REDUCED-COMPLEXITY ML DETECTION FOR CODED MIMO SYSTEMS USING AN ABSOLUTE-VALUE SEARCH

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ABSTRACT

Spatial multiplexing of data streams on multiple transmit antennae is a well known means of increasing the data-rate in a bandwidth constrained system. At the receiver, maximum likelihood (ML) detection offers the best performance. However, for large constellations and/or large number of antennae the complexity of the decoder grows exponentially. Sphere-decoding is one suboptimal decoding method that has been proposed to reduce complexity by reducing the size of the search space for determining the optimum transmitted symbol vector. In this paper, we present an alternative approach, using an absolute-value based distance for searching, that keeps the size of the search space the same, but reduces the complexity by reducing the number of multiplications required to evaluate the quantities being compared in the search space. This method has vastly reduced complexity compared to ML. Simulation results will be presented with a bit-interleaved coded modulation (BICM) system to show that this method has less than 0.5 dB performance loss as compared to ML detection.

1. INTRODUCTION

In recent years there has been increasing interest in building multiple-antennae wireless systems capable of improving the channel capacity in a fixed bandwidth [1]. One of the ways of doing so is spatial multiplexing, where multiple coded data streams are transmitted simultaneously over two or more antennae and received with the same or greater number of antennae. The optimal receiver in this scenario is a maximum-likelihood (ML) receiver that finds the soft-metric for each bit in a transmitted symbol vector by searching for the best log-likelihood ratio over all possible transmitted symbol combinations. In practice, the log-likelihood ratio is approximated by the minimum Euclidean distance metric [2]. Since a MIMO system transmits a N_T dimensional symbol vector, where N_T is the number of transmit antennae, the complexity of this search, in terms of number of multiplications required, grows exponentially. This complexity is considered to be too high for a practical implementation. Hence, most implementations use simpler, but suboptimal methods based on linear filtering, like minimum-mean-squared-error (MMSE) or zero-forcing (ZF) receivers. These receivers do not exploit the full available channel diversity and therefore suffer in performance as compared to a ML receiver. Hence there has been recent interest in reduced-complexity methods that can approach the ML performance [3]. Recently, sphere-decoding methods [3, 4, 5, 6] have been proposed that attempt to reduce the search space by searching candidate vectors that lie within a certain radius. There are several limitations to these methods, such as varying degree of complexity depending on SNR and choice of radius [5, 6]. The conclusion in [5] was that the complexity of sphere-decoding for *coded* systems is still too high, since a large number of candidates need to be searched to provide good soft metrics for the decoder. In [7] another reduced-search space method for uncoded systems is described, along with a modified absolute-value metric that has an error floor for low bit-error rates. Thus there is still a need for reduced complexity MIMO decoding for coded systems that do not suffer an appreciable performance loss.

In this paper we take a different approach, where the size of the search space remains the same, but a metric based on absolute-value instead of Euclidean distance is used to initially search for the "best" transmitted symbol vector. Once this is found, the actual soft-metric used by the Viterbi decoder could be either based on the absolute-value search metric, or the Euclidean distance metric. The latter method has slightly better performance at lower SNR. Even though the number of comparisons remains the same, the number of multiplications used in the evaluation of each of these compared values is reduced drastically, hence reducing the overall complexity. The performance of this proposed method will be demonstrated via simulations to be very close to the optimal ML method for a BICM system

The rest of the paper is organized as follows. Section 2 describes the signal model, channel assumptions and briefly summarizes the ML, MMSE and ZF methods. Section 3 describes the absolute-value search algorithm and compares the complexity with that of ML. Section 4 presents simulation results comparing the performance of the proposed methods with ML and weighted zero-forcing (WZF). Finally, conclusions are presented in Section 5.

2. BACKGROUND

We assume, without loss of generality, a 2 x 2 spatially multiplexed system, using BICM. On each spatial stream, the data bits are coded by a convolutional code, interleaved, and then mapped into symbols which are chosen from a predetermined constellation C of size $|C| = 2^Q$, where Q is the number of bits per symbol. Hence the received signal model at time k can be written as:

$$\underline{r}_k = H_k \underline{a}_k + \underline{n}_k \tag{1}$$

where $\underline{r}_k = [r_{1,k} \ r_{2,k}]^T$ is the received symbol vector, H_k is the 2 x 2 channel matrix composed of independent, complex Gaussian coefficients, $\underline{a}_k = [a_{1,k} \ a_{2,k}]^T$ is the symbol vector transmitted simultaneously from both antennae and $\underline{n}_k = [n_{1,k} \ n_{2,k}]^T$ is the additive noise, assumed to be complex, Gaussian and independent on each received stream with variance σ_n^2. Each of the symbols $a_{1,k}$ and $a_{2,k}$ belongs to the constellation C and are normalized to have unit variance. Thus the transmitted vector $\underline{a}_k$ contains $2Q$ coded bits. We assume that the channel matrix H_k is known at the receiver for each k and that the channel could be either fast-fading, i.e. H_k is independent for every k, or quasi-static, i.e. H_k stays unchanged for the duration of the packet. Henceforth, for simplicity, we will drop the time index k from the above equation and work with the following signal and channel model:

$$\underline{r} = H\underline{a} + \underline{n} \tag{2}$$

where $\underline{r} = [r_1 \ \ r_2]^T$, $H = [H_{11} \ \ H_{12}; H_{21} \ \ H_{22}]$, $\underline{a} = [a_1 \ a_2]^T$ and $\underline{n} = [n_1 \ n_2]^T$.

2.1. ML decoding

Let a_1 and a_2 be two symbols from the constellation C. Define $X_1 = r_1 - H_{11}a_1 - H_{12}a_2$ and $X_2 = r_2 - H_{21}a_1 - H_{22}a_2$. Define distance $d_E(a_1, a_2)$ as follows:

$$d_E(a_1, a_2) = |X_1|^2 + |X_2|^2 \tag{3}$$

Since a_1 and a_2 are each drawn from the same constellation C, there are $|C|^2$ possible values for $d_E(a_1, a_2)$ that have to be searched for determining the soft-metrics for each of the bits in a_1 and a_2. This is done as follows. Let the Q bits that are mapped to the symbol a_1 transmitted on the first antenna be denoted as $b_0, b_1 \cdots b_{Q-1}$. Let us define the subset of constellation points C_q^p as the set of symbols from the defined constellation C such that $b_q = p$ where p is either 0 or 1.The first step in computing the soft-metric for b_q is to find four symbols $a_{1,q}^{(0)}$, $a_{1,q}^{(1)}$, $a_{2,q}^{(0)}$ and $a_{2,q}^{(1)}$ as follows:

$$\left[a_{1,q}^{(0)}\, a_{2,q}^{(0)}\right] = \arg \min_{\substack{[a_1 \in C_q^0] \\ [a_2 \in C]}} d_E(a_1, a_2) \tag{4}$$

$$\left[a_{1,q}^{(1)} a_{2,q}^{(1)}\right] = \arg \min_{\substack{[a_1 \in C_q^1] \\ [a_2 \in C]}} d_E(a_1, a_2) \tag{5}$$

Then the soft-metric $m(b_q)$ can be defined as:

$$m(b_q) = d_E(a_{1,q}^{(0)}, a_{2,q}^{(0)}) - d_E(a_{1,q}^{(1)}, a_{2,q}^{(1)}) \tag{6}$$

The soft-metrics for each bit in the symbol a_2 transmitted on the second antenna are computed in the same way. These soft metrics are then deinterleaved and used in a soft-decision Viterbi decoder to decode the transmitted bits [2]. It is immediately obvious from equations (3)-(5) above that the complexity of the ML search is proportional to $|C|^2$. There are two factors of interest here, one is the size of the search space, which is $|C|^2$, and the other is the actual computation of each of the values in this search space.It should be noted here that unlike an uncoded system where the search is for a single symbol vector for every bit in a symbol, in a 2 x 2 coded system the search is for 2 symbol vectors per bit, or $4Q$ vectors per transmitted symbol vector, according to equations (4) and (5) above. Hence, it is not possible to reduce the search space as in [7] without incurring a substantial loss in performance. This was the same conclusion reached in [5] as well. Our approach, described in Section 3 will be to keep the size of the search space the same, while reducing the complexity of computation for each of the values in the space.

