OPTICAL CDMA DETECTION BY ORTHOGONAL MATCHING PURSUIT

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ABSTRACT

In this paper we present a novel optical CDMA multi-user detector employing the orthogonal matching pursuit algorithm. The proposed system is compared with most of the receiver structures in the literature. It is shown by simulation results that the proposed detection architecture is a very promising candidate with its low computational complexity, and high detection performance. It is also shown to be more robust to low SNR and near-far effect when compared to most of the well-known optical CDMA receiver architectures.

1. INTRODUCTION

Although CDMA has initially been employed in wireless communication systems, it was proposed to be used in optical systems first by [1]. Optical code division multiple access (O-CDMA) facilitates the large bandwidth of optical communication systems in order to employ the flexibility of CDMA systems.

The main difference of O-CDMA systems from wireless CDMA is the code structure. Optical systems are mainly intensity modulated; hence the signal space is non-negative. Therefore, the chips in the CDMA system are alternating '1's and'0's, instead of '-1's and '1's. So, good wireless CDMA codes may not be suitable for optical CDMA. As a result, optical orthogonal codes (OOC) are proposed initially in [2] and recently in [3].

The main challenge in the O-CDMA systems is in the detection process. Recently many different O-CDMA receiver architectures are proposed, in order to decrease system complexity, cost, and increase the interference rejection property. The best solution in terms of performance is achieved by maximum likelihood (ML) solution [4]. However the ML solution has a formidable computational complexity and it is known to be NP-hard. Hence some sub-optimal detectors such as correlation receivers, optical hard limiters, and chip level detectors have been proposed. A detailed comparison of these algorithms was given in [5]. An interference canceler has been presented in [6]. Lately algorithms such as expectation-maximization [7] and serial search [8] have also been proposed. However none of these algorithms are cost-efficient, simple and can provide good and robust performance at the same time, and this delays the wide range implementation of O-CDMA systems.

In this paper we propose the application of basis selection (BS) algorithms, in particular the orthogonal matching pursuit (OMP) algorithm for multi-user detection of O-CDMA signals. The BS algorithms are about the selection of a basis set for signal decomposition by determining a small, possibly the smallest, subset of vectors chosen from a large redundant set of vectors to match given data. This problem has various applications such as time/frequency representations [9], speech coding [10], and spectral estimation [11]. Recently, we applied these algorithms to the smart antenna systems with reasonable complexity and good performance [12]. The main motivation behind this work is to approach heuristically to the ML solution, without the complexity of ML solution. The OMP algorithm is selected since it provides a good tradeoff between detection performance and computational complexity [13].

The paper is organized as follows: In Section 2, the system model is given, basis selection problem is discussed and the OMP algorithm is summarized. Simulation results are presented in Section 3 and conclusions are given in Section 4.

2. SYSTEM MODEL

An O-CDMA system with K users is considered as shown in Fig. 1. Sticking to the notation of [2], OOC's are de-

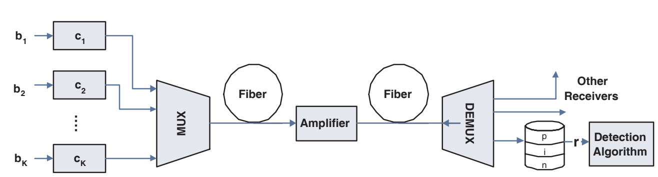

Fig. 1. Multiuser O-CDMA system model

fined as $\Phi(N, w, \lambda_a, \lambda_b)$, where N is the code length, w is the code weight, λ_a is the maximum correlation of a code sequence with its shifted versions, and λ_b is the maximum cross-correlation of a code sequence with any shifted version of any other code in the code set. The symbol period is denoted by T_s, the chip period is T_c with the relation $N = T_s/T_c$. The spreading code for user k is denoted by $\{c_k(l)\}_{l=0}^{N-1}$.

