A NEW LOW-COMPLEXITY DEMAPPER FOR HIGH-PERFORMANCE ITERATIVE MIMO: INFORMATION-THEORETIC AND BER ANALYSES

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ABSTRACT

Space-time bit interleaved coded modulation with iterative detection has been recognized as a method for achieving near-capacity performance using multiple-input multiple-output (MIMO) systems. However, the maximum-likelihood/joint-detection, required for this method, exhibits prohibitive implementation complexity at high rates (≥15 bps/Hz). With a view to enabling practical implementations of high spectral-efficiency wireless communications, this paper introduces a new single-stream detection approach using spatial-filtering and soft-cancellation. This method also exhibits superior performance relative to the conventional multi-stream detection approach as long as there are as many receive elements as there are transmit streams. The proposed method is evaluated in terms of bit error rate performance and information-theoretic considerations using an EXIT (Extrinsic Information Transfer) chart method.

1. INTRODUCTION

Recently, space-time bit-interleaved coded modulation (STBICM) using MIMO has been proposed as a means of realizing high-rate (≥15 bps/Hz) wireless communications with near-capacity performance [1]. However, due to the complexity of the demapper (or the inner decoder, in BICM parlance), this method is not amenable to practical implementation. The demapper is required to perform a joint-detection considering the spatial coupling among all of the transmitted streams, and consequently, has an exponential complexity in the number of streams. To reduce the complexity, an approximate demapper using list sphere detection [1] has been proposed. Notwithstanding the complexity reduction, the sphere detection methods still suffer from significant complexity for high rate systems. Further, they also introduce certain *ad hoc* fine-tuning requirements such as the adaptation of the sphere's search radius and handling of missing candidates from the list.

In this paper, we introduce a new low-complexity high-performance single-stream or single-input single-output (SISO) demapper using spatial-filtering and soft-cancellation (SC), and refer to it as the STBICM-SC approach. Here the demapper operates on a per-stream basis, and consequently, no longer has an exponential complexity in the number of streams. Further, the use of soft-cancellation in conjunction with spatial-filtering actually enhances the performance compared to the conventional multi-stream demapper approach as long as there are as many receive elements as there are transmit streams. We note that a similar single-stream approach has been applied in the context of iterative multi-user detection [2].

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2. STBICM SYSTEM MODEL

Consider a MIMO system with N_t transmit and N_r receive antennas. The information bit sequence is first encoded and interleaved. The interleaved coded sequence is then split into binary-valued vectors $\mathbf{x} = [\mathbf{x}_1, \ldots, \mathbf{x}_{N_t}]^{\mathrm{T}}$ of size $N_t M \times 1$ with $\mathbf{x}_i = [x_{i1}, \ldots, x_{iM}]$. The vector $\mathbf{x}$ is mapped into a $N_t \times 1$ symbol vector $\mathbf{s} = [s_1, \ldots, s_{N_t}]^{\mathrm{T}}$ for transmission on N_t antennas. The symbols $s_i = \mathrm{map}(\mathbf{x}_i)$ are chosen from a complex constellation of size 2^M. The symbols are transmitted through the $N_r \times N_t$ matrix channel $\mathbf{H}$. The channel is assumed to be flat Rayleigh-fading and spatially independent with unity gain for each channel coefficient. The vector of N_r received complex symbols corrupted by AWGN is given as

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n} \tag{1}$$

where $\mathbf{n}$ represents the noise vector whose elements are complex zero-mean Gaussian with variance $\sigma_n^2 = N_0/2$ per real dimension.

At the receiver, as shown in Fig. 1, the demapper takes the observation $\mathbf{y}$, knowledge of the channel $\mathbf{H}$, and *a priori* knowledge L_{A1} (available in the 2nd pass from the decoder output) on the inner coded bits, and computes new (extrinsic) information L_{E1} for each of the $N_t M$ coded bits per channel use. The L-values are the computed log-likelihood ratios. The extrinsic information is deinterleaved and becomes the *a priori* input L_{A2} to the outer decoder. The outer decoder further refines the extrinsic information L_{E2} on the coded bits given its knowledge of the temporal coupling of the bits. This extrinsic information is then reinterleaved and fed back as *a priori* knowledge L_{A1} to the inner detector thus completing one cycle or iteration. With each iteration, the *a priori* information is improved. The information exchange continues until the desired performance is achieved. The bits are detected by making hard decisions on L'_{D2} which represents the LLR value of the information bits. The outer soft-in soft-out decoder may be implemented using the BCJR algorithm [3].

