [x_1(n), \cdots, x_{Lt}(n)]^T$, while there are L_t transmissions taking place at each time index n. As a result, for each transmission in time n, the L_t symbols are sent through L_t transmission antennas in different permutations with different amounts of phase shifts. It was shown in [3] that the full-rate $(L_t \times L_t)$ real orthogonal design is only possible for $L_t = 2, 4, 8$. For example, for $L_t = 2$, the transmitted signals is given by

$$\mathbf{X}_n \triangleq \begin{bmatrix} \mathbf{x}_{n1} & \mathbf{x}_{n2} \end{bmatrix} \triangleq \begin{bmatrix} x_1(n)e^{j\theta(n)} & -x_2(n)e^{j\theta(n)} \\ x_2(n) & x_1(n) \end{bmatrix}. \quad (1)$$

Each column vector of $\mathbf{X}_n$ corresponds to symbols being transmitted in one transmission during time n, and each row corresponds to symbols being transmitted through an antenna. Let $c_{p,q}(n)$ denote the channel coefficient between the pth receive antenna, $p = 1, \cdots, L_r$, and the qth transmit antenna, $q = 1, \cdots, L_t$, at time n, and $\mathbf{c}_p(n) \triangleq [c_{l,1}(n), \cdots, c_{l,L_t}(n)]$ be the channel vector corresponding to the pth receive antenna at time n. The received signal $y_{p,m}(n)$ at the pth antenna for the mth transmission, $m = 1, \cdots, L_t$, in time n is given by

$$y_{p,m}(n) = \mathbf{c}_p(n)\mathbf{x}_{n,m} + n_{p,m}(n),$$

where $n_{p,m}(n) \sim \mathcal{CN}(0, 2N_0)$. We define noise vector $\mathbf{n}_p(n) \triangleq [n_{p,1}(n), \cdots, n_{p,L_t}(n)]$. The received signal for the pth antenna at time n can be represented as

$$\mathbf{y}_p(n) \triangleq [y_{p,1}(n), \cdots, y_{p,L_t}(n)] = \sqrt{\frac{2E_s}{L_t}}\mathbf{c}_p(n)\mathbf{X}_n + \mathbf{n}_p(n). \quad (2)$$

Concatenating the received signals from the L_r antennas, the signal matrix $\mathbf{Y}_n \triangleq [\mathbf{y}_1^H(n), \cdots, \mathbf{y}_{L_r}^H(n)]^H$ is given by

$$\mathbf{Y}_n = \sqrt{\frac{2E_s}{L_t}}\mathbf{C}_n\mathbf{X}_n + \mathbf{N}_n, \quad (3)$$

where $\mathbf{C}_n \triangleq [\mathbf{c}_1^H(n), \cdots, \mathbf{c}_{L_r}^H(n)]^H$ and $\mathbf{N}_n \triangleq [\mathbf{n}_1^H(n), \cdots, \mathbf{n}_{L_r}^H(n)]^H$. Note that, matrices $\mathbf{M}^*$, $\mathbf{M}^T$ and $\mathbf{M}^H$ denote the complex conjugate, transpose and Hermitian of matrix $\mathbf{M}$, respectively.

3. CORRELATED FADING CHANNEL MODEL

For spatially and temporally (ST) uncorrelated channel matrix $\mathbf{C}_n$, the exact PEP formula for SO-STTC codes has been derived for both Rayleigh and Ricean fading channels in [4]. For spatially correlated fading channels, approximate PEP evaluations have also been widely studied, *e.g.*, [5]. In this work, we use the channel model developed in [5] for ULA antennas to evaluate the PEP in fading scattering environments.

