SOFT DEMODULATION AND UNEQUAL POWER ALLOCATION FOR DIGITAL MODULATION SCHEMES

T. Brüggen, C. Schulte-Hillen, and P. Vary

Institute of Communication Systems and Data Processing (ind), RWTH Aachen University, Germany
{brueggen,vary}@ind.rwth-aachen.de

ABSTRACT

The quality of digital transmission of speech, audio and video signals over a noisy channel can be improved in many cases by error concealment, specifically by *softbit source decoding* (SBSD). It applies parameter estimation exploiting *a priori* knowledge on parameter level and reliability information on bit level. In this contribution we present a novel soft demodulation (SDM) method for higher order digital modulation schemes based on SBSD. The soft demodulator makes a soft decision on signal point level using a *mean square* (MS) estimator and can also exploit *a priori* knowledge about the source. Furthermore, we present a novel concept for *unequal error protection* (UEP) by higher order *modulation with unequal power allocation* (MUPA). Usually, the bits representing the quantized source codec parameters are protected by channel coding with UEP according to the individual bit error sensitivities. For communication systems where channel coding is not applicable, we propose the MUPA method with periodically time-varying power allocation. The average transmitted energy per bit remains unaffected. The significant SNR gains of the SDM combined with MUPA in comparison to systems with hard decision and a fixed signal constellation are demonstrated by simulation.

1. INTRODUCTION

In digital transmission systems the digital representations of the source encoded parameters of speech, audio, or video signals are transmitted over a channel using digital modulation, e.g., 16-QAM as specified in the ITU-T recommendation V.32. The demodulation process influences the transmission quality significantly. An optimized demodulator with the capability to conceal errors due to channel noise is an important step to a robust transmission system.

In this paper we introduce a soft demodulation (SDM) method (Sec. 3) which is based on the error concealment by *softbit source decoding* (SBSD) [1]. SBSD requires reliability information about each bit. Besides, SBSD can exploit *a priori* knowledge (AK) on parameter level which is, e.g., determined by the correlation between the source parameters. In the literature (e.g., [1]) SBSD has been applied to *binary phase shift keying* (BPSK) as example using *log-likelihood* values (L-values), i.e., reliability information on bit level. However, in transmission systems with higher order modulation schemes a single channel output value is not associated to a specific bit exclusively. The inphase and the quadrature component of the complex channel symbols are assigned to bit patterns of M bits. Consequently, a bitwise estimation of the transmitted bit pattern at the receiver is difficult. The SDM adopts the principles of SBSD, but estimates the source encoded parameters on the level of the signal points, which are the modulation symbols in the signal space.

Furthermore, we present the novel concept called *modulation with unequal power allocation* (MUPA) with periodically time-varying power allocation for *unequal error protection* (UEP) in higher order modulation schemes (Sec. 4). MUPA is based on a modulation scheme with Gray mapping, e.g., QPSK or 16-QAM and changes the distances between the signal points of the modulation constellation. According to the mapping of the bit patterns, the distance between modulation signal points which differ in the *most significant bit* (MSB) is made larger, other distances between signal points differing, e.g., in the *least significant bit* (LSB) are made smaller. For each channel SNR a set of weights exists, which is calculated once in advance and determines the distances between the modulation signal points.

Hierarchical modulation and coding is examined in [2] with respect to the design of wireless systems. In [3] differently weighted bits in *pulse code modulation* (PCM) transmission have been discussed. However, an approximate solution only for PCM is derived which is based on the simplifying assumption that only single bit errors occur. In [4] the desired behavior is achieved by bitwise control of BPSK modulation amplitudes. This approach called *source-adaptive power allocation* is based on the neighborhood relations between the *unquantized* and quantized source parameter values. A fixed step size is used for the calculation of the *discrete* symbol amplitudes. In contrast to [3] and [4], we present an approach with rigorous optimization of the weights w_i to reduce the *mean square error* (MSE) and allowing continuous signal point amplitudes for higher order modulation schemes. As example we consider 16-QAM-MUPA with $M = 4$ bit quantizers. The performance improvements by SDM (Sec. 5.1) and MUPA (Sec. 5.2) compared to systems with hard decision demodulation and a fixed signal constellation are verified by simulation.

