EXACT SYMBOL ERROR RATE AND TOTAL DEGRADATION PERFRORMANCE OF NONLINEAR M-QAM FADING CHANNELS

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ABSTRACT

In this paper, we derive the exact symbol error rate (SER) and total degradation (TD) performances of coherent M-ary QAM constellations over nonlinear fading channels with maximum ratio combining (MRC) diversity. We analyze the combined effect of nonlinear distortion introduced by the high power amplifier (HPA) and multipath fading. Our results are used to optimize system parameters, such as ring ratios of circular QAM constellations and HPA-backoffs. Comparisons among five popular 16-ary constellations are also made for various nonlinear fading channels.

1. INTRODUCTION

In satellite mobile communications, there is a growing need to overcome the severe nonlinear distortion introduced by the on-board HPA as well as the performance degradation by multipath fading. In order to design efficient modulation schemes, it is very important to evaluate the exact SER performance of different QAM constellations over nonlinear channels. While a fair amount of research works have been carried out to calculate the exact or approximate SER in linear channels [1] [2], very few works have been done on evaluating the combined effect of nonlinear distortion and multipath fading. The authors in [5] have presented some analytical results on the SER performance of 16-rectangular QAM signals over nonlinear fading channels. However, in this work [5], only the performance of one constellation was addressed, where the variation of system parameters, such as the ring ratio and the HPA-backoff, were not considered. Besides, the method in [5] did not provide any means to evaluate the total nonlinear degradation. It is worth mentioning that to design an efficient communication system, which involves nonlinearity, the knowledge of TD performance is an important requirement [3]. This is because TD performance provides a way to quantify the total nonlinear degradation considering both the performance loss due to the constellation distortion and power loss due to inefficient use of the HPA. In this paper, we extend the work in [1] [2] to a nonlinear fading diversity scenario and derive the precise SER and TD performances. The method is used to optimize ring ratios and HPA-backoffs. Comparisons are then made among different 16-ary QAM constellations. This comparative analysis can be used as a reference for signaling constellation design in nonlinear fading channels.

The rest of the paper is organized as follows. In sections 2 and 3, we describe the system model and derive the SER expressions. Following this, the method is applied to 16*QAM constellation in section 4. In section 5, we extend the method to calculate TD and make a comparison among five candidate 16-ary QAM constellations.

2. SYSTEM MODEL

We consider in this paper a simplified satellite mobile channel model (Fig. 1).

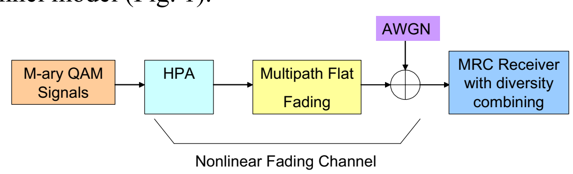


Fig.1. Nonlinear flat fading system model

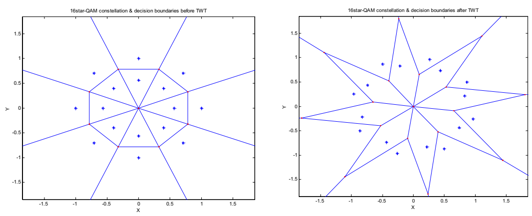
The equivalent baseband model is expressed as:

$$y = \alpha e^{j\psi} \bullet A(r)\exp\left\{ j(\phi(r)+\theta)\right\} + n$$

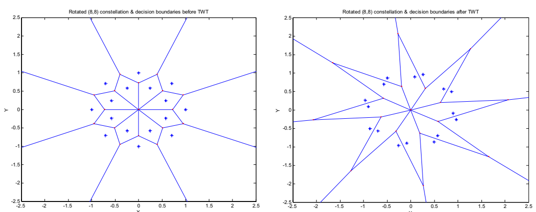
where y is the received signal, r and θ are, respectively, the amplitude and phase of the transmit signal after modulation, $\alpha e^{j\psi}$ is the complex fading gain, n is the White Gaussian Noise. $A(r)$ and $\Phi(r)$ are the amplitude to amplitude (AM/AM) and amplitude to phase (AM/PM) transfer functions of the HPA model [3][4], which are given by:

$$A(r) = \frac{2r}{1+r^2} \quad (2.1) \quad \& \quad \Phi(r) = \frac{\pi}{3} * \frac{r^2}{1+r^2} \quad (2.2)$$