Consider one dimension ULA antennas at both the transmitter and the receiver sides, where the far-field assumption applies. The channel matrix can be described via the array steering and response vectors given, respectively, by

$$\mathbf{a}_t(\theta_t) \triangleq [1, e^{j2\pi\theta_t}, \cdots, e^{j2\pi(L_t-1)\theta_t}]^T, \quad (4)$$

$$\mathbf{a}_r(\theta_r) \triangleq [1, e^{j2\pi\theta_r}, \cdots, e^{j2\pi(L_r-1)\theta_r}]^T, \quad (5)$$

where θ represents the angle variable that is related to the physical angle measured with respect to the horizontal axis via $\theta = d\sin(\phi)/\lambda$, and where λ is the wavelength of propagation and d is the antenna spacing. The channel matrix in a multipath scattering environment can be expressed as

$$\mathbf{C}_n = \sum_{l=1}^{L} \alpha_l(n)\mathbf{a}_r(\theta_{r,l})\mathbf{a}_t(\theta_{t,l})^T, \quad (6)$$

where L is the number of paths and $\alpha_l(n)$ is the complex Gaussian distributed channel gain with $E[\alpha_k(n)^*\alpha_l(m)] = \sigma_l^2 J_0(2\pi f_d(m-n))\delta(k-l)$, f_d the normalized Doppler shift and $\sum_{l=1}^{L}\sigma_l^2 = 1/L_r$. The function $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind. For convenience, we express $\mathbf{C}_n$ as

$$\mathbf{C}_n = \mathbf{A}_r\mathbf{D}_n\mathbf{A}_t^T, \quad (7)$$

where $\mathbf{A}_r \triangleq [\mathbf{a}_r(\theta_{r,1}), \cdots, \mathbf{a}_r(\theta_{r,L})]$, $\mathbf{A}_t \triangleq [\mathbf{a}_t(\theta_{t,1}), \cdots, \mathbf{a}_t(\theta_{t,L})]$ and $\mathbf{D}_n \triangleq \mathrm{diag}\{\alpha_1(n), \cdots, \alpha_L(n)\}$.

For the PEP analysis, we need to find the covariance matrix of the concatenated channel vector $\underline{\mathbf{c}}(n) \triangleq [\mathbf{c}_1(n), \cdots, \mathbf{c}_{L_r}(n)]$. By operator $vec(\mathbf{C})$, we stack the columns of $\mathbf{C}$ to form a concatenated vector. Based on the identity $vec(\mathbf{ABC}) = (\mathbf{C}^T \otimes \mathbf{A})vec(\mathbf{B})$, where $\otimes$ is the Kronecker product, we obtain

$$\begin{aligned}\underline{\mathbf{c}}(n) &= vec(\mathbf{C}_n^T)^T = vec(\mathbf{A}_t\mathbf{D}_n\mathbf{A}_r^T)^T \\ &= vec(\mathbf{D}_n)^T(\mathbf{A}_r^T \otimes \mathbf{A}_t^T). \end{aligned} \quad (8)$$

It can be shown that the cross-covariance matrix $\mathbf{R}_{\underline{\mathbf{c}}}(m-n) \triangleq E[\underline{\mathbf{c}}^H(m)\underline{\mathbf{c}}(n)]$ is equal to

$$\begin{aligned}\mathbf{R}_{\underline{\mathbf{c}}} &= (\mathbf{A}_r^* \otimes \mathbf{A}_t^*)E[vec(\mathbf{D}_m^*)vec(\mathbf{D}_n)^T](\mathbf{A}_r^T \otimes \mathbf{A}_t^T) \\ &= R_{m,n}\sum_{l=1}^{L}\sigma_l^2[\mathbf{a}_r^*(\theta_{r,l})\mathbf{a}_r^T(\theta_{r,l})] \otimes [\mathbf{a}_t^*(\theta_{t,l})\mathbf{a}_t^T(\theta_{t,l})] \\ &\triangleq R_{m,n}\Phi_{\underline{\mathbf{c}}}, \end{aligned} \quad (9)$$

