

SINGLE-SYMBOL DECODABLE DIFFERENTIAL SPACE-TIME MODULATION BASED ON QO-STBC

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ABSTRACT

We show that Minimum-Decoding-Complexity Quasi-Orthogonal Space-Time Block Code (MDC-QOSTBC) can be used to form a new differential space-time modulation (DSTM) scheme to provide full transmit diversity with single-symbol decodable complexity. This is the first known single-symbol decodable DSTM not based on Orthogonal STBC. We derive the DSTM code design criteria and use them to construct customized constellation sets that provide full diversity and maximum coding gain. Our new DSTM scheme can provide higher code rate and better decoding performance than DSTM schemes based on rate-1/2 Orthogonal STBC. It also has a lower decoding complexity than the other DSTM schemes, as its maximum likelihood decoding only requires the joint detection of two real symbols.

1. INTRODUCTION

Space-time modulation or transmit diversity can be used to reduce the fading effects caused by multi-path signal propagation in wireless communications effectively. Early space-time transmit diversity schemes were designed for coherent detection, with channel estimates assumed available at the receiver. However, the complexity and cost of channel estimation grow with the number of transmit and receive antennas. Therefore the availability of transmit diversity schemes that do not require channel estimation is desirable. To this end, several differential space-time modulation (DSTM) schemes have been designed.

The first generation of DSTM has been designed based on unitary matrices [1][2]. These DSTMs have high maximum-likelihood (ML) decoding complexity. Just as coherent transmit diversity schemes, a DSTM scheme with low decoding complexity is desired. A DSTM based on Sp(2) has been proposed in [3], with ML decoding performed by sphere decoding. A DSTM based on O-STBC has been proposed in [4]. Its ML decoding can be achieved by simple linear processing, but such scheme has a limitation on its maximum achievable code rate. As a result, two DSTMs have recently been proposed in [5][6] to achieve a higher code rate than [4]. The DSTM in [5] is designed based on Quasi Orthogonal STBC (QO-STBC), whereas the DSTM in [6] is based on a newly proposed non-linear STBC. Both of them can provide a higher code rate than DSTM based on O-STBC, and their ML decoding can be performed by jointly detecting two complex symbols, hence achieving much lower decoding complexity than many of the existing DSTM schemes besides those based on O-STBC.

Due to the unitary requirement, the DSTMs mentioned above mostly employ phase-shift keying (PSK) constellation, which is less efficient as the constellation size increase. As a result, DSTMs based on quasi-unitary matrices have been proposed. In [7], a DSTM based on O-STBC with Amplitude-PSK (A-PSK) constellation is first proposed. In subsequent publications [8][9], DSTM based on O-STBC with general QAM constellation is designed. They are shown to have better performance than similar DSTM schemes with PSK constellations. Likewise, a DSTM based on QO-STBC with non-constant matrix norm is proposed in [10].

In this paper, we propose a new DSTM scheme that is single-symbol decodable and is based on quasi-unitary matrices constructed from the Minimum-Decoding-Complexity QO-STBC (MDC-QOSTBC) described in [11][12]. The proposed scheme can provide full transmit diversity with higher code rate than DSTM schemes based on O-STBC [4][8][9], and with lower decoding complexity than the rest DSTM schemes.

2. REVIEW OF DSTM

2.1. Signal Model

Consider a communication system with N_T transmit and N_R receive antennas. Let \mathbf{H}_t be the $N_R \times N_T$ channel gain matrix at a time t . Thus the ij^{th} element of \mathbf{H}_t is the channel coefficient for the signal path from the j^{th} transmit antenna to the i^{th} receive antenna. Let \mathbf{X}_t be the $N_T \times N_T$ square codeword transmitted at a time t . Then, the received signal matrix \mathbf{R}_t is

$$\mathbf{R}_t = \mathbf{H}_t \mathbf{X}_t + \mathbf{N}_t \quad (1)$$

where \mathbf{N}_t is the AWGN. At the start of the transmission, we transmit a known unitary codeword \mathbf{X}_0 . The codeword \mathbf{X}_t transmitted at a time t is then differentially encoded by

$$\mathbf{X}_t = a_{t-1}^{-1} \mathbf{X}_{t-1} \mathbf{U}_t \quad (2)$$

where a_{t-1} is the “magnitude” of \mathbf{U}_{t-1} and \mathbf{U}_t is a quasi-unitary matrix (such that $\mathbf{U}_t \mathbf{U}_t^H = a_t^2 \mathbf{I}$) called the code matrix, that contains information of the transmitted data. To ensure that the total average transmission power is maintained, it is required that

$$E(\mathbf{X}_t \mathbf{X}_t^H) = E(\mathbf{U}_t \mathbf{U}_t^H) = \mathbf{I} \quad (3)$$