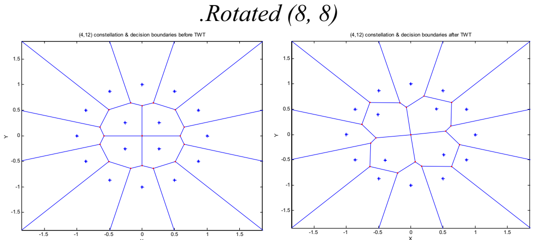
The HPA causes sever nonlinear distortion on the input signals especially when working at the saturation region. This is illustrated in Fig.2, where the constellations and decision regions of five 16-ary QAM signals before and after nonlinear amplifier distortion are shown.



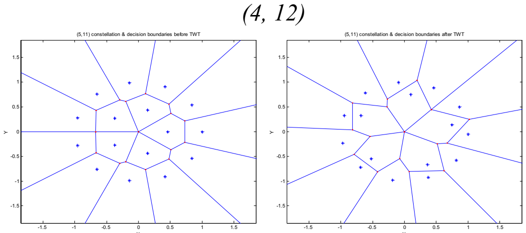
*16*QAM*



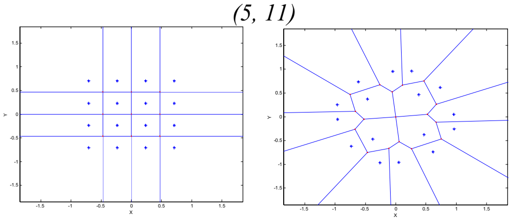
.Rotated (8, 8)



(4, 12)



(5, 11)



16 rectangular-QAM

Fig.2. 16 ary-QAM constellations and decision regions before and after amplifier distortion

3. SER DERIVATION IN NONLINEAR FADING DIVERSITY CHANNELS

For the received M-ary QAM constellation, there are two general types of decision regions (Fig. 3).

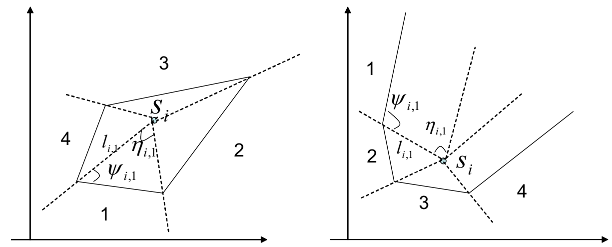


Fig.3. Close region and open region.

Craig in [1] derived the average SER expression in AWGN channel:

$$P_s(\gamma) = \sum_{i=1}^{M}\sum_{j=1}^{G_i}\frac{P(s_i)}{2\pi}\int_0^{\eta_{i,j}}\exp\left\{-\frac{b_{i,j}\gamma\sin^2\psi_{i,j}}{\sin^2(\theta+\psi_{i,j})}\right\}d\theta \quad (3.1)$$

Where $P(s_i)$ is the prior probability of the transmitted symbol s_i, M represents the number of symbols in the M-ary signals; G_i is the total number of sub-regions for symbol s_i, γ is the SNR per symbol; $b_{i,j}$ is a scaling factor, which is equal to $l_{i,j}^{\;2}/E_s$,where E_s is the average output signal energy; $l_{i,j}$, $\eta_{i,j}$ and $\psi_{i,j}$ are the geometrical parameters corresponding to symbol s_i, sub-region j, as illustrated in Fig. 3.

Now we consider satellite nonlinear systems. In this system, the output signal constellations are severely distorted by the amplifier (Fig. 2). Thus equations (3.1) is also applicable for nonlinear AWGN channels, but $b_{i,j}, \eta_{i,j}$ and $\psi_{i,j}$ are then the corresponding sub-region parameters for signal constellations and decision regions after nonlinear amplifier distortion, which is determined by the input constellations and the HPA-backoffs.