where $R_{m,n} \triangleq J_0(2\pi f_d(m-n))$ characterizes the temporal correlation and $\Phi_{\underline{\mathbf{c}}} \triangleq \sum_{l=1}^{L}\sigma_l^2[\mathbf{a}_r^*(\theta_{r,l})\mathbf{a}_r^T(\theta_{r,l})]\otimes[\mathbf{a}_t^*(\theta_{t,l})\mathbf{a}_t^T(\theta_{t,l})]$ forms the spatial correlation matrix of the channel. Note that the rank of the spatial covariance matrix, $\Phi_{\underline{\mathbf{c}}}$ is $\min(L, L_rL_t)$.

4. ANALYSIS OF PAIRWISE ERROR PROBABILITIES

4.1. PEP and Moment Generating Function (MGF)

We use the maximum likelihood (ML) metric to evaluate the PEP. Form (2), the log likelihood of the transmitted codeword $\mathbf{X} \triangleq [\mathbf{x}_1, \cdots, \mathbf{x}_N]$ of length N is given by

$$m(\mathcal{Y}, \mathbf{X}) = \sum_{n=1}^{N}\sum_{p=1}^{L_r}\|\mathbf{y}_p(n) - \sqrt{\frac{2E_s}{L_t}}\mathbf{c}_p(n)\mathbf{X}_n\|^2, \quad (10)$$

where $\mathcal{Y} \triangleq \{\mathbf{Y}_1, \cdots, \mathbf{Y}_n\}$ denotes the set of received signals. Conditioned on $\mathcal{C} \triangleq \{\mathbf{C}_1, \cdots, \mathbf{C}_N\}$, the probability of choosing $\widehat{\mathbf{X}}$ when $\mathbf{X}$ was truly transmitted is given by

$$P(\mathbf{X} \to \widehat{\mathbf{X}}) = Pr\{m(\mathcal{Y}, \mathbf{X}) > m(\mathcal{Y}, \widehat{\mathbf{X}})|\mathcal{C}\}. \quad (11)$$

After some manipulations, the PEP can be shown to be

$$\begin{aligned}&Pr(\mathbf{X} \to \widehat{\mathbf{X}}|\mathcal{C}) = \\ &Pr\left\{\sum_{n=1}^{N}\sum_{p=1}^{L_r}\mathbf{z}_p(n)\begin{bmatrix}\Delta_n\Delta_n^H & \Delta_n \\ \Delta_n^H & \mathbf{0}\end{bmatrix}\mathbf{z}_p^H(n) < 0|\mathcal{C}\right\}, \end{aligned} \quad (12)$$

where $\mathbf{z}_p(n) \triangleq [\sqrt{\frac{2E_s}{L_t}}\mathbf{c}_p(n), \mathbf{n}_p(n)]$ and $\Delta_n \triangleq \mathbf{X}_n - \widehat{\mathbf{X}}_n$. It is clear that vector $\mathbf{z}_p(n)$ is complex Gaussian distributed. The PEP can be further written in matrix form as

$$\begin{aligned}&Pr(\mathbf{X} \to \widehat{\mathbf{X}}|\mathcal{C}) = \\ &Pr\left\{[\sqrt{\tfrac{2E_s}{L_t}}\tilde{\mathbf{c}}, \tilde{\mathbf{n}}]\begin{bmatrix}\widetilde{\Delta}\widetilde{\Delta}^H & \widetilde{\Delta} \\ \widetilde{\Delta}^H & \mathbf{0}\end{bmatrix}\begin{bmatrix}\sqrt{\frac{2E_s}{L_t}}\tilde{\mathbf{c}}^H \\ \tilde{n}^H\end{bmatrix} < 0|\mathcal{C}\right\}, \\ &\triangleq Pr\{Q(\mathcal{C}) < 0|\mathcal{C}\}, \end{aligned} \quad (13)$$