If we assume that the channel remains unchanged during two consecutive code periods, i.e. $\mathbf{H}_t = \mathbf{H}_{t-1}$, then the received signal \mathbf{R}_t at a time t can be expressed as

$$\mathbf{R}_t = a_{t-1}^{-1} \mathbf{R}_{t-1} \mathbf{U}_t + \tilde{\mathbf{N}}_t \quad (4)$$

where $\tilde{\mathbf{N}}_t = -a_{t-1}^{-1} \mathbf{N}_{t-1} \mathbf{U}_t + \mathbf{N}_t$. Since $\tilde{\mathbf{N}}_t$ is not independent of a_{t-1} and \mathbf{U}_t , and in order to make use of the linear property of

STBC during the decoding process, we employ the near-optimal differential decoder proposed in [8] to estimate \mathbf{U}_t , as follows:

$$\begin{aligned}\hat{\mathbf{U}}_t &= \arg \min_{\mathbf{U}_t \in \mathcal{U}} \|\mathbf{R}_t - a_{t-1}^{-1} \mathbf{R}_{t-1} \mathbf{U}_t\|^2 \\ &= \arg \min_{\mathbf{U}_t \in \mathcal{U}} \text{tr} \left[a_{t-1}^{-2} \mathbf{R}_{t-1}^H \mathbf{R}_{t-1} \mathbf{U}_t \mathbf{U}_t^H - 2a_{t-1}^{-1} \text{Re}(\mathbf{R}_t^H \mathbf{R}_{t-1} \mathbf{U}_t) \right]\end{aligned}\quad (5)$$

where \mathcal{U} denotes the set of all possible code matrices, tr represents the trace of a matrix, $\|\cdot\|^2$ represents the Frobenius norm and a_{t-1} can be estimated from the previous decision $\hat{\mathbf{U}}_{t-1}$ (this may lead to error propagation, but it has been shown in [8-10] that the effect of such error propagation is very small). It has been shown in [8] that the performance loss of the near-optimal differential decoder compared to the optimal differential decoder is small, but the near-optimal differential decoder could lead to a significant reduction in decoding complexity. Hence in this paper, we adopt the above-described signal model and decoder for our proposed DSTM scheme.

2.2. Diversity and Coding Gain

The decoding performance of DSTM with optimal differential decoder has been analyzed in [2]. Specifically, the transmit diversity level that can be achieved by DSTM is given by:

$$\text{Min}[\text{rank}(\mathbf{U}_k - \mathbf{U}_l)] \quad \forall k \neq l. \quad (6)$$

In order to achieve full transmit diversity, the minimum rank in (6) has to be equal to N_T . For a full-rank DSTM code, its *coding gain* is defined in [2][4] as

$$\text{Min} \left[N_T \times \det \left((\mathbf{U}_k - \mathbf{U}_l)(\mathbf{U}_k - \mathbf{U}_l)^H \right)^{1/N_T} \right] \quad \forall k \neq l \quad (7)$$

In order to achieve optimum decoding performance, the coding gain has to be maximized. Since the performance of the near-optimal differential decoder has been shown to be similar to that of the optimal decoder [8], we adopt the above design criteria for designing our DSTM in this paper.