In a nonlinear fading channel, the received SNR is random. The average SER is then given by:

$$P_s = \int_0^{\infty} p_\gamma(\gamma)\,P_s(\gamma)d\gamma \qquad (3.2)$$

where $p_\gamma(\gamma)$ is the probability density function (PDF) of the received SNR per symbol, $P_s(\gamma)$ is the channel conditional SER given by equation (3.1).

When MRC diversity reception is used, the total instantaneous SNR is the sum of the instantaneous SNR per branch. By averaging the multi-channel conditional SER over the joint PDF of the instantaneous SNR sequence, the average SER is given by [2]:

$$P_s = \sum_{i=1}^{M}\sum_{j=1}^{G_i}\frac{P(s_i)}{2\pi}\int_0^{\eta_{i,j}}\prod_{l=1}^{L}\int_0^{\infty}p_{\gamma_l}(\gamma_l)\exp\left\{-\frac{b_{i,j}\gamma_l\sin^2\psi_{i,j}}{\sin^2(\theta+\psi_{i,j})}\right\}d\gamma_l d_\theta$$

where L is the total number of diversity branches. γ_l is the instantaneous SNR in the l-th branch ($l = 1,2,...,L$). $p_{\gamma_l}(\gamma_l)$ is the PDF of SNR in the l-th branch.

Particularly, for nonlinear Ricean fading, the PDF of SNR distribution is, as shown in [2],

$$p_{\gamma_l}(\gamma_l) = \frac{K_l+1}{\Lambda_l}\exp\left[-\left(K_l + \frac{K_l+1}{\Lambda_l}\gamma_l\right)\right] I_0\left(2\sqrt{\frac{K_l(K_l+1)}{\Lambda_l}\gamma_l}\right)$$

where K_l is the Rice factor. $\Lambda_l = E[\gamma_l]$ is the average SNR per symbol in the l-th branch. According to [2], the SER expression in can be simplified in this case to

$$P_s = \sum_{i=1}^{M}\sum_{j=1}^{G_i}\frac{P(s_i)}{2\pi}\int_0^{\eta_{i,j}}\prod_{l=1}^{L}\left\{\left[\frac{\sin^2(\theta+\psi_{i,j})}{\sin^2(\theta+\psi_{i,j})+\frac{\Lambda_l}{K_l+1}b_{i,j}\sin^2\psi_{i,j}}\right]\exp\left[\frac{-K_l\Lambda_l b_{i,j}\sin^2\psi_{i,j}}{(K_l+1)\sin^2(\theta+\psi_{i,j})+\Lambda_l b_{i,j}\sin^2\psi_{i,j}}\right]\right\}d\theta$$

For nonlinear Rayleigh fading [2],

$$p_{\gamma_l}(\gamma_l) = \frac{1}{\Lambda_l}e^{-\gamma_l/\Lambda_l}$$

$$P_s = \sum_{i=1}^{M}\sum_{j=1}^{G_i}\frac{P(s_i)}{2\pi}\int_0^{\eta_{i,j}}\prod_{l=1}^{L}\frac{\sin^2(\theta+\psi_{i,j})}{\sin^2(\theta+\psi_{i,j})+\Lambda_l b_{i,j}\sin^2\psi_{i,j}}d\theta$$

4. 16*QAM AND ITS PERFORMANCE

In this section, our method is applied to calculate the exact SER of 16*QAM constellation when the HPA is operated at the saturation region. For analysis, we first consider a fixed input constellation and keep the ratio between the radius of the outer and inner constellation circles (defined as the ring ratio) to be 1.7654. This value is optimum in AWGN channel [2]. Fig. 4 illustrates the combined effect of multipath fading and amplifier nonlinearity as well as MRC diversity. Clearly, our theoretical results agree with Monte Carlo simulations very well. The SNR requirements to achieve a SER of 10^{-4} in various channel conditions have been summarized in table 1. It is shown that both the nonlinearity and fading severely degrade the overall performance, while MRC diversity effectively combats fading and improve the performances. Table 1 also demonstrates that for both the nonlinear and linear Ricean fading channels, the advantage we get from MRC diversity decreases as the Rice factor K increases. This is an expected result since as the Rice factor increases (i.e. the line of sight (LOS) becomes stronger), there is more correlation and less diversity between the instantaneously received SNR on the various diversity branches.