where $\tilde{\mathbf{c}} \triangleq [\underline{\mathbf{c}}(1), \cdots, \underline{\mathbf{c}}(N)]$, $\tilde{\mathbf{n}} \triangleq [\underline{\mathbf{n}}(1), \cdots, \underline{\mathbf{n}}(N)]$, $\underline{\mathbf{n}}(N) \triangleq [\mathbf{n}_1(n), \cdots, \mathbf{n}_{L_r}(n)]$ and

$$\widetilde{\Delta} \triangleq \mathrm{diag}(\mathbf{I}_{L_r} \otimes \Delta_1, \cdots, \mathbf{I}_{L_r} \otimes \Delta_N), \quad (14)$$

where $\mathbf{I}_p$ denotes the identity matrix of $p \times p$.

Therefor, the PEP in (13) is equal to evaluating the probability of $Q(\mathcal{C})$ less than zero. $Q(\mathcal{C})$ has a Gaussian quadratic form, since vector $[\tilde{\mathbf{c}}, \tilde{\mathbf{n}}]$ is zero-mean complex Gaussian distributed. Thus, the PEP can be obtained by evaluating the residues of the moment generating function (MGF) of $Q(\mathcal{C})$ [6], which is given by

$$Pr(\mathbf{X} \to \widehat{\mathbf{X}}) = -Res\sum_{RHP}\frac{1}{s}M_Q(s). \quad (15)$$

To this end, we have to find the covariance matrix of $[\tilde{\mathbf{c}}, \tilde{\mathbf{n}}]$. We will focus on $E(\tilde{\mathbf{c}}^H\tilde{\mathbf{c}})$ only, since $\tilde{\mathbf{c}}$ and $\tilde{\mathbf{n}}$ are uncorrelated and $E(\tilde{\mathbf{n}}^H\tilde{\mathbf{n}}) = 2N_0\mathbf{I}_{NL_rL_t}$. Covariance matrix $\mathbf{R}_{\tilde{c}} \triangleq E(\tilde{\mathbf{c}}^H\tilde{\mathbf{c}})$ is a block matrix with its submatrix on the nth row and the mth column equal to $[\mathbf{R}_{\tilde{\mathbf{c}}}]_{n,m} \triangleq \mathbf{R}_{\underline{\mathbf{c}}}(m-n) = E[\underline{\mathbf{c}}^H(m)\underline{\mathbf{c}}(n)]$. From (9), we have $[\mathbf{R}_{\tilde{\mathbf{c}}}]_{n,m} = R_{m,n}\Phi_{\underline{\mathbf{c}}}$. Thus,

$$\mathbf{R}_{\tilde{\mathbf{c}}} = \begin{pmatrix}R_{1,1}\Phi_{\underline{\mathbf{c}}} & \cdots & R_{1,N}\Phi_{\underline{\mathbf{c}}} \\ \vdots & \ddots & \vdots \\ R_{N,1}\Phi_{\underline{\mathbf{c}}} & \cdots & R_{N,N}\Phi_{\underline{\mathbf{c}}}\end{pmatrix} \triangleq \mathbf{R}_{\mathbf{c}} \otimes \Phi_{\underline{\mathbf{c}}}, \quad (16)$$

where the mth row and the nth column of the temporary covariance matrix $\mathbf{R}_{\mathbf{c}}$ is $[\mathbf{R}_{\mathbf{c}}]_{m,n} \triangleq J_0(2\pi f_d(m-n))$.