3. NEW DSTM BASED ON MDC-QOSTBC

3.1. Minimum-Decoding-Complexity QO-STBC

The method to construct the K dispersion matrices of an MDC-QOSTBC, denoted as $\{\mathbf{A}, \mathbf{B}\}$, have been described in [11][12]. For example, a full-rate MDC-QOSTBC, \mathbf{C} , for $N_t = T = K = 4$ is shown in (8).

$$\begin{aligned}\mathbf{C} &= \sum_{i=1}^K c_i^R \mathbf{A}_i + j c_i^I \mathbf{B}_i \\ &= \frac{1}{\sqrt{4}} \begin{bmatrix} c_1^R + j c_3^R & -c_2^R + j c_4^R & -c_1^I + j c_3^I & c_2^I + j c_4^I \\ c_2^R + j c_4^R & c_1^R - j c_3^R & -c_2^I + j c_4^I & -c_1^I - j c_3^I \\ -c_1^I + j c_3^I & c_2^I + j c_4^I & c_1^R + j c_3^R & -c_2^R + j c_4^R \\ -c_2^I + j c_4^I & -c_1^I - j c_3^I & c_2^R + j c_4^R & c_1^R - j c_3^R \end{bmatrix}\end{aligned}\quad (8)$$

where the factor $1/\sqrt{4}$ is to ensure that (3) can be satisfied, the symbols c_i represent the transmitted symbols and the superscript R and I represent the real and imaginary part of a transmitted symbol respectively. It has been shown in [13] that,

$$\mathbf{C}\mathbf{C}^H = \frac{\alpha}{K} \begin{bmatrix} \mathbf{I}_{N_t/2} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{N_t/2} \end{bmatrix} + \frac{\beta}{K} \begin{bmatrix} \mathbf{0} & \mathbf{I}_{N_t/2} \\ \mathbf{I}_{N_t/2} & \mathbf{0} \end{bmatrix} \quad (9)$$

where $\alpha = \sum_{i=1}^K |c_i|^2$, $\beta = 2 \sum_{i=1}^{K/2} -c_i^R c_{i+1}^I + c_{K/2+i}^R c_{K/2+i+1}^I$, and the minimum determinant of its codeword distance matrix is:

$$\det_{\min} = \frac{1}{K} \left[(\Delta_i^R)^2 - (\Delta_i^I)^2 \right]^{N_t} \quad (10)$$

where Δ_i , $1 \leq i \leq 4$, represents the possible error in the i^{th} transmitted constellation symbol. That is, $\Delta_i = c_i - e_i$ if the receiver decides erroneously in favor of e_i if c_i is transmitted. In order to achieve full transmit diversity and optimum coding gain in accordance with the design criteria defined in Section 2.2, the value of (10) has to be non-zero and maximized.

3.2. New DSTM Based on MDC-QOSTBC

As we have shown earlier, the use of a quasi-unitary code matrix, \mathbf{U}_t , is essential. Since $\beta \neq 0$ in (9), MDC-QOSTBC is generally not a quasi-unitary matrix, i.e. $\mathbf{C}\mathbf{C}^H \neq \gamma \mathbf{I}$, where γ is a constant value. In order to apply the MDC-QOSTBC in (8) as a DSTM code, i.e. use \mathbf{C} as \mathbf{U}_t in (2), \mathbf{C} has to be a quasi-unitary matrix, i.e. the value of β in (9) has to be zero for all possible values of c_i and the average value of α in (9) has to be constrained. In order to have β equal to zero, we can see from (9) that some form of correlation between the real and imaginary parts of the transmitted symbols is required. This calls for the use of *specifically designed constellation set* for the MDC-QOSTBC such that their codewords are always quasi-unitary.

Based on the above idea, a constellation set \mathcal{M} which consists of M constellation points (each represented as $x_k + jy_k$, $1 \leq k \leq M$) should be designed with the following requirements:

- (i) Power Criterion: $E(x_k^2 + y_k^2) = 1$
- (ii) Quasi-Unitary Criterion: $x_k y_k = v$ (11)
- (iii) Performance Optimization Criterion:

$$\text{maximize Min} \left\{ \left[(\Delta x_{kl})^2 - (\Delta y_{kl})^2 \right]^{N_t} \right\}$$

where v can be any constant value, and $\Delta x_{kl} = x_k - x_l$, $\Delta y_{kl} = y_k - y_l$ for all $1 \leq k \neq l \leq M$. In complying with the Power Criterion (11)(i), we ensure that the transmitted codewords have a constant average power and hence $E(\alpha) = K$. In complying with the Quasi-Unitary Criterion (11)(ii), we ensure that \mathbf{C} is always a quasi-unitary matrix, i.e. we achieve $\beta=0$. With the Performance Optimization Criterion (11)(iii), we ensure that the minimum determinant value in (10) is maximized.