2. SYSTEM MODEL

The baseband model of the transmission system is shown in Fig. 1. The source generates zero-mean Gaussian distributed values u_τ with time instant τ and variance $\sigma_u^2 = 1$. These source parameters are correlated by a 1[st] order recursive low-pass filter

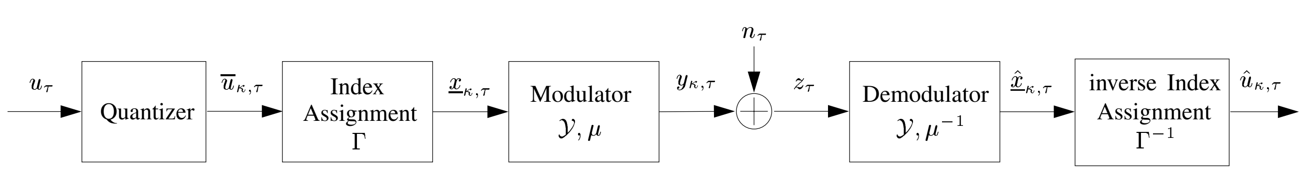

Figure 1: Baseband system model with hard decision demodulation

with the coefficient a and can be modeled as 1st order Markov process. Next, u_τ is quantized to $\overline{u}_{\kappa,\tau} \in \{\overline{u}_\kappa | \kappa = 1, 2, ...2^M\}$ with the reproduction levels $\overline{u}_\kappa$. The index assignment (IA) utilizes the one-to-one mapping function Γ and maps each value $\overline{u}_{\kappa,\tau}$ to a unique bit pattern $\underline{x}_{\kappa,\tau} \in \{\underline{x}_\kappa | \kappa = 1, 2, ...2^M\}$ with $\underline{x}_\kappa = (x_\kappa^{(1)}, x_\kappa^{(2)}, ...x_\kappa^{(M)})$:

$$\underline{x}_{\kappa,\tau} = \Gamma(\overline{u}_{\kappa,\tau}) \ . \quad (1)$$

The bits $x_\kappa^{(i)} \in \{-1, +1\}$, $i = 1, 2, ...M$, contain the MSB $x_\kappa^{(1)}$ and the LSB $x_\kappa^{(M)}$. The modulator assigns each bit pattern $\underline{x}_{\kappa,\tau}$ by the one-to-one mapping function μ to a complex modulation signal point $y_{\kappa,\tau}$ of the signal constellation set $\mathcal{Y} = \{y_\kappa | \kappa = 1, 2, ...2^M\}$:

$$y_{\kappa,\tau} = \mu(\underline{x}_{\kappa,\tau}) = y_{\kappa,\tau}^{(I)} + \mathrm{j} y_{\kappa,\tau}^{(Q)} \quad (2)$$

with the inphase (I) component $y_{\kappa,\tau}^{(I)}$ and the quadrature (Q) component $y_{\kappa,\tau}^{(Q)}$ of $y_{\kappa,\tau}$. The transmission of the obtained complex symbols $y_{\kappa,\tau} \in \mathcal{Y}$ is described by a complex channel with zero-mean additive white Gaussian noise (AWGN) $n_\tau = n_\tau^{(I)} + \mathrm{j} n_\tau^{(Q)}$ with $n_\tau^{(I)} \in \mathbb{R}$ in the inphase dimension and $n_\tau^{(Q)} \in \mathbb{R}$ in the quadrature dimension. We consider an AWGN channel with independent transmission of the two components with the variance $\sigma_n^2 = N_0/2$ in each dimension and the noise power spectral density N_0. As we set the energy per bit E_b to 1, the channel symbols are normalized to an average energy of $\mathrm{E}\{||y_{\kappa,\tau}||^2\} = M$.

At the receiver, the symbol is estimated from the disturbed complex samples $z_\tau = (y_{\kappa,\tau}^{(I)} + n_\tau^{(I)}) + \mathrm{j}(y_{\kappa,\tau}^{(Q)} + n_\tau^{(Q)})$. After hard decision demodulation, the inverse IA returns the estimated $\hat{u}_{\kappa,\tau}$ corresponding to the estimated quantized value $\overline{u}_{\kappa,\tau}$ by a codebook table lookup. The hard demodulator and the inverse IA can be replaced by a soft demodulator.