We are now ready to find the MGF of $Q(\mathcal{C})$, denoted by $M_Q(s)$. From, (13), it is given by

$$M_Q(s) = \det\left(\mathbf{I} + s\begin{bmatrix} \widetilde{\Delta}\widetilde{\Delta}^H & \widetilde{\Delta} \\ \widetilde{\Delta}^H & \mathbf{0} \end{bmatrix}\begin{bmatrix} \frac{2E_s}{L_t}\mathbf{R}_{\tilde{\mathbf{c}}} & \mathbf{0} \\ \mathbf{0} & 2N_0\mathbf{I}_{NL_rL_t} \end{bmatrix}\right)^{-1}$$

$$= \det\left(\mathbf{I} + \tfrac{2E_s}{L_t}(s - 2N_0s^2)\widetilde{\Delta}\widetilde{\Delta}^H(\mathbf{R_c}\otimes\Phi_{\underline{\mathbf{c}}})\right)^{-1}. \quad (17)$$

The dimension of $\mathbf{R_c}\otimes\Phi_{\underline{\mathbf{c}}}$ is $d_{\mathbf{R}_{\tilde{c}}} = N\min(L, L_rL_t)$. Expressing $\widetilde{\Delta}\widetilde{\Delta}^H(\mathbf{R_c}\otimes\Phi_{\underline{\mathbf{c}}}) = \mathbf{Q\Lambda Q}^H$ in its singular value decomposition (SVD) form, the MGF becomes

$$M_Q(s) = \det\left(\mathbf{I}_d + \frac{2E_s}{L_t}(s - 2N_0s^2)\Lambda\right)^{-1}, \quad (18)$$

where $d = \min(\text{rank}(\widetilde{\Delta}), d_{\mathbf{R}_{\tilde{c}}})$ is the dimension of Λ. Substituting (18) back to (15), we have

$$Pr(\mathbf{X}\to\widehat{\mathbf{X}}) = \sum_{i=1}^{d}\frac{1}{\sqrt{\frac{\text{SNR}}{L_t}\lambda_i(\frac{\text{SNR}}{L_t}\lambda_i + 2) + \frac{\text{SNR}}{L_t}\lambda_i + 2}}\prod_{\substack{j=1\\ j\neq i}}^{d}\frac{\lambda_i}{\lambda_i - \lambda_j}, \quad (19)$$

where the input signal to noise ratio (SNR) is $\frac{E_s}{2N_0}$.

To put the PEP, which is a general form for spatially and temporally correlated fading channels, into a proper context, we shall elaborate the MGF in (17) for some specific types of channels. These will be conducted in the next two subsections.

4.2. Spatially Correlated Fading Channels

We first focus on spatially correlated fading channels and discuss two extreme cases to characterize the upper and lower performance bounds of STCs in realistic spatially correlated channels. We present results for these two cases using the MGF approach.

A. Fast Fading Channels

In this case, the temporal correlation matrix $\mathbf{R}_{\underline{\mathbf{c}}} = \mathbf{I}_N$. Thus, from (14) and (16), we have

$$\widetilde{\Delta}\widetilde{\Delta}^H(\mathbf{R_c}\otimes\Phi_{\underline{\mathbf{c}}}) = \text{diag}((\mathbf{I}_{L_r}\otimes\Delta_1\Delta_1^H)\Phi_{\underline{\mathbf{c}}}, \cdots, (\mathbf{I}_{L_r}\otimes\Delta_1\Delta_1^H)\Phi_{\underline{\mathbf{c}}}). \quad (20)$$

Substituting (20) back into (17), we obtain

$$M_Q(s) = \prod_{n=1}^{N}\det\left(\mathbf{I} + \frac{2E_s}{L_t}(s - 2N_0s^2)(\mathbf{I}_{L_r}\otimes\Delta_n\Delta_n^H)\Phi_{\underline{\mathbf{c}}}\right)^{-1} \quad (21)$$

Expressing $(\mathbf{I}_{L_r}\otimes\Delta_n\Delta_n^H)\Phi_{\underline{\mathbf{c}}} = \mathbf{Q}_n\Lambda_n\mathbf{Q}_n^H$, $n = 1,\cdots,N$, we get $\Lambda = \text{diag}\{\Lambda_1,\cdots,\Lambda_N\}$. Hence, the PEP formula (19) directly applies to the case.