In the decoding process, the near-optimal differential decoder in (5) can be simplified to:

$$\begin{aligned}\hat{\mathbf{U}}_t &= \arg \min_{c_1, \dots, c_K \in \mathcal{M}} \text{tr} \left[a_{t-1}^{-2} \mathbf{R}_{t-1}^H \mathbf{R}_{t-1} \left(\sum_{i=1}^K \frac{|c_i|^2}{K} \mathbf{I}_{N_t} \right) - 2a_{t-1}^{-1} \text{Re} \left(\mathbf{R}_t^H \mathbf{R}_{t-1} \left(\sum_{i=1}^K c_i^R \mathbf{A}_i + j c_i^I \mathbf{B}_i \right) \right) \right] \\ &= \arg \min_{c_1, \dots, c_K \in \mathcal{M}} \sum_{i=1}^K \text{tr} \left[a_{t-1}^{-2} \mathbf{R}_{t-1}^H \mathbf{R}_{t-1} \left(\frac{|c_i|^2}{K} \mathbf{I}_{N_t} \right) - 2a_{t-1}^{-1} \text{Re} \left(\mathbf{R}_t^H \mathbf{R}_{t-1} (c_i^R \mathbf{A}_i + j c_i^I \mathbf{B}_i) \right) \right]\end{aligned}\quad (12)$$

Due to the linear property and code structure of MDC-QOSTBC, the detection of the transmitted symbols can be further simplified as:

$$\hat{c}_i = \arg \min_{c_i \in \mathcal{M}} \text{tr} \left[a_{i-1}^{-2} \mathbf{R}_{i-1}^H \mathbf{R}_{i-1} \left(\frac{|c_i|^2}{K} \mathbf{I}_{N_i} \right) - 2a_{i-1}^{-1} \text{Re} \left(\mathbf{R}_{i-1}^H \mathbf{R}_{i-1} (c_i^R \mathbf{A}_i + j c_i^I \mathbf{B}_i) \right) \right] \quad (13)$$

where the \mathbf{A}_i and \mathbf{B}_i are the dispersion matrices of an MDC-QOSTBC.

As shown in (13), the differential decoding of the proposed differential MDC-QOSTBC modulation scheme is single-symbol decodable, i.e. its decoding can be achieved by the detection of one complex symbols (i.e. joint detection of two real symbols, c_i^R and c_i^I), and each of the transmit symbols can be detected in parallel. So the proposed DSTM scheme does **not** increase the ML decoding complexity as compared to the original coherent MDC-QOSTBC scheme. Furthermore, it has a lower ML decoding complexity than the DSTMs reported in [1-3, 5-6, 10], which generally require a larger joint detection search space for the same spectral efficiency and antenna number.

3.3. Design of Constellation Set

To design a constellation set \mathcal{M} that satisfy all the three criteria in (11), we propose the following approach. As shown in Figure 1, the (x_k, y_k) solutions to the Power Criterion in (11)(i) can be modeled as points on the multiple concentric circles with radius r_1, r_2, \dots, r_L , where r_i are subject to the constraint $\sum_{i=1}^L r_i^2 = L$. Next, the solution loci to the Quasi-Unitary Criterion in (11)(ii) can be represented as hyperbola. Hence the intersection points of the hyperbola with the circles will be the valid constellation points that satisfy the criteria (11)(i) and (ii). By such modeling, every circle will have at most four intersection points with the hyperbola, namely A, B, C and D, as shown in Figure 1. However, due to the geometrical symmetry of the hyperbola, some combinations of the intersection points cannot be used simultaneously as they will lead to a zero value for the Performance Optimization Criterion in (11)(iii), hence only constellation points A and C, or B and D, can be used at the same time as indicated by different marker-pairs (x and o) in Figure 1. Furthermore, in order to maximize the Euclidean distance between the constellation points, constellation pairs (A, C) and (B, D) are used alternately on the adjacent circles.