The baseband representation of the received O-CDMA signal after coherent reception is given by

$$s(t) = \sum_{n=-\infty}^{\infty} \sum_{k=1}^{K} A_k \bar{c}_k(t - nT_s) b_k(n) + d(t), \quad (1)$$

where $d(t)$ is the dark current. The transmitted amplitude for user k is A_k. The symbol sequence of the kth user, $b_k(n)$ is independent and identically distributed and modulated by on-off keying (OOK). In the equation above, the effective spreading waveform of the kth user is represented by $\bar{c}_k(t)$. This is formed by convolution of the spreading waveform with the fiber dispersion impulse response as $\bar{c}_k(t) = c_k(t) \star h(t)$. The spreading waveform can be given as

$$c_k = \sum_{l=1}^{N} c_i(l) \psi(t - lT_c), \quad (2)$$

where $\psi(t)$ is the chip pulse shape of duration T_c, where $\|\psi(t)\| = 1$. For simplicity, we will consider rectangular pulse shapes. Both A_k and dark current are modeled as Poisson processes as a result of the p-i-n diode receiver structure.

In order to apply the OMP algorithm to O-CDMA detection, we introduce the following notation for the discrete time model for single bit duration

$$\mathbf{r} = \sum_{k=1}^{K} A_k b_k \mathbf{c}_k + \mathbf{d}, \quad (3)$$

where b_k is the transmitted bit of the k^{th} user, $\mathbf{c}_k$ is a column vector of length N formed by OOC chip sequence of user k. If we define the dictionary matrix $\mathcal{D} = [\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c_K}]$, the received vector $\mathbf{r}$ is a linear combination of the columns of $\mathcal{D}$ with coefficients $A_k b_k$, k=1...K. These coefficients are actually the transmitted bits multiplied by the channel gains. For the case of OOK, it is straight forward to recover b_k from $A_k b_k$. Once b_k are recovered, the detection for all K users is accomplished.

In this work, we limit ourselves to the chip-synchronous case. This is a reasonable assumption considering that the actual transmission duration in O-CDMA systems is small when compared to T_c. We also assume that the delays of the users are known at the receiver. This is not a critical assumption since the system is not mobile. However the case can easily be extended to a more general case where delays are also estimated, by adding new columns to $\mathcal{D}$, which are cyclic-chip shifted versions of the chip sequences.

2.1. The Orthogonal Matching Pursuit Algorithm

The basis selection problem can be viewed as finding the sparsest solution to a linear system of equations. More precisely, if we form a matrix $\mathbf{A} = [a_1, a_2, \ldots, a_K]$ by normalizing the columns of the dictionary $\mathcal{D}$, the problem can be stated as finding an $\mathbf{x}$, with at most r non-zero entries such that

$$\|\mathbf{Ax} - \mathbf{r}\| \leq \epsilon, \quad (4)$$

for $\epsilon \geq 0$, and $r > 1$. For $\epsilon = 0$, i.e. the perfect representation case, the problem reduces to solving the system $\mathbf{Ax} = \mathbf{r}$.

Finding the sparsest solution to (4) in an overcomplete dictionary ($N < K$) using an exhaustive search is infeasible. In order to solve this problem, suboptimal methods based on sequential and parallel basis selection are proposed [14]. In this paper, we consider a sequential basis selection algorithm, the orthogonal matching pursuit (OMP) algorithm which is proposed in [15]. The OMP algorithm is selected since it provides a good compromise between detection performance and computational complexity [13].

The OMP algorithm is also called modified matching pursuit algorithm in [13], from which we take the notation presented here. Let the residual vector after the p^{th} iteration be denoted by b_p, with $b_0 = \mathbf{r}$. Let P_{S_p} denote the orthogonal projection matrix onto the range space of S_p, and $P_{S_p}^{\perp} = I - P_{S_p}$ denote its orthogonal complement, with $P_{S_0} = 0$ and $P_{S_0}^{\perp} = I$. The projection matrix on the space spanned by a_k, with $\|a_k\| = 1$, is $P_{a_k} = a_k a_k^T$. The algorithm terminates after r iterations.