B. Slow Fading Channels

In this case, channel coefficients remain unchanged for the entire block of length N. Thus, $\tilde{\mathbf{c}} = [\underline{\mathbf{c}}(1),\cdots,\underline{\mathbf{c}}(N)] = \underline{\mathbf{c}}[1,\cdots,1]$. Following the same procedure from (13) to (17), one can show that the MGF for slow fading is

$$M_Q(s) = \det\left(\mathbf{I} + \frac{2E_s}{L_t}(s - 2N_0s^2)\sum_{n=1}^{N}(\mathbf{I}_{L_r}\otimes\Delta_n\Delta_n^H)\Phi_{\underline{\mathbf{c}}}\right)^{-1} \quad (22)$$

Expressing $\sum_{n=1}^{N}(\mathbf{I}_{L_r}\otimes\Delta_n\Delta_n^H)\Phi_{\underline{\mathbf{c}}} = \mathbf{Q}\Lambda\mathbf{Q}^H$ and using (19) gives the PEP for slow fading channels directly.

4.3. Temporally Correlated Fading Channels

In this case, channel coefficients are assumed to be spatially uncorrelated. Then, the cross-covariance matrix between channel vectors $\mathbf{c}_p(m)$ and $\mathbf{c}_k(n)$, on the pth and kth receive antennas, respectively, becomes

$$E(\mathbf{c}_p(m)^H\mathbf{c}_k(n) = J_0(2\pi f_d(m-n))\delta(p-k)\sigma_{r,p}^2\Sigma_t,$$

where $\sigma_{r,p}^2$ is the channel covariance associated with the pth receive antenna, and $\Sigma_t \triangleq \text{diag}\{\sigma_{t,1}^2,\cdots,\sigma_{t,L_t}^2\}$ is the channel covariance matrix of the transmit antenna, with $\sigma_{t,q}^2$ being the channel covariance associated with the qth transmit antenna. Let $\bar{\mathbf{c}}_p = [\mathbf{c}_p(1),\cdots,\mathbf{c}_p(N)]$ denote the channel coefficients of the pth receive antenna for time index from 1 to N. Then, we have $E(\bar{\mathbf{c}}_p^H\bar{\mathbf{c}}_k) = \sigma_{r,p}^2\mathbf{R_c}\otimes\Sigma_t$. Let $\bar{\mathbf{z}}_p = [\sqrt{\frac{2E_s}{L_t}}\bar{\mathbf{c}}_p, \bar{\mathbf{n}}_p]$, where $\bar{\mathbf{n}}_p = [\mathbf{n}_p(1),\cdots,\mathbf{n}_p(N)]$. We can immediately evaluate its covariance matrix as

$$E(\bar{\mathbf{z}}_p^H\bar{\mathbf{z}}_p) = \begin{bmatrix} \sigma_{r,p}^2\mathbf{R_c}\otimes\Sigma_t & \mathbf{0} \\ \mathbf{0} & 2N_0\mathbf{I}_{NL_t} \end{bmatrix}. \quad (23)$$

Define concatenated vector $\tilde{\mathbf{z}} = [\bar{\mathbf{z}}_1,\cdots,\bar{\mathbf{z}}_{L_r}]$. The PEP formula (12) can be rearranged into the form of

$$Pr(\mathbf{X}\to\widehat{\mathbf{X}}|\mathcal{C}) = Pr\left\{\tilde{\mathbf{z}}\left(\mathbf{I}_{L_r}\otimes\begin{bmatrix} \bar{\Delta}\bar{\Delta}^H & \bar{\Delta} \\ \bar{\Delta}^H & \mathbf{0} \end{bmatrix}\right)\tilde{\mathbf{z}}^H < 0|\mathcal{C}\right\}, \quad (24)$$

where $\bar{\Delta} \triangleq \text{diag}\{\Delta_1,\cdots,\Delta_N\}$, which is of the Gaussian quadratic form. By applying the property of spatial uncorrelatedness, the MGF is given by