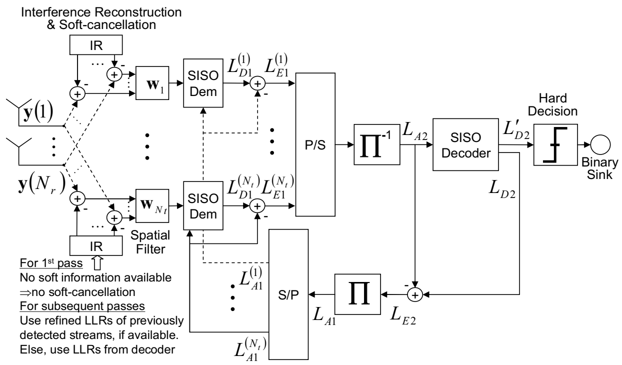

Fig. 1. STBICM-SC receiver using a SISO demapper.

2.1 MIMO Demapper

This section presents the conventional multi-stream or MIMO demapper LLR calculation. Using a standard Max-log approximation [1], the LLR calculation for the n th ($n = 1,\ldots,M$) bit of the k th ($k = 1,\ldots,N_t$) stream in a channel use is given as

$$L_{D1}(k,n) \approx \frac{1}{2}\max_{\mathbf{X}_{kn,+1}}\left\{-\frac{1}{\sigma_n^2}\|\mathbf{y}-\mathbf{H}\mathbf{s}(\mathbf{x})\|^2+\mathbf{x}^T\mathbf{L}_{A1}\right\} - \frac{1}{2}\max_{\mathbf{X}_{kn,-1}}\left\{-\frac{1}{\sigma_n^2}\|\mathbf{y}-\mathbf{H}\mathbf{s}(\mathbf{x})\|^2+\mathbf{x}^T\mathbf{L}_{A1}\right\} \tag{2}$$

where $\mathbf{X}_{kn,b}$ denotes all possible $N_t M \times 1$ bit vectors $\mathbf{x}$ whose entry corresponding to the n th bit of the k th stream has value b, $\mathbf{s}(\mathbf{x})$ is the mapping from bits to symbol, and $\mathbf{L}_{A1}$ is the $N_t M \times 1$ vector containing *a priori* values corresponding to bit vector $\mathbf{x}$. The extrinsic LLR $L_{E1}(k,n)$ is calculated by subtracting the *a priori* value $L_{A1}(k,n)$ from $L_{D1}(k,n)$. Perfect channel knowledge is assumed for the analysis herein. From (2), it follows that for a specified bit position, each of the two terms in the LLR computation requires hypothesizing over $2^{(MN_t-1)}$ bit vectors. This exponential complexity in the product of M and N_t is prohibitive for a high spectral-efficiency MIMO system. In this case, a reduced complexity demapper employing a sphere detection approach is used [1].

2.2 SISO Demapper for STBICM-SC

The single-stream demapper works in conjunction with a max-SINR spatial filter and a soft-canceller. These are described first. The max-SINR spatial filter is essentially a matched filter that maximizes the ratio of the average power of the desired stream to that of the sum of the "other-stream" (like multiple access interference) interference and noise. The SINR for the k th stream is maximized by applying a weight vector $\mathbf{w}_k$ to the received signal $\mathbf{y}$. The max-SINR filter is described next. Separating the received signal vector into a desired k th stream and interfering streams, and applying the filter to this decomposed signal, we have