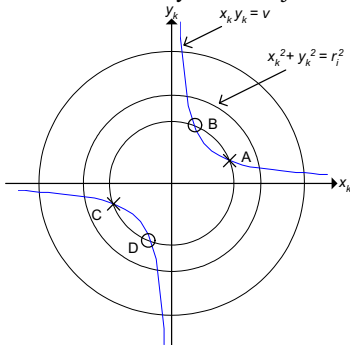


Figure 1 Solution loci of the DSTM constellation design criteria

By the above modeling, we are able to reduce the variables to be optimized in (11)(iii) from $M(x_k, y_k)$ pairs to r_1, r_2, \dots, r_L and v , where $L = M/2$ as every circle can provide two constellation points. Through computer optimization, the optimum coding gain of the proposed DSTM based on (7) versus v for $M = 4$ and 8 are shown in Figure 2. It shows that as v

decreases the coding gain increases, and $v = 0$ gives the best coding gain for both values of M .

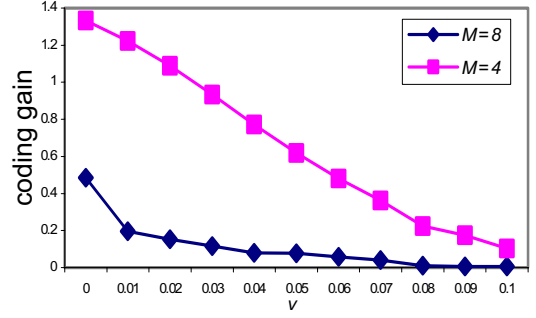


Figure 2 Optimization of coding gain

In Figure 3, we show two constellation sets \mathcal{M}_1 and \mathcal{M}_2 that have been optimized with $M = 4$ and 8 constellation points respectively with $v=0$. By using rate-1 MDC-QOSTBC for four transmit antennas, the constellation set \mathcal{M}_1 and \mathcal{M}_2 give a spectral efficiency of 2 bps/Hz and 3 bps/Hz respectively; by using rate-3/4 MDC-QOSTBC for eight transmit antennas with \mathcal{M}_1 , a spectral efficiency of 1.5 bps/Hz can be obtained.

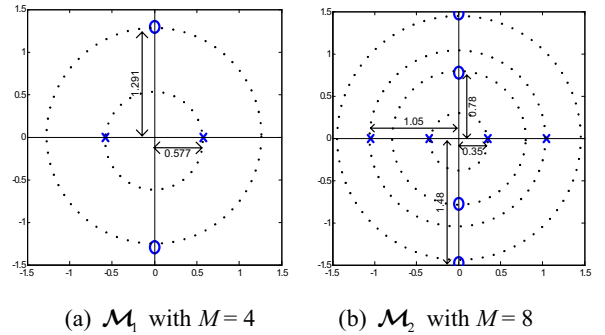


Figure 3 Optimum constellation sets designed for DSTM

4. PERFORMANCE

In our simulations, we assume that the channel is flat fading and quasi-static, i.e. the fading is constant within one frame and varies independently from one frame to another. Every frame from each antenna includes 132 symbols, which is chosen to be the same as that used in [10] so that their results can be compared.

In Figure 4, we compare the block error rate (BLER) performance of our proposed DSTM based on MDC-QOSTBC with the DSTM based on O-STBC [8][9] under the same spectral efficiency. For the case of eight transmit antennas, it is clear that our proposed DSTM, based on rate-3/4 MDC-QOSTBC with constellation set \mathcal{M}_1 , performs better (1.5 dB gain at BLER = 10^{-4}) than the DSTM based on rate-1/2 O-STBC with 8QAM constellation, both having the same spectral efficiency of 1.5 bps/Hz. For the case of four transmit antennas, our proposed DSTM based on rate-1 MDC-QOSTBC with constellation set \mathcal{M}_1 again performs better (3 dB gain at BLER = 10^{-4}) than the DSTM based on rate-1/2 O-STBC with 16QAM constellation under a common spectral efficiency of 2 bps/Hz. Finally, for four transmit antennas, our proposed DSTM with rate-1 MDC-QOSTBC with constellation set \mathcal{M}_2 performs comparably with

the DSTM based on rate-3/4 O-STBC with 16QAM constellation under 3 bps/Hz. These results suggest that our proposed DSTM scheme can offer good performance gain over DSTM schemes based on rate-1/2 O-STBC.