$$M_Q(s) = \prod_{p=1}^{L_r}\det\left(\mathbf{I} + s\begin{bmatrix} \bar{\Delta}\bar{\Delta}^H & \bar{\Delta} \\ \bar{\Delta}^H & \mathbf{0} \end{bmatrix}E(\bar{\mathbf{z}}_p^H\bar{\mathbf{z}}_p)\right)^{-1} =$$

$$\prod_{p=1}^{L_r}\det\left(\mathbf{I} + \tfrac{2E_s}{L_t}(s - 2N_0s^2)\sigma_{r,p}^2\bar{\Delta}\bar{\Delta}^H(\mathbf{R_c}\otimes\Sigma_t)\right)^{-1} (25)$$

Similarly, we can express $\sigma_{r,p}^2\bar{\Delta}\bar{\Delta}^H(\mathbf{R_c}\otimes\Sigma_t) = \mathbf{Q}_p\Lambda_p\mathbf{Q}_p^H$, $p = 1,\cdots,L_r$, in its SVD, and have $\Lambda = diag\{\Lambda_1,\cdots,\Lambda_{L_r}\}$. Then, formula (19) can be directly used for the evaluation of PEP in this case.

We will present below MGFs for temporally correlated (but spatially uncorrelated) fast and slow fading channels. These two cases are the most widely studied cases for STC designs. Their exact PEPs can also be found in [4] using a different MGF-based approach.

A. Fast Fading Channels

In this case, $\mathbf{R_c} = \mathbf{I}_N$. Thus, formula (25) can be simplified to

$$M_Q(s) = \prod_{p=1}^{L_r}\prod_{n=1}^{N}\det\left(\mathbf{I} + \frac{2E_s}{L_t}(s - 2N_0s^2)\sigma_{r,p}^2\Delta_n\Delta_n^H\Sigma_t\right)^{-1} (26)$$

B. Slow Fading Channels

In this case, $\bar{\mathbf{c}}_p = [\mathbf{c}_p(1), \cdots, \mathbf{c}_p(N)] = \mathbf{c}_p[1, \cdots, 1]$. Then, formula (25) reduces to

$$M_Q(s) = \prod_{p=1}^{L_r} \det\left(\mathbf{I} + \frac{2E_s}{L_t}(s - 2N_0 s^2)\sigma_{r,p}^2 \sum_{n=1}^{N} \Delta_n \Delta_n^H \Sigma_t\right)^{-1} \quad (27)$$

5. SIMULATION RESULTS

We present results for the 2×2 SO-STTC codes described in [2]. A rate $r = 1$ BPSK 2-state code (Fig. 7, [2]) was adopted in our simulation study. The PEPs for an error event path of length $N = 2$ with the all-zero path as the correct one are shown for various channel conditions. Following the setup provided in [4], the corresponding transmitted and detected symbols were $x_1(1) = x_2(1) = 1$, $x_1(2) = x_2(2) = 1$, $\hat{x}_1(1) = 1$, $\hat{x}_2(1) = -1$, $\hat{x}_1(2) = -1$, $\hat{x}_2(2) = 1$ and $\theta(1) = \hat{\theta}(1) = 0$, $\theta(2) = 0$, $\hat{\theta}(2) = \pi$. The numbers of transmit and receive antennas were $L_t = 2$, $L_r = 3$, respectively. The number of transmission paths was $L = 4$. For temporally correlated fading channels, the normalized Doppler frequency was $f_d = 0.1$.

First, we show the PEPs for spatially correlated fading channels in Fig. 1. Three cases are given in this figure, which are fasting fading, temporally correlated fading and slow fading, respectively. Obviously, the PEP curve of the temporally correlated fading falls between the two extreme cases. This curve will move up towards the curve of fast fading as f_d increases and down towards the curve of slow fading as f_d decreases. Therefore, STC should take the fading speed into account for realistic channels.