2.2. Linear Receivers: MMSE and ZF

The ML detector described above does not explicitly separate the data on each spatial stream before computing the soft-metrics for each bit in the transmitted symbol vector. The more commonly used, lower-complexity approach, is to use a linear receiver first to separate the data and then proceed to evaluate the soft-metrics independently on each stream. Let G be a linear transformation of the received vector $\underline{r}$ and $\hat{\underline{a}}$ be the estimate of the transmitted symbol vector $\underline{a}$. Then:

$$\hat{\underline{a}} = G\underline{r} \tag{7}$$

$$\text{MMSE Receiver:} \quad G = H^H(HH^H + \sigma_n^2 I)^{-1} \tag{8}$$

$$\text{ZF Receiver:} \quad G = (H^H H)^{-1} H^H \tag{9}$$

The soft-metrics can now be computed independently on each of the symbols $\hat{a}_1$ and $\hat{a}_2$ on the 2 antennae. A modification of the above [8] is to weigh the soft-metrics on each antenna by the variance of the noise on each antenna after filtering with G. This gives the so-called weighted MMSE and ZF solutions (WMMSE, WZF).

Search Method	No. of operations/symbol vector		
	Complex multiplications	Complex additions	Real additions
Euclidean distance	$N_T^2\|C\| + N_T\|C\|^{N_T}$	$N_T^2\|C\|^{N_T}$	$N_T - 1$
Absolute value	$N_T^2\|C\|$	$N_T^2\|C\|^{N_T}$	$2N_T - 1$

Table 1. Complexity comparison for N_T x N_T system.

3. ABSOLUTE-VALUE SEARCH METRIC

Let us define a metric d_A as follows:

$$d_A(a_1, a_2) = |\text{Re}(X_1)| + |\text{Im}(X_1)| + |\text{Re}(X_2)| + |\text{Im}(X_2)| \tag{10}$$

where a_1, a_2, X_1 and X_2 are as defined previously. Comparing equations (3) and (10) we see that we have essentially used the following approximation for any complex number Y, that holds for high SNR:

$$|Y|^2 \approx (|\text{Re}(Y)| + |\text{Im}(Y)|)^2 \tag{11}$$

There are two possible ways of using the metric d_A defined above.

1. **Method 1**: Metric d_A is used instead of d_E in *both* searching and computing the soft metrics, i.e. in equations (4) to (6).

2. **Method 2**: Metric d_A is used for searching, but then the Euclidean distance d_E is used for computing the soft-metrics, i.e d_A is used in equations (4) and (5) but d_E is used in equation (6), using the arguments from the search in (4) and (5).

The benefit of Method 2 above is that the performance at low to moderate SNRs is improved, since we are using the approximation of the Euclidean distance only in the search but the actual metric used by the Viterbi decoder is still the Euclidean distance metric.

3.1. Complexity comparison

Table 1 shows the complexity of the two search methods in terms of number of complex multiplications, complex additions and real additions required to compute the metric values in the search space. These values are for a N_T x N_T system with a constellation size $|C|$ on each antenna. For the Euclidean-distance search, we first need $N_T^2|C|$ complex multiplications to compute all possible entries of $H\underline{a}$ and then $N_T|C|^{N_T}$ complex multiplications to compute the Euclidean distance between the received vector and each possible transmitted symbol vector. For the absolute value search, we just need $N_T^2|C|$ complex multiplications to compute all possible entries of $H\underline{a}$, and all further computations involve only additions. Method 2 would require an additional N_T complex multiplications. The number of complex and real additions required for both searches is about the same. Hence we see that the complexity of the Euclidean distance search is primarily due to the number of complex multiplications required. With the absolute-value search (Method 1 or 2), the number of complex multiplications required is linear with respect to the constellation size $|C|$. In most practical implementations of MIMO considered thus far, N_T is small, say 2 or 3, while $|C|$ could be large. In such cases the reduction in complexity is substantial. For example, a 2 x 2, 64 QAM system would need 8448 complex multiplications for the Euclidean distance search and only 256 complex multiplications for the absolute-value search, which is a 97% reduction in complexity.

The reduction in complexity is even more when one considers a quasi-static fading channel where the matrix H stays the same for N_S symbols. This is the case in many practical systems like 802.11a where it is assumed that the channel does not change over one packet. The absolute-value search still requires only $N_T^2|C|$ complex multiplications for all N_S symbols, but the Euclidean distance search requires $N_S N_T|C|^{N_T} + N_T^2|C|$ complex multiplications. For comparison, the MMSE/ZF receivers require $N_T^2 N_S$ multiplications for N_S symbols. Hence, as the packet size increases, the number of multiplications required for the absolute-value search method approaches that of MMSE/ZF and when $N_S > |C|$, the number of multiplications required for the entire packet becomes *less*. The only difference is in the complexity of the comparator that is required: for absolute-value search it is $|C|^{N_T}$ whereas for the MMSE/ZF receiver it is $N_T|C|$. Usually the complexity of the comparator is less compared to multipliers, and hence for large packets the absolute-value search method approaches the low complexity of MMSE/ZF with a performance close to that of ML, as will be shown in the next section.