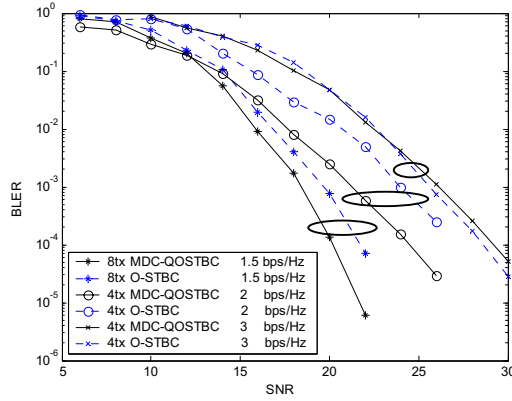


Figure 4 Simulated block error rate (BLER) of DSTM based on O-STBC and MDC-QOSTBC

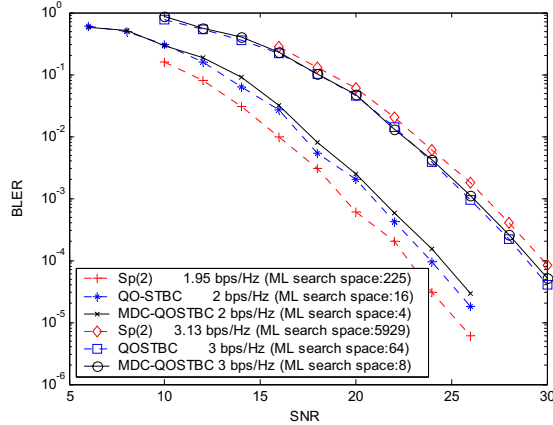


Figure 5 Simulated block error rate (BLER) of DSTM for four transmit and one receive antennas

In Figure 5, we first compare the BLER performance of DSTM based on QO-STBC [5][10] with our MDC-QOSTBC DSTM. Under a spectral efficiency of 3 bps/Hz, our MDC-QOSTBC DSTM performs comparably as the DSTM based on QO-STBC in [10], but the number of symbols required for joint detection in our DSTM scheme is half of that required by the other scheme. Likewise can be said for the case of spectral efficiency of 2 bps/Hz, where the DSTM in comparison is based on QO-STBC from [5]. Next, comparing the BLER performance of Sp(2) DSTM and our proposed MDC-QOSTBC DSTM, it can be seen that our MDC-QOSTBC DSTM with spectral efficiency of 3 bps/Hz has a 0.5dB performance gain at BLER 10^{-4} than the Sp(2) DSTM [3] with spectral efficiency of 3.13 bps/Hz; furthermore, our MDC-QOSTBC DSTM is single-symbol decodable (with a decoding search space of 8), while the Sp(2) DSTM has a decoding search space of 5929. At a lower spectral efficiency, our MDC-QOSTBC DSTM with spectral efficiency of 2 bps/Hz does not perform as good as the Sp(2) DSTM with 1.95 bps/Hz, but the decoding complexity of our MDC-QOSTBC DSTM remains significantly lower. These results suggest that our MDC-QOSTBC DSTM is able to offer significant reduction in decoding complexity over the DSTM schemes of [3], [5] and [10] at comparable decoding performance.

5. CONCLUSIONS

We have proposed a new single-symbol decodable differential space-time modulation (DSTM) scheme based on Minimum-Decoding-Complexity Quasi-Orthogonal STBC (MDC-QOSTBC). This is the first single-symbol decodable DSTM not based on O-STBC. The main idea is to force the MDC-QOSTBC codeword to be a quasi-unitary matrix by using a specially designed constellation set for the codeword. The design criteria for the corresponding constellation set are derived. We then present the solutions of two constellation sets that meet the design criteria and also achieve full diversity and maximum coding gain. Our proposed DSTM scheme has the merits of MDC-QOSTBC, hence it achieves higher code rate (rate 1 for four transmit antennas and rate 3/4 for eight transmit antennas) than a DSTM based on O-STBC, and lower ML decoding complexity (joint detection of two real symbols) than other DSTMs based on QO-STBC (joint detection of two complex symbols), with better or comparable decoding performance.

6. REFERENCES

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