$$\mathbf{w}_k^H\mathbf{y} = \mathbf{w}_k^H\mathbf{h}_k s_k + \mathbf{w}_k^H\mathbf{H}_{\bar{k}}\mathbf{s}_{\bar{k}} + \mathbf{w}_k^H\mathbf{n} \tag{3}$$

where $\mathbf{h}_k$ is the k th column vector of channel matrix $\mathbf{H}$ (representing the channel response of the k th stream) and s_k is the desired symbol of the k th stream. The remaining symbols on other streams are collected in vector $\mathbf{s}_{\bar{k}}$. The collection of the channel responses for these interfering streams is denoted by $\mathbf{H}_{\bar{k}}$. The SINR for the k th stream is then computed as

$$SINR_k = \frac{E_s\mathbf{w}_k^H\mathbf{h}_k\mathbf{h}_k^H\mathbf{w}_k}{\left(E_s\mathbf{w}_k^H\mathbf{H}_{\bar{k}}\mathbf{H}_{\bar{k}}^H\mathbf{w}_k + 2\sigma_n^2\mathbf{w}_k^H\mathbf{w}_k\right)} = \frac{\mathbf{w}_k^H\mathbf{A}\mathbf{w}_k}{\mathbf{w}_k^H\mathbf{B}\mathbf{w}_k}$$

where the interference-plus-noise covariance vector $\mathbf{B} = \left(E_s\mathbf{H}_{\bar{k}}\mathbf{H}_{\bar{k}}^H + 2\sigma_n^2\mathbf{I}_{N_r}\right)$ and the desired stream covariance matrix $\mathbf{A} = E_s\mathbf{h}_k\mathbf{h}_k^H$. Then, the spatial-filter $\mathbf{w}_k$ that maximizes $SINR_k$ is known to be the eigenvector corresponding to the maximum eigenvalue of $\mathbf{B}^{-1}\mathbf{A} = \left(E_s\mathbf{H}_{\bar{k}}\mathbf{H}_{\bar{k}}^H + 2\sigma_n^2\mathbf{I}_{N_r}\right)^{-1}\left(\mathbf{h}_k\mathbf{h}_k^H\right)$ [5].

Next, we describe the mechanics of the soft-cancellation process. In the first pass, no *a priori* information is available for symbol reconstruction, and hence, there is no opportunity for cancellation. In the subsequent passes, as *a priori* information becomes available, the interfering symbols are reconstructed and cancelled. The soft-symbol reconstruction, given the *a priori* bit-wise LLRs, is performed using the following expression.

$$\hat{s} = \sum_{i=1}^{2^M} a_i \Pr(s = a_i) = \sum_{i=1}^{2^M} a_i \prod_{n=1}^{M} \frac{1}{1+\exp\left(-b_n^{(i)}L_n\right)} \tag{4}$$

where $\{a_i\}$ represents the 2^M symbols of the chosen constellation, $\Pr(s = a_i)$ is the probability that the symbol in question takes the value a_i, L_n is the LLR for the n th bit, and $b_n^{(i)}$ is the value of the n th bit representing symbol a_i. During the 2nd pass, interference reconstruction uses the interleaved decoder LLR $\Pi(L_{D2})$. This is followed by cancellation, nulling, and demapping. At the end of demapping, a new set of M LLR values $L_{D1}^{(1)}$ is available for the 1st stream. The soft-symbol estimate for the 1st stream based on this updated LLR, as opposed to the older decoder LLR, is used by cancellers of subsequent streams to remove the contribution of the 1st stream. This process of using updated LLRs from the demapper for symbol reconstruction continues as demapping proceeds sequentially from one stream to the next. Notice that the last demapped stream has the benefit of using updated LLRs to reconstruct all of its interfering streams whereas as the first stream has only the older decoder LLR to reconstruct its interference. Naturally, the demapping process is ordered from the most reliable stream to the least so that the least reliable stream benefits from the best cancellation leading to the maximum possible diversity. The reliability metrics are computed once for all streams using the decoder LLRs, and is given as $\prod_{n=1}^{M}\tanh^2(L_n/2)$. After forming the soft-symbol values for all of the interfering symbols, their channel filtered versions are subtracted from the received signal vector as

$$\underline{\mathbf{y}}_k = \mathbf{y} - \mathbf{H}_{\bar{k}}\hat{\mathbf{s}}_{\bar{k}} \tag{5}$$

where $\hat{\mathbf{s}}_{\bar{k}}$ is the collection of the reconstructed soft-symbols for the interfering streams, and $\underline{\mathbf{y}}_k$ is the "cleaned" vector that becomes the input to the spatial-filter and the demapper for the k th stream.