3. SOFT DEMODULATION (SDM)

In the following we develop the soft demodulation according to the SBSD. For a detailed derivation of SBSD we refer to the literature, e.g., [1]. The estimation of the transmitted quantized source codec parameter of SBSD is replaced for soft demodulation by an estimation on signal point level. Consequently, the estimation of a modulation symbol at the receiver requires *a posteriori* probabilities providing information about all symbols $y_\kappa \in \mathcal{Y}$, $\kappa = 1, 2, ...2^M$, which have possibly been sent. With a 1st order Markov model for the source codec parameters, the *a posteriori* probabilities $P(y_{\kappa,\tau}|z_\tau)$ can be calculated as

$$P(y_{\kappa,\tau}|z_\tau, Z_{\tau-1}) = C \cdot p(z_\tau|y_{\kappa,\tau}) \cdot P(y_{\kappa,\tau}|Z_{\tau-1}) \quad (3)$$

with $Z_{\tau-1} = (z_{\tau-1}, z_{\tau-2}...)$ consisting of all received values from the beginning of the transmission until the time instant $\tau - 1$.

The channel transition probabilities $p(z_\tau|y_{\kappa,\tau})$, $\kappa = 1, 2...2^M$, are calculated on symbol level, since the two channel output values are associated to *one* symbol. We utilize the real-valued geometric distance $D_{z_\tau,y_\kappa} = ||z_\tau - y_\kappa||$ between the complex received symbol z_τ and all signal points $y_\kappa \in \mathcal{Y}$. The AWGN of the channel causes the distance D_{z_τ,y_κ}, and the conditional probability density $p(z_\tau|y_{\kappa,\tau})$ becomes

$$p(z_\tau|y_{\kappa,\tau}) = p(D_{z_\tau,y_\kappa}) = \frac{1}{\sqrt{2\pi}\sigma_n} \cdot \exp\left(\frac{-D_{z_\tau,y_\kappa}^2}{2\sigma_n^2}\right) \ . \quad (4)$$

The probabilities $P(y_{\kappa,\tau}|Z_{\tau-1})$ in (3) are determined by the available *a priori* knowledge. If no *a priori* knowledge (NAK) is used, the probability density $p(z_\tau|y_{\kappa,\tau})$ of (4) equals the *a posteriori* probability [1]:

$$P(y_{\kappa,\tau}|z_\tau) = C \cdot p(z_\tau|y_{\kappa,\tau}) \quad (5)$$

with the normalization constant

$$C = \frac{1}{\sum_{i=1}^{2^M} p(z_\tau|y_{i,\tau})} \ . \quad (6)$$

In the following we will use the factor C to normalize the *a posteriori* probabilities $P(y_{\kappa,\tau}|z_\tau, ...)$ that $\sum_\kappa P(y_{\kappa,\tau}|z_\tau, ...) = 1$ is fulfilled. Note, that C does not have to equal the term given in (6) in all cases.

With *a priori* knowledge of 0th order (AK0), the probabilities of occurrence $P(y_{\kappa,\tau})$ of the signal points $y_{\kappa,\tau} \in \mathcal{Y}$ determine the *a posteriori* probabilities. Since Γ and μ are one-to-one mapping functions, it holds

$$P(y_{\kappa,\tau}) = P(\mu(\Gamma(\overline{u}_{\kappa,\tau}))) \quad (7)$$

with the probabilities of occurrence $P(\overline{u}_{\kappa,\tau})$, $\kappa = 1, 2, ...2^M$, of the quantizer reproduction levels $\overline{u}_{\kappa,\tau}$ mapped to the symbols $y_{\kappa,\tau} \in \mathcal{Y}$. For (3) it holds

$$P(y_{\kappa,\tau}|z_\tau) = C \cdot p(z_\tau|y_{\kappa,\tau}) \cdot P(\mu(\Gamma(\overline{u}_\kappa))) \quad (8)$$

For 1st order *a priori* knowledge (AK1) the correlation of the source parameters u_τ (Fig. 1) is exploited. The *a posteriori* probabilities can be calculated recursively, because the source parameters can be modelled as a 1st order Markov process [1]:

$$P(y_{\kappa,\tau}|z_\tau, Z_{\tau-1}) = C \cdot p(z_\tau|y_{\kappa,\tau}) \cdot \sum_{\vartheta=1}^{2^M} P(y_{\kappa,\tau}|y_{\vartheta,\tau-1}) \cdot P(y_{\vartheta,\tau-1}|z_{\tau-1}, Z_{\tau-2}) \ . \quad (9)$$