4. SIMULATION STUDY

In this section we present simulation results comparing the ML decoder with the absolute-value decoder for a BICM system using the constraint length 7, rate 1/2 convolutional coder with generators [133,171]. This is the same code used in the 802.11a standard [9]. We consider a 2 x 2 system with 16QAM constellation on each antenna. Data is transmitted in packets of 500 bytes each. The channel is assumed to be fast-fading, i.e. a different channel matrix is generated for each transmitted symbol vector. Figure 1 shows the bit error rate (BER) and Figure 2 shows the packet-error rate (PER) performance of the ML decoder, the absolute-value decoder using Method 1, and the absolute-value search with Method 2. We see that the performance difference between ML and absolute-value with Method 1 is about 0.5 dB. The absolute-value search with Method 1 is about 0.2 dB worse than with Method 2. For comparison, the WZF receiver is also shown. This receiver has a performance loss of about 2 dB compared to the ML decoder.

In this example, the search space is 256 values, which is not considered too large for a practical implementation of a comparator. The absolute-value search requires only 64 complex multiplications per symbol vector, whereas the ML search requires 1088 complex multiplications. Hence there is a 94% reduction in complexity, and only a 0.5 dB loss in performance.

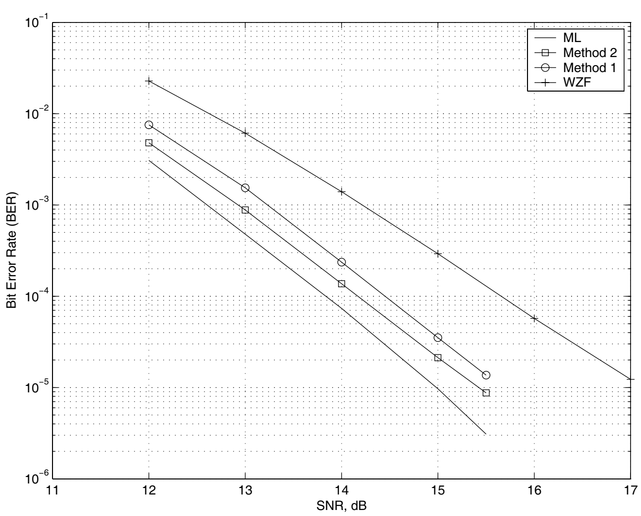

Fig. 1. BER performance of ML, Method 1, Method 2 and WZF.

5. CONCLUSIONS

In this paper we have used an absolute-value search to reduce the computational complexity of ML decoding for a MIMO system. The number of multiplications required becomes linear in $|C|$ instead of exponential and hence this method is very useful in cases where $|C|$ is large. Simulation results have been presented to show that the reduction in complexity does not affect the performance appreciably, showing less than 0.5 dB loss for a BICM, 2 x 2 system with 16QAM on each antenna. Future extensions of this work will look at applying sphere-decoding techniques to the absolute-value metric to further reduce the complexity by reducing the size of the search space as well.

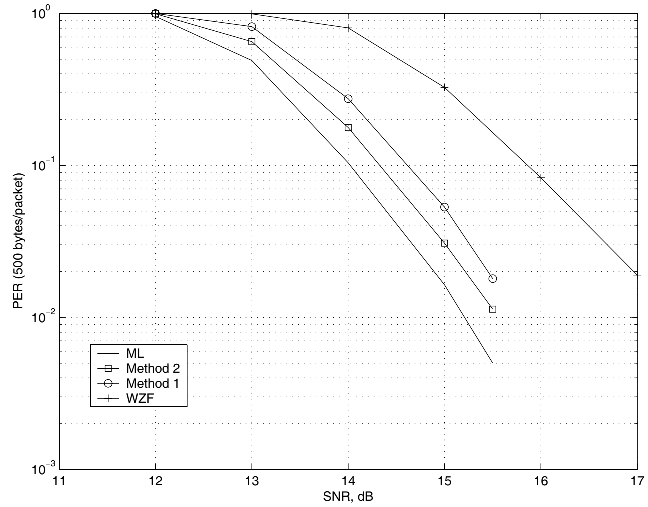

Fig. 2. PER performance of ML, Method 1, Method 2 and WZF.

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