When cancellation takes place, the covariance matrix of the residual interference is expressed as $E_s \mathbf{H}_{\bar{k}}\left(\mathbf{I}_{N_t-1} - \boldsymbol{\rho}\right)\mathbf{H}_{\bar{k}}^H$. Here, $\boldsymbol{\rho}$ notionally represents the fidelity of the interference reconstruction and depends on the quality of the *a priori* information. For perfect knowledge ($|L| \to \infty$), $\boldsymbol{\rho} \to \mathbf{I}_{N_t-1}$, and results in perfect cancellation. In this case, the spatial-filtering that follows realizes the maximum diversity benefit since no dimensions are sacrificed for the nulling of interfering streams. Conversely, when there is no *a priori* information ($L = 0$), $\boldsymbol{\rho} \to \mathbf{O}_{N_t-1}$, and results in 100% retention of interference (or no diversity gain). The metric $\boldsymbol{\rho}$ is a diagonal matrix with its i th diagonal element ρ_i ($0 \le \rho_i \le 1$) representing the fidelity of the i th symbol reconstruction. The fidelity of the symbol reconstruction using the LLR values of the M bits that constitute the i th symbol is given as $\rho_i = \prod_{n=1}^{M} \tanh^2\left(L_n/2\right)$. Note that ρ_i equals one, only when all M bits are perfectly known. If knowledge of any of the bits is totally unreliable, then ρ_i equals zero. Lastly, when cancellation takes place, the spatial-filter should use no more than the required spatial resources to remove the residual interference so that diversity is maximized. Therefore, when cancellation is possible, $\mathbf{B} = \left(E_s \mathbf{H}_{\bar{k}}\left(\mathbf{I}_{N_t-1} - \boldsymbol{\rho}\right)\mathbf{H}_{\bar{k}}^H + 2\sigma_n^2 \mathbf{I}_{N_r}\right)$ should be used in the spatial-filter design equation described earlier. The per-stream fidelity should be updated in the same manner as the reconstructed symbols are using LLRs from previous demappers.

Assuming that the effect of interfering streams has been removed through spatial-filtering, the MIMO demapper may now be approximated as a single-stream or SISO demapper with the LLR calculation given as

$$L_{D1}(k,n) \approx \frac{1}{2}\max_{\underline{\mathbf{x}}_{n,+1}}\left\{-\frac{1}{\sigma_{\underline{n}_k}^2}\left\|\mathbf{w}_k^H \underline{\mathbf{y}}_k - \mathbf{w}_k^H \mathbf{h}_k s(\underline{\mathbf{x}})\right\|^2 + \underline{\mathbf{x}}^T \underline{\mathbf{L}}_{A1}^{(k)}\right\} - \frac{1}{2}\max_{\underline{\mathbf{x}}_{n,-1}}\left\{-\frac{1}{\sigma_{\underline{n}_k}^2}\left\|\mathbf{w}_k^H \underline{\mathbf{y}}_k - \mathbf{w}_k^H \mathbf{h}_k s(\underline{\mathbf{x}})\right\|^2 + \underline{\mathbf{x}}^T \underline{\mathbf{L}}_{A1}^{(k)}\right\} \qquad (6)$$

where $\underline{\mathbf{X}}_{n,b}$ denotes the set of $M \times 1$ bit vectors $\underline{\mathbf{x}}$ whose n th bit value is b, $\underline{\mathbf{L}}_{A1}^{(k)}$ is an $M \times 1$ vector containing the *a priori* information for the k th stream, and $\sigma_{\underline{n}_k}^2$ is the noise variance per real dimension of the filtered noise $\mathbf{w}_k^H \mathbf{n}$.