Noting that the SER performance changes when the ring ratio varies, we obtained the optimum ring ratios by searching the value that minimizes the SER. Fig. 5 shows the optimum ring ratios for different SNRs. It is observed that in a nonlinear Ricean or Rayleigh fading channel, there exists an optimum ring ratio which minimizes the SER for asymptotically large SNR (i.e. 3.444 for L=1). These optimum ring ratios are the same for the nonlinear Rayleigh and Ricean fading channels with different Rice factors, but they are different when different order of diversities are employed. The values of the asymptotical optimum ring ratios are summarized in table 2. The results for the nonlinear cases are new.

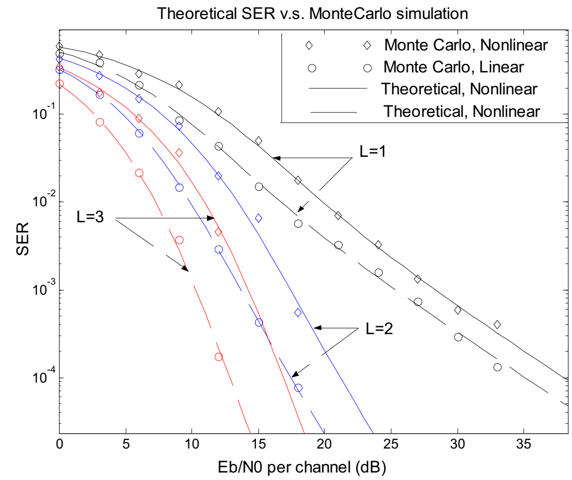


*Fig.4. Theoretical SER v.s. Monte Carlo simulations: 16*QAM in nonlinear and Linear Ricean fading (K=5dB)*

		AWGN channel	Fading channel		
			Rayleigh	Rice, K=10dB	Rice, K=20dB
L=1	Li	14.2	42.6	19.4	14.6
	NL	18.6	45.5	23.3	19.0
L=2	Li		23.3	13.1	11.4
	NL		26.8	17.3	15.8
L=3	Li		16.9	10.6	9.5
	NL		20.6	14.9	13.9

Table 1: Eb/N0 requirement (dB) for to achieve a SER of 10^{-4} (Li: Linear channel, NL: Nonlinear channel)

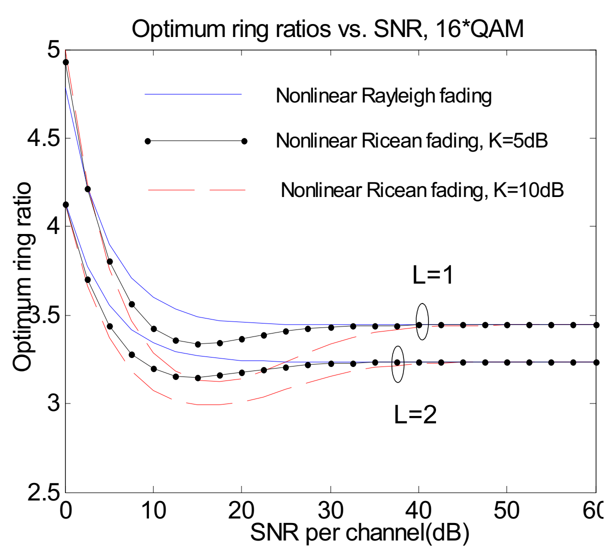


*Fig.5. 16*QAM optimum ring ratios versus SNR*

Diversity Order	AWGN	Linear Ricean or Rayleigh	Nonlinear Ricean or Rayleigh
L=1	1.765	1.951	3.444
L=2		1.879	3.234
L=3		1.846	3.128
L=4		1.828	3.064

*Table 2: Asymptotically optimum ring ratios for 16*QAM*

5. COMPARISON BETWEEN DIFFERENT POPULAR 16ARY SIGNAL CONSTELLATIONS

In this section, performances of five candidate 16-ary constellations are compared. These are 16*QAM, Rotated (8, 8), (4, 12), (5, 11) and 16-rectangular-QAM. They are appropriate for satellite mobile communications.