The *minimum mean square error* (MMSE) between $\overline{u}_\kappa$ and $\hat{u}_\tau$ is an appropriate and established error criterion [1, 5]. The last step of the soft demodulation algorithm is the estimation of the source parameter by a so-called *mean square* (MS) estimator which is based on the MMSE [1]:

$$\hat{u}_\tau = \sum_{\kappa=1}^{2^M} \overline{u}_\kappa \cdot P\left(\Gamma^{-1}(\mu^{-1}(y_{\kappa,\tau}))|z_\tau, ...\right) \ . \quad (10)$$

4. UNEQUAL POWER ALLOCATION

In this section we develop the *modulation with unequal power allocation* (MUPA) for higher order modulation schemes. MUPA distributes the transmission energy unequally to the modulation signal points by applying the so-called weights w_i, $i = 1, 2, ...M$, which determine the distances between the signal points. In the example of Fig. 2 a conventional 16-QAM scheme with Gray labeling (Fig. 2 a)) [6] is compared to the modified scheme (Fig. 2 b)) with unequal weights w_i, $i \in \{1, 2, 3, 4\}$. The distances of the signal points are changed by the weights w_i such that, e.g., w_1 is responsible for the error events of the MSB (e.g., signal points labeled 1001 and 0001).

From now on, we skip the time index τ, because the following calculations are independent of τ. We assume perfect knowledge of σ_n^2 at the transmitter and at the receiver.

4.1. Approach

If a hard decision demodulator selects the wrong signal point out of $\mathcal{Y}$ due to AWGN, the distortion d between the quantized source sample and the estimated sample occurs. Our goal is to minimize the expected value $\mathrm{E}\{d^2\}$, i.e., the MSE, by optimizing the weights w_i, $i \in \{1, 2, ...M\}$. The specific distortion with regard to the quantizer representation levels is $d_{\kappa,\eta} = |\overline{u}_\kappa - \hat{u}_\eta|$, $\kappa, \eta = 1, 2, ...2^M$. With the focus on the modulation, $\mathrm{E}\{d^2\}$ depends on the probability of occurrence $P(y_\kappa)$ of the signal points in the signal constellation set and on the probability that

$y_\kappa = \mu(\Gamma(\overline{u}_\kappa)) \in \mathcal{Y}$ was sent and $\hat{y}_\eta = \mu(\Gamma(\hat{u}_\eta)) \in \mathcal{Y}$ is received, i.e., the transition probability $P(\hat{y}_\eta|y_\kappa)$. Consequently, the MSE

$$\mathrm{E}\{d^2\} = \sum_{\kappa=1}^{2^M}\sum_{\eta=1}^{2^M} d^2_{\kappa,\eta} \cdot P(y_\kappa) \cdot P(\hat{y}_\eta|y_\kappa) \tag{11}$$

has to be minimized. The probability of occurrence $P(y_\kappa)$ equals $P(\mu(\Gamma(\overline{u}_\kappa)))$ of the quantizer reproduction levels. The transition probability $P(\hat{y}_\eta|y_\kappa)$ is the probability, that the channel noise exceeds the real-valued geometric distance from y_κ to the decision bound between y_κ and $\hat{y}_\eta$, i.e., the weight w_i. In this case, the demodulator selects $\hat{y}_\eta$ due to the channel noise although y_κ was sent. Consequently, for AWGN channels $P(\hat{y}_\eta|y_\kappa)$ can be calculated as Gaussian probability:

$$P(\hat{y}_\eta|y_\kappa) = \int_{w_i}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_n} \cdot \exp\left(\frac{-\xi^2}{2\sigma_n^2}\right) \,\mathrm{d}\xi \tag{12}$$

with w_i corresponding to $\hat{y}_\eta$ and y_κ. If the upper limit of the integral is set to infinity, only the nearest neighbor of y_κ is considered. Otherwise, the upper limit has to be adjusted according to the specific signal constellation set.

The minimization of (11) is constrained by the transmission energy. As additional signal energy increases interference, e.g., in transmission systems for mobile communications, we restrict the transmission energy by normalizing the average bit energy to 1. Hence, the average energy $\mathrm{E}\{E_y\}$ per complex symbol $y_\kappa \in \mathcal{Y}$ has to be normalized to M:

$$\mathrm{E}\{E_y\} = \sum_{\kappa=1}^{2^M} E_y^{(\kappa)} \cdot P(y_\kappa) = M\ . \tag{13}$$

with the energy $E_y^{(\kappa)} = ||y_\kappa||^2$, $\kappa = 1, 2, ...2^M$, depending on the weights w_i, $i = 1, 2, ...M$. Recapitulating, the problem to calculate weights w_i minimizing the mean square error $\mathrm{E}\{d^2\}$ is described by the non-linear system of the two equations, (11) and the energy constraint (13).