The OMP algorithm selects k_p in the p^{th} iteration by finding the vector best aligned with the residual obtained by projecting b onto the orthogonal complement of the range space S_{p-1}, that is

$$k_p = \arg\max_{l} |a_l^T b_{p-1}|, \quad l \notin I_{p-1}. \quad (5)$$

With the initial values, $\hat{a}_{k_p}^0 = a_{k_p}$, $q_0 = 0$, applying the orthogonalization process as

$$\hat{a}_{k_p}^l = \hat{a}_{k_p}^{l-1} - (q_{l-1}^T \hat{a}_{k_p}^{l-1}) q_{l-1}, \quad l = 1, 2, ..., p, \quad (6)$$

and normalization as

$$q_p = \frac{\hat{a}_{k_p}^p}{\|\hat{a}_{k_p}^p\|}, \quad (7)$$

we can write

$$P_{S_p} = P_{S_{p-1}} + q_p q_p^T. \quad (8)$$

The residual b_p is updated as

$$b_p = P_{S_p}^{\perp} b_{p-1} = b_{p-1} - (q_p^T b_{p-1}) q_p. \quad (9)$$

The algorithm terminates when either $p = r$, or $\|b_p\| \leq \epsilon$. In order to use the OMP algorithm in O-CDMA detector, we set the algorithm parameter $r = K$.

In the detector, the OMP algorithm detects the information bit that belongs to one user at each iteration. Hence, for the case where the number of active users are $L < K$, only L iterations are performed, decreasing the computational complexity of the detector.

3. SIMULATION RESULTS

In the simulations, we build our OOC as discussed in [2], using the parameter set $\Phi(511, 3, 1, 1)$. There are 85 distinct codes available for this set. The main imperfections considered are the dark current, the Poisson distributed output of the p-i-n diode and the multiple user interference. Fiber dispersion is assumed to be negligible. Due to space limitations complexity analysis is not included in this work, however we would like to mention that the number of floating point operations required for the OMP algorithm is an order of magnitude less than the interference canceler proposed in [6]. At the receiver, we assume for an interval of T_c, the average effect of dark current is on the order of 5 photons.

Other than the proposed OMP method, four different receiver structures are considered. These are the decorrelator receiver, the optical hard-limiter, the chip level detector, and the interference canceler. The details of these algorithms can be found in [5, 6]. The decorrelator receiver simply sums the desired chip durations, and compares the result with a threshold. The optical hard-limiter limits the energy in each chip duration to a single transmitted bit energy, hence canceling some of the interference. The chip-level detector investigates each chip position of the code in the optical domain. Finally the interference canceler repeatedly applies decorrelator, every time taking the last iteration's decisions into consideration.

First we investigate the error performances of the mentioned algorithms, as the number of users increase. As depicted in Fig. 2, the bit error rates (BER) for hard limiter and decorrelator deteriorate very fast as the number of users increases. The OMP based detector's error performance is logarithmically affected by the increase while others affected almost linearly.

Next we investigate the effect of the average number of photons received at the receiver. As seen in Fig. 3, especially for low number of photons, which we can interpret as low signal to noise ratio (SNR), OMP based detector performs better that the other methods.

Finally in Fig. 4, we investigate the effect of unequal received power from different users. It is well know that wire-

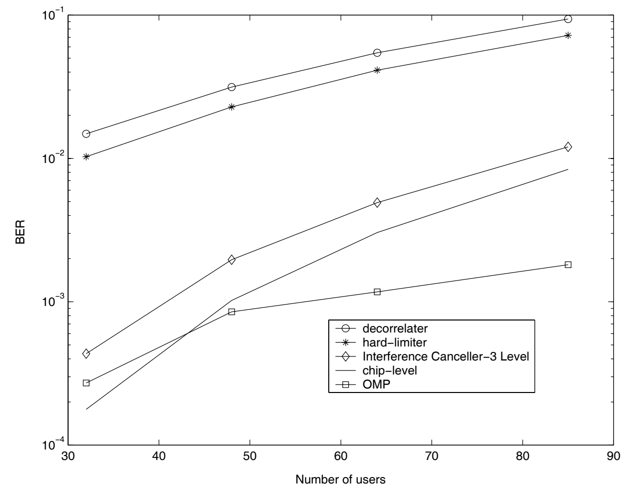

Fig. 2. Dependence of BER to interference for different detection methods.