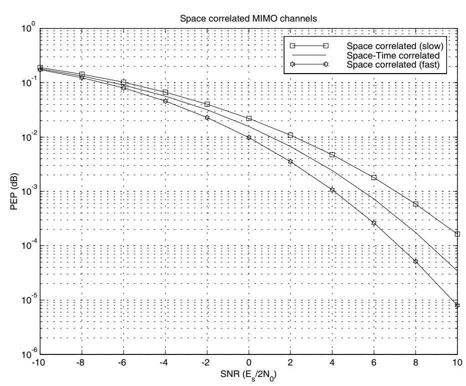

Fig. 1. PEPs for spatially correlated channels, where $N = 2$, $Lt = 2$, $Lr = 3$, $L = 4$ and $f_d = 0.1$.

Fig. 2 presents results for two extreme cases: the space-time correlated fading v.s. the space-time uncorrelated fading (fast fading) channels. This example is to investigate the performance of a space-time code, which was originally designed for uncorrelated channels, in realistic space-time correlated fading channels. The SNR degradation is about $4dB$ at PEP $\sim 10^{-4}$. Thus, a space-time code designed for correlated fading channels can potentially provide better performance in realistic channels than one without considering it.

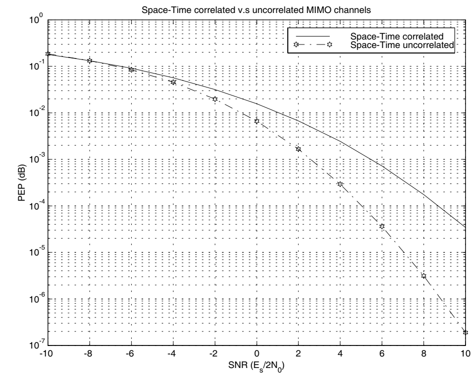

Fig. 2. Comparisons of PEPs for space-time correlated v.s. uncorrelated channels. The systems parameters are $N = 2$, $Lt = 2$, $Lr = 3$, $L = 4$ and $f_d = 0.1$.

6. CONCLUSION AND FUTURE WORK

Exact PEP formulas for spatially and temporally correlated fading channels were derived in this work. Using the MGF approach, the evaluation of a PEP can be converted to evaluating the residues of a MGF with respect to its right half-plane poles. This technique greatly simplifies the exact expressions of PEPs for STC design and extends the exact evaluations of PEPs to both spatially and temporally correlated fading channels. In the future, we will study effective STC design for spatially and temporally correlated fading channels based on the results derived in this work.

7. REFERENCES

[1] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communication: Performance criterion and code construction," *IEEE Trans. on Information Theory*, vol. 44, pp. 744–765, Mar. 1998.

[2] H. Jafarkhani and N. Seshadri, "Super-Orthogonal space-time trellis codes," *IEEE Trans. on Information Theory*, vol. 49, pp. 937–950, Apr. 2003.

[3] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-Time block coding from orthogonal designs," *IEEE Trans. on Information Theory*, pp. 1456–1467, July 1999.

[4] M. K. Simon and H. Jafarkhani, "Performance evaluation of super-orthogonal space-time trellis codes using a moment generating function-based approach," *IEEE Trans. on Signal Processing*, vol. 51, no. 11, pp. 2739–2751, Nov. 2003.

[5] J. H. Kotecha, Z. Hong, and A. M. Sayeed, "Coding and diversity gain tradeoff in space-time codes for correlated MIMO channels," in *Globecom*. San Franciso, Dec 2003.

[6] S. L. Miller, M. L. Honig, and L. B. Milstein, "Performance analysis of MMSE receivers for DS-CDMA in frequency-selective fading channels," *IEEE Trans. on Communications*, vol. 48, no. 11, pp. 1919–1929, Nov. 2000.