4.2. Lagrange Multiplier Method (LMM)

An approach to solve a minimization problem with a constraint is the LMM with the *Lagrange Multiplier* λ. For this purpose we set up the *Lagrange* equation combining (11) with (13)

$$\begin{aligned} L(w_1, w_2, ...w_M, \lambda) \quad &= \quad \left(\sum_{\kappa=1}^{2^M}\sum_{\eta=1}^{2^M} d^2_{\kappa,\eta} \cdot P(y_\kappa) \cdot P(\hat{y}_\eta|y_\kappa)\right) \\ &- \quad \lambda\left(M - \mathrm{E}\{E_y\}\right) . \end{aligned} \tag{14}$$

This equation describes a non-linear function $L : \mathbb{R}^{M+1} \rightarrow \mathbb{R}$ with $M+1$ unknown variables, i.e., the M weights w_i and the additional *Lagrange multiplier* λ. Next, we calculate the M+1 partial derivatives $\partial L(w_1, w_2...M, \lambda)/\partial w_i$ and $\partial L(w_1, w_2...M, \lambda)/\partial \lambda$ of (14).

4.3. Newton Algorithm

In the next LMM step the roots w_i and λ of the $M+1$ derivatives have to be calculated. Due to the transcendent function $\exp(x)$, we employ the $(M+1)$-dimensional *Newton* algorithm [7] to calculate the roots. For convenience, we insert λ into the set of variables by renaming $(w_1, ...w_M, \lambda)^T$ to $\underline{v} = (v_1, ...v_{M+1})^T$ with $v_i = w_i$ for $i = 1, 2, ...M$, and $v_{M+1} = \lambda$. We define $L_i'(\underline{v}) = \partial L(\underline{v})/\partial v_i$, $i = 1, 2, ...M+1$, and $\underline{L}'(\underline{v}) = (L_1'(\underline{v}), ...L_{M+1}'(\underline{v}))^T$ for the vector of the first partial derivations. With this notation, the *Newton* iteration rule is given by

$$\underline{v}_{k+1} = \underline{v}_k - J_L^{-1}(\underline{v}_k) \cdot \underline{L}'(\underline{v}_k) \tag{15}$$

with the iteration counter k and the inverse *Jacobian* matrix $J_L^{-1}(\underline{v}_k)$ containing the first partial derivations of $L'(\underline{v})$, i.e., the second partial derivations of $L(\underline{v})$, $L''(\underline{v}) = \partial L_i'(\underline{v})/\partial v_j$, for all combinations of $i, j = 1, 2, ...M+1$. The number of iterations can be reduced by adapting λ in each iteration k according to the difference between $\underline{v}_{k+1}$ and $\underline{v}_k$.

When the exit condition $||\underline{v}_{k+1} - \underline{v}_k|| < \epsilon$ is fulfilled, the LMM supplies the weights w_i as optimized solution. It is very difficult to check this solution for minimum analytically due to the non-linearity of (11) and (13). For simplification, we take other sets of weights with w_i^* fulfilling (13) and close to w_i. If the $\mathrm{E}\{d^2\}$ of (11) is larger with the w_i^* than with the w_i, we suppose that the optimized w_i are the solution for a minimum. The simulation results in the next section show that this strategy is successful.

5. SIMULATION RESULTS

In this section, the performance of soft demodulation (SDM) (Sec. 5.1) and MUPA (Sec. 5.2) is evaluated in terms of the *parameter signal-to-noise ratio* (parameter SNR) between the original codec parameters u_τ (Fig. 1) and its reconstruction $\hat{u}_\tau$. As the parameter SNR is an important measure for, e.g., the audio or video quality, we utilize it to compare the performance of SDM and MUPA with hard decision demodulation and modulation schemes with fixed signal constellations. We use

- a source signal with the Gaussian probability density function (pdf), $\sigma_u^2 = 1$, and correlated source parameters (a = 0.9),
- a symmetric *Lloyd-Max* quantizer (LMQ) which is pdf-optimized for the source signal,
- Natural Binary (NB) as IA function Γ,
- the 16-QAM modulation scheme of Fig. 2 with μ: Gray.