2.3 Discussion on Complexity

In contrast to the MIMO demapper, the SISO demapper in the STBICM-SC considers only the coupling between the M bits of a symbol resulting in a $O(2^M)$ complexity. Additionally, it also requires one matrix-inverse computation and Eigenvalue decomposition per stream. The complexity of these operations are polynomial, and depending on the implementation, varies from $O(N_r^2)$ to $O(N_r^3)$. As discussed earlier, the MIMO demapper considers the coupling between all N_t streams resulting in $O(2^{MN_t})$ complexity. Therefore, the savings using STBICM-SC

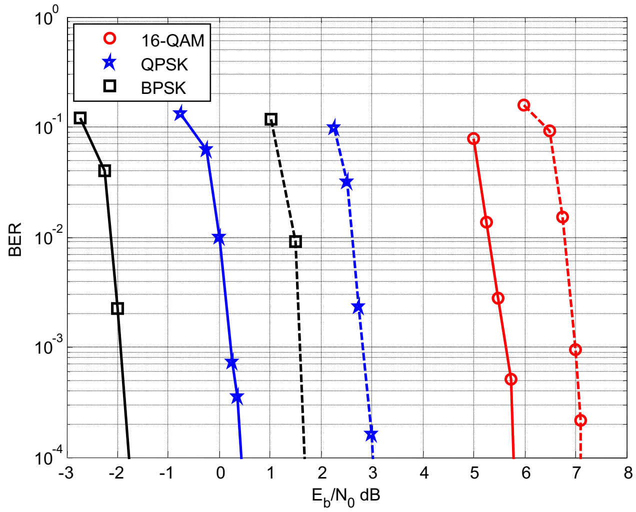

Fig. 2. BER performance of an 8×8 system; STBICM (dash line) and STBICM-SC (solid line).

is enormous for high-rate MIMO with large MN_t. For example, consider a 16 bps/Hz system using a ½ rate turbo code, with 8 transmit and receive elements and 16-QAM modulation. The complexity of the MIMO demapper is of order $2^{32} \approx 4 \times 10^9$, and that of the STBICM-SC demapper is of the order 10^2 to 10^3.

3. PERFORMANCE ANALYSIS

3.1 BER Performance

Fig. 2 shows the BER performance for the conventional and proposed STBICM approaches. A rate-½ turbo code, as specified in [1], is used with an 8×8 MIMO configuration with various gray-mapped constellations. The conventional MIMO demapper for $MN_t > 8$ is realized using the list sphere detection approach [1]. Four iterations were used along with an interleaver size of 9216 information bits. From the figure, we observe that the STBICM-SC approach outperforms the conventional approach in all cases; turbo cliffs are reached at lower E_b/N_0 values using STBICM-SC. E_b in E_b/N_0 denotes energy per information bit.

3.2 EXIT Chart Analysis

An EXIT (extrinsic information transfer) chart [4] analysis is provided to gain insight into the superior performance of the STBICM-SC method. EXIT charts are a useful way to predict performance of concatenated coding schemes using iterative decoding. They display the mutual information (MI) transfer function (TF) of both demapper and decoder on the same plot. Fig. 3 shows the EXIT charts for the SISO and MIMO demappers corresponding to the BPSK curves shown in Fig. 2. The axis labels $I_{a,q}$ and $I_{e,q}$ denote the MI between transmitted bits and the *a priori* and *a posteriori* extrinsic LLR values, respectively, and $q = dem$ and $q = dec$ indicates the source of the LLRs to be the demapper or decoder, respectively. As a useful benchmark, we also plot two extreme cases of cancellation; one with