The amplifier nonlinearity can be reduced by operating the HPA at a point with a large output backoff (OBO) rather than in the saturation region. The OBO is defined as the ratio between the amplifier output saturation power (P_{sat}) and the average amplifier output signal power (P_{out})

$$OBO(dB) = 10\log(P_{sat} / P_{out}) \quad (5.1)$$

However, a large OBO will also decrease the efficiency of the power amplifier. Thus there exists an optimum operating point which well balances the output power and the nonlinear distortion. To quantify the total influence, we resort to the concept of TD, defined as the sum (in decibels) of the OBO and the increment in the ratio Eb/N0 required to achieve a given SER (i.e. 10^{-4}) with respect to the case of a perfect linear amplifier[3].

$$\text{TD (dB)} = \text{OBO (dB)} + \Delta\text{SNR (dB)} \quad (5.2)$$

Fig. 6 shows the TD performances of these five 16-ary constellations operating at different OBOs. For each constellation, the figure shows the optimum operating OBO that gives the lowest TD. Fig. 7 compares the SER performances of these five constellations over nonlinear fading diversity channels. The HPA is set to work at the optimum operating OBO for each constellation. We can observe that the orders of these error rate curves are not necessarily the same for different channel conditions. However, (4, 12) and (5, 11) perform relatively better than the Rotated (8, 8) and 16*QAM in our comparisons.

6. CONCLUSION

We have presented the precise symbol error rate and total degradation performances of several popular constellations over various nonlinear fading diversity channels. The combined effect of nonlinear distortion, fading and diversity combining is illustrated. Five popular 16-ary QAM constellations are compared. We also reported optimum system parameters, including optimum ring ratios for circular constellations and optimum operating point for the power amplifiers.

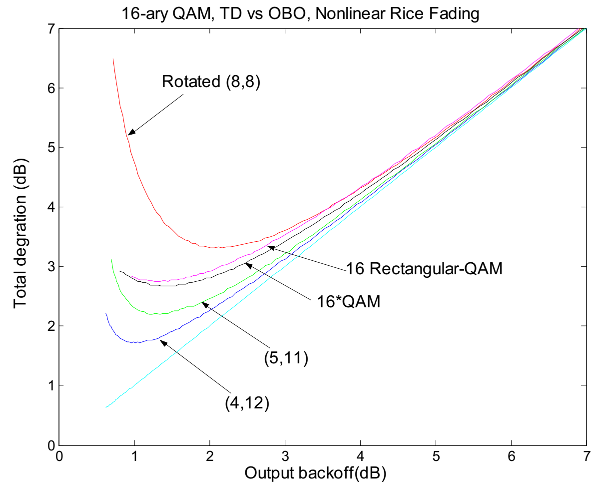


Fig.6: 16-ary QAM total degradation (dB) versus OBO in Ricean fading channels (K=5dB)

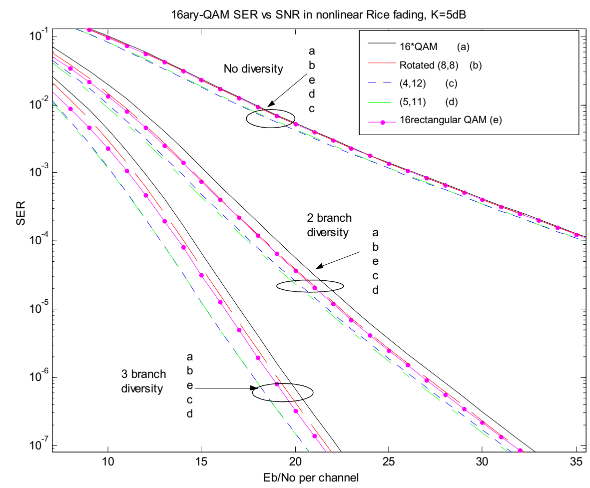


Fig.7. 16-ary QAM SER performances in nonlinear Ricean fading (K=5dB) with MRC diversity combining.

7. REFERENCES

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