5.1. Soft Demodulation (SDM)

At first, we set $w_i = 1.0\ \forall\ i$. The exploitation of the source parameter correlation in the SDM working on signal point level causes performance improvements in a wide range of E_b/N_0 values (see Fig. 3). As example, we specify the maximum parameter SNR gain of 6.52 dB which is achieved at E_b/N_0 = 4 dB by SDM with 1st order *a priori* knowledge (AK1) in comparison to hard decision demodulation. SDM with no or 0th order *a priori* knowledge (NAK and AK0, respectively) still attains parameter SNR gains of 0.8 to 0.9 dB and 1.5 to 1.8 dB, respectively, for E_b/N_0 values in the range of 3 to 6 dB.

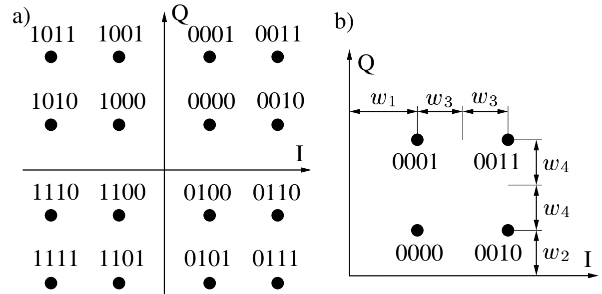

Figure 2: a) Gray mapped bit patterns of 16-QAM signal points b) first quadrant of a) but with unequal weights w_i as example for all quadrants

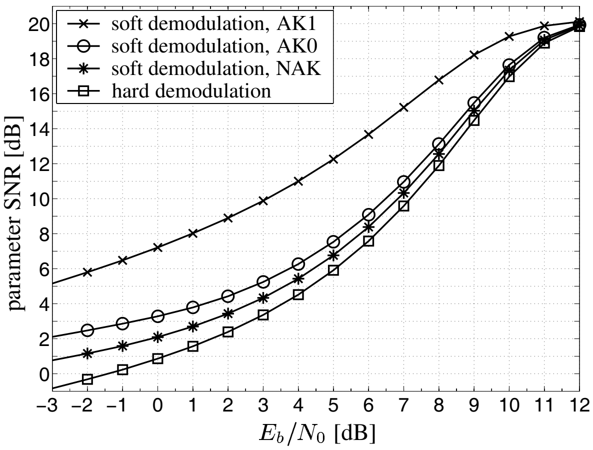

Figure 3: Parameter SNR for soft demodulation with different orders of *a priori* knowledge compared to hard decision demodulation (LMQ, 4 bits, Γ: NB, μ: Gray (Fig. 2), $w_i = 1.0 \; \forall \; i$)

5.2. Weighted Signal Points

According to (13) the 16-QAM of Fig. 2 is constrained by

$$\sum_{\kappa=1}^{16} E_y^{(\kappa)} \cdot P(y_\kappa) = 4 \; . \tag{16}$$

The calculation of $P(\hat{y}_\eta|y_\kappa)$ in (12) is executed only with the nearest neighbors of each signal point y_κ to reduce the computational complexity of the LMM. However, this simplification could cause computational problems at very low E_b/N_0 values.

If the LMM/Newton algorithm provides a *local* minimum instead of a *global* minimum, the MSE of (11) is minimized in the sense of the local minimum. Although the algorithm could provide suboptimal solutions in the sense of *minimum* MSE and we only consider the next neighbors of each signal point, significant performance improvements can be reached with the optimized weights compared to transmission systems without weighted signal points.

SDM with AK1 reaches the highest performance in terms of parameter SNR (Fig. 3), but can still be improved by MUPA. Although the weights w_i are calculated with the assumption of a hard decision demodulator, Fig. 4 demonstrates the higher performance of SDM with MUPA in case of AK1 by unequal power allocation of the signal points. As example, the maximum parameter SNR gain of 1.82 dB is achieved for SDM with AK1 at E_b/N_0 = 2 dB. The performance of SDM with less *a priori* information (NAK and AK0) or hard decision demodulation is also improved by MUPA (Fig. 5). The parameter SNR gains outperform the ones for AK1. Compared to the corresponding system with a fixed signal constellation, i.e., $w_i = 1.0 \; \forall \; i$, and with the focus on the maximum values, SDM with AK0 is improved at most by 4.32 dB at E_b/N_0 = 4 dB, SDM with NAK by 4.39 dB at E_b/N_0 = 4 dB, and hard decision demodulation by 3.55 dB at E_b/N_0 = 5 dB.