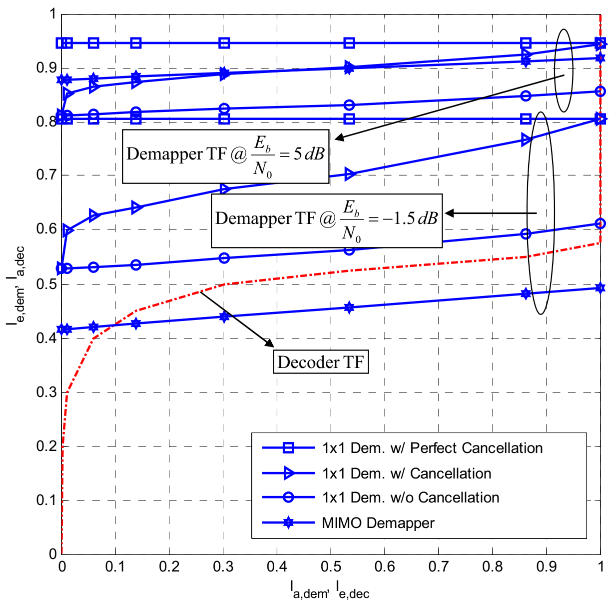

Fig. 3. EXIT chart for 8×8 BPSK

perfect knowledge of interference resulting in perfect cancellation, and the other with no cancellation at all. Note that all SISO demappers shown here use spatial-filtering regardless of the choice or quality of cancellation. Our proposed method (denoted as 1x1 Dem w/ cancellation in Fig. 3) lies in between these two extremes of perfect cancellation and no cancellation since it uses cancellation based on the quality of the soft information. That is why when there is no information about the other bits (represented by $I_{a,dem} = 0$), the SC method's performance matches that of the no-cancellation method. Conversely, when there is perfect knowledge of the other bits (represented by $I_{a,dem} = 1$), the performance of the SC method matches that of the perfect-cancellation case.

The demapper TFs are shown for two E_b/N_0 values of -1.5 dB and 5 dB. At an E_b/N_0 of -1.5 dB, it is seen that a tunnel opens up between the demapper and decoder TFs for the STBICM-SC demapper whereas it is pinched off at {0.1, 0.42} for the MIMO demapper. This corroborates the superior BER performance in Fig. 2 (see BPSK curve) for the STBICM-SC method whose turbo cliff occurs below an E_b/N_0 of -1.5 dB. The 2nd E_b/N_0 value of 5 dB allows both methods to converge to a low BER solution since the intersection points for both are sufficiently close to the {1, 1} point. For both E_b/N_0 values, the perfect-cancellation method has the best performance. The performance of the proposed method and the no-cancellation method remains the same relative to the perfect cancellation method. They collectively move up or down depending on whether the E_b/N_0 increases or decreases, respectively. However, the performance of the MIMO demapper method relative to the SC method is not constant and depends on the actual E_b/N_0 value. At low E_b/N_0 values, its performance is worse than the SC method. But as the E_b/N_0 increases, it begins to surpass the SC method. These observations suggest the following: 1) in a moderate to high E_b/N_0 region, useful self-information can be gleaned from other bits, and therefore, a joint-decision approach (equivalent to the MIMO demapper) is better, and 2) in the low E_b/N_0 region, when knowledge of the other bits is not very reliable, it is better to remove their effects (through a combination of cancelling and nulling), and then perform a single-user detection (equivalent to the SISO demapper).

Finally, we comment on the effect of the outer codec and N_r in achieving this superior performance. First, the turbo code considered here has a very sharp transition in that the output MI $I_{e,dec}$ shows a sharp increase from 0.1 to 1 as the input MI $I_{a,dec}$ increases from 0.42 to 0.58. If we use a code with a more gradual transition, it is possible that an intersection point near {1, 1} will not occur at a low E_b/N_0 value but only at a high E_b/N_0 value where the MIMO demapper performs better. Second, as the number of receive elements decreases given a fixed number of transmit streams, the performance superiority will decrease.

4. CONCLUSION

Low complexity demapping using spatial-filtering and soft-cancellation has been proposed for STBICM. When there are as many receive antennas as there are transmit streams, the BER performance of the proposed approach using a SISO demapper is superior to that of the conventional STBICM approach. Insight into this superior performance is provided from an information-theoretic perspective using an EXIT chart analysis.

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