less CDMA systems are critically affected by unequal received powers. To the best of authors' knowledge all of the proposed detector architectures investigate the case when all users transmit with equal powers. However, this is not necessarily the case. In our investigation, we keep the power of the desired user the same, while multiplying the powers of the interferers with the coefficient indicated in the x-axis of Fig. 4. As can be seen in the figure, although the OMP based detector cannot perform as well as optical chip level detector, it out-performs the rest up to a near far ratio of 6. We should also mention that it is easier to implement the OMP based detector since it does not require operations accomplished in an optical chip duration.

4. CONCLUSIONS

The main problem about the optical hard-limiters is that, once the received energy for a single chip duration is limited, the possibility of interference cancelation by estimating other user's signals is eliminated. The decorrelator structure treats the interference as noise, hence loses performance for congested systems.

Although chip-level detectors perform well, they need to work in an optical chip duration, which is practical (but not cost-efficient) for only all-optical systems. Also it is well known that chip-level detectors are highly affected by dark current and the increase in the number of users in the channel.

In order to overcome these issues, in this paper we proposed a novel OMP based optical CDMA detector. The proposed detector is more robust to users with unequal received power than the interference cancelers, and more robust to multi-user interference than the decorrelator and optical hard limiters. Considering its simple implemention,

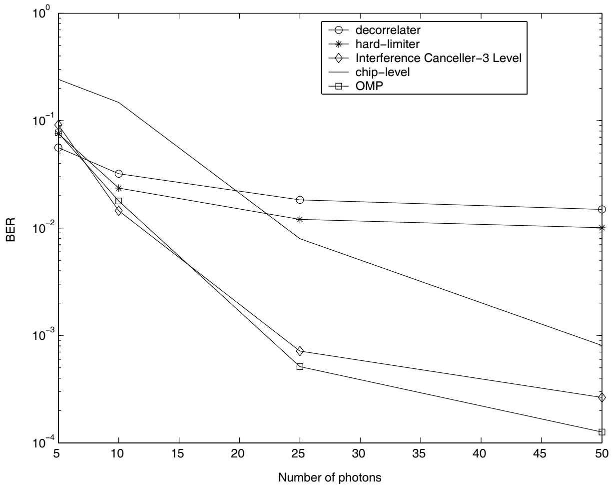

Fig. 3. The effect of number of photons received for different detection methods on BER.

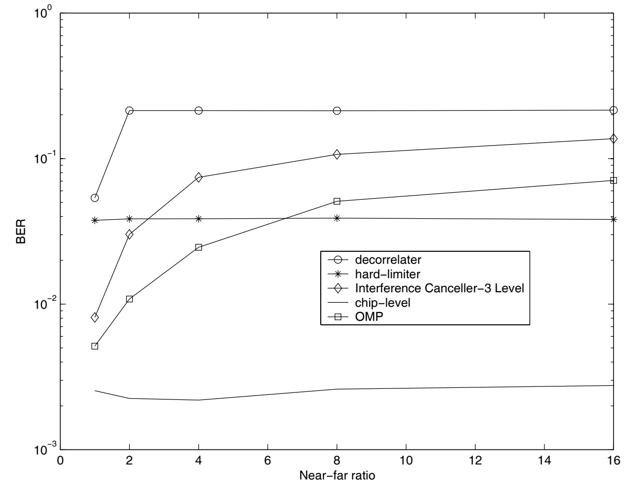

Fig. 4. The near-far effect for different detection methods on BER.

the OMP based detector is a promising candidate for low-cost O-CDMA system architecture.

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