Further simulations have shown, that even if transmitter and receiver choose the wrong (but the same, due to, e.g., a small feedback) set of weights because of an imprecise estimation of the E_b/N_0, the MUPA technique improves the parameter SNR. In this case the performance loss is reduced, if the weights of a higher E_b/N_0 are chosen instead of weights for a lower E_b/N_0.

6. CONCLUSIONS

The soft demodulation (SDM) presented in this paper enhances the SBSD for the use with higher order modulation schemes by source parameter estimation on modulation signal point level. Parameter SNR gains of up to 6.52 dB are achieved for SDM with AK1 with 16-QAM and 4 bit LMQ compared to systems with hard decision demodulation. MUPA adjusts the energy of the modulation symbols by varying the distances between the signal points according to the differing bits of the assigned bit patterns. MUPA combined with SDM outperforms systems solely based on SDM by up to 1.82 dB with AK1 and up to 4.39 dB with NAK.

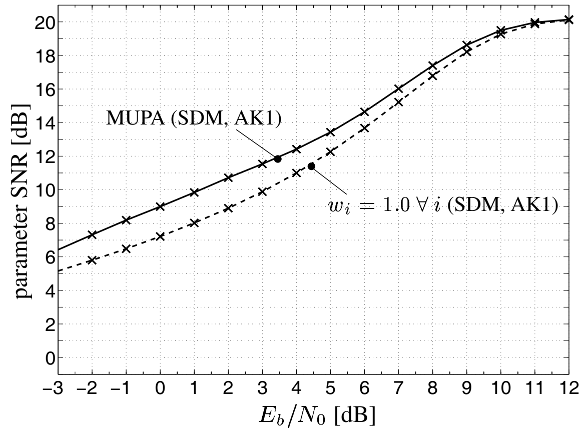

Figure 4: 16-QAM (SDM, AK1) with $w_i = 1.0 \; \forall \; i$ (dashed line) and with MUPA (solid line)

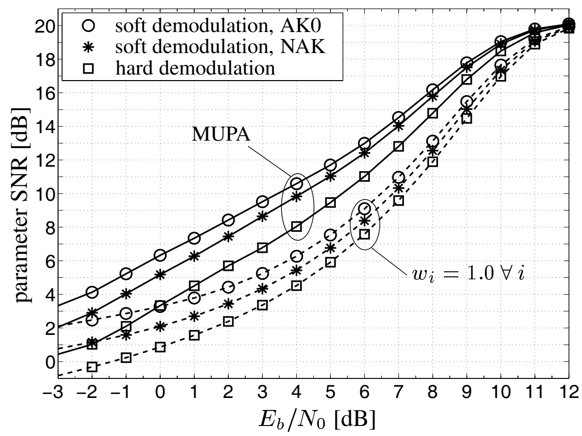

Figure 5: 16-QAM with $w_i = 1.0 \; \forall \; i$ (dashed lines) and with MUPA (solid lines)

REFERENCES

[1] T. Fingscheidt and P. Vary, "Softbit Speech Decoding: A New Approach to Error Concealment," *IEEE Trans. Speech Audio Processing*, pp. 240–251, Mar. 2001.

[2] M. Speth and A. Seeger, "Wireless Transmission Using Hierarchical Modulation and Coding: Implications for System Design," in *Proc. of the 3rd ITG Conference on Source and Channel Coding*, München, Germany, Jan. 2000, pp. 193–198.

[3] Edward Bedrosian, "Weighted PCM," *IRE Transactions on Information Theory*, vol. IT-4, pp. 45–49, Mar. 1958.

[4] N. Görtz and E. Bresch, "Source-Adaptive Power Allocation for Digital Modulation," *IEEE Commun. Lett.*, vol. 7, pp. 569–571, Dec. 2003.

[5] J. L. Melsa and D. L. Cohn, *Decision and Estimation Theory*, McGraw-Hill, 1978.

[6] W. T. Webb, L. Hanzo, *Modern Quadrature Amplitude Modulation*, Pentech Press, 1994.

[7] W. H. Press, S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, *Numerical Recipes in C*, Cambridge University Press, 1992.