PROBABILITY DISTRIBUTION ESTIMATION FOR AN INTEGRATED CODING AND EQUALIZATION SCHEME IN OPTICAL COMMUNICATIONS SYSTEMS

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ABSTRACT

We develop a practical method to accurately estimate the distribution of electrical current in optical communications systems in the presence of polarization mode dispersion (PMD)-induced intersymbol interference (ISI) and amplified spontaneous emission (ASE) noise. We then introduce an integrated coding and equalization scheme that use these estimated probabilities of the received current given a transmitted sequence. Simulation results with all-order PMD and ASE noise show the effectiveness of the scheme.

1. INTRODUCTION

Electrical-domain equalization techniques such as linear adaptive filters have been demonstrated to be effective in mitigating the effects of polarization mode dispersion (PMD) in optical communications systems [1]. These equalizers use a feedback or feedforward structure and their coefficients are updated such that the mean square error (MSE) or another error statistics is minimized. Maximum-likelihood detection based techniques, such as maximum likelihood sequence estimation (MLSE) or maximum a posteriori (MAP) detection, are recently proposed for PMD mitigation [2]–[4]. MLSE bases its decision on the observation of a sequence of received signals, and searches for the best path through a trellis that maximizes the joint probability of received signals. MAP detector, on the other hand, makes decisions on a symbol-by-symbol basis and is optimum in the sense that it minimizes the probability of bit errors. Both the MAP detector and the maximum likelihood sequence (MLS) estimator are superior to equalizers that rely on error metrics such as the MSE, as they directly minimize the errors in a symbol or sequence.

Amplified spontaneous emission (ASE) noise is the dominant noise source in optical communication systems. At the end of optical fiber propagation at receiver, the ASE noise generated by the amplifiers installed in the fiber accumulate and can significantly increase the bit-error-rate (BER). Forward error-correction (FEC) coding has proved to be an effective way to increase the power margin due to noise in optical communications systems as well (see e.g. [8], [9]).

These two solutions, coding and equalization, are typically employed in cascade form and are designed independent from each other. A scheme where the two are integrated and designed together, however, is much more desirable and is expected to provide important performance gains. Joint coding and equalization techniques that use a matched filter to generate the sufficient statistics [10] cannot be used for optical communications systems as these channels do not have the additive white Gaussian noise structure. The propagation and the receiver structure in an optical communications channel lead to nonlinear and non-Gaussian channel characteristics, and the photodetector that converts light into electrical current leads to signal-dependent noise term in the receiver. Our previous work [4], which derives an analytical formula for the probability distribution of the filtered electrical current in the presence of PMD and ASE noise, enables the design of such an integrated scheme for optical communications systems.

By building on the work in [4], in this paper, we develop an integrated coding and equalization (ICE) scheme. In addition, we provide a practical method to estimate the distribution of the filtered electrical current given an accurate receiver model in the presence of both all-order PMD and ASE noise as the formulation given in [4] requires the knowledge of noise-free optical signals at two polarization states, which is not easily measurable. We show that the entire probability density function (pdf) of the filtered electrical current, including its low-probability tails, can be computed using the measurements of the averaged electrical current as a function of time. Hence, complicated measurements of the optical signal can be avoided, greatly simplifying the implementation of maximum-likelihood based equalization and soft decoding implementations for optical communications systems. We then demonstrate the effectiveness of our proposed integrated coding and equalization scheme, which uses the estimated probabilities by using a simple convolutional code as an example.

2. ELECTRICAL CONDITIONAL PDF ESTIMATION

In [4], we consider the problem of accurate receiver modeling for optical communications systems, where all-order PMD and ASE noise are the dominant distortions in the systems. We assume that the receiver consists of an optical filter, an ideal square-law photodetector that converts the optical signal into an electrical current, and a low-pass electrical filter. If both the noise-free optical signal $S_x(t)$, $S_y(t)$, and the ASE noise $N_x(t)$, $N_y(t)$ in the two orthogonal states of polarization x and y are known, the characteristic function of the output electrical current $y(t)$ after the low-pass electrical filter can be written as [4], [12]:

$$\Phi_y(\xi) = \prod_{k=1}^{2N} \frac{1}{1-2i\lambda_k\xi} \ \exp\left[i\xi \sum_{k=1}^{2N} \frac{\lambda_k(u_x^2(k)+u_y^2(k))}{1-2i\lambda_k\xi} \right] \quad (1)$$

where λ_k are defined by the Fourier coefficients of the optical filter, the electrical filter, and the covariance matrix of the noise [4].

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Here, $u_x(k)$ and $u_y(k)$ are the Fourier coefficients of the optical signal $S_x(t)$ and $S_y(t)$ up to a linear transformation, and N is the number of Fourier coefficients in each expansion [4]. Inversion of the characteristic function $\Phi_y(\xi)$ yields the pdf of the received electrical current $y(t)$

$$f_y(y(t)) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \Phi_y(\xi)\exp\left[-iy(t)\xi\right] d\xi. \tag{2}$$

It is obviously difficult to directly measure the pdf in an experimental or real system, especially in the low probability tails because of the time that would be required to accumulate the required statistics. Instead, we would like to calculate the pdf directly given an optical channel and a receiver model. To compute $f_y(y(t))$ using Eqn. (2), we need to know the noise-free optical signal in the two polarization states $S_x(t)$, and $S_y(t)$, and the total noise spectral density $2\sigma^2$. The total noise spectral density can easily be measured using an optical spectrum analyzer. However, in practice, it is difficult to directly measure the noise-free optical signal. Nevertheless, one can fairly readily measure the first order moment $\langle y(t)\rangle$ of the electrical current as a function of time t. The expectation of $y(t)$ can be calculated using the first order moment-generating property of the characteristic function given in Eqn. (1), which is written as

$$\langle y(t)\rangle = -i\,\frac{\partial \Phi_{y(t)}}{\partial \xi}\mid_{\,\xi=0} \; = 2\sum_{k=1}^{2N}\lambda_k + \sum_{k=1}^{2N}\lambda_k[u_x^2(k)+u_y^2(k)] \tag{3}$$

where $\langle\cdot\rangle$ denotes expectation. The first term we define as $\overline{y}_n$ in Eqn. (3) is the mean filtered electrical current due to noise and the second term defined as $\overline{y}_s$ is the filtered noise-free electrical current. An estimate of the first term $\overline{y}_n$ can be obtained by transmitting a sequence of all zeros and taking its time average. Similarly, after repeated transmission of a known sequence, observation of the ensemble average for the received electrical current yields an estimate $\langle y(t)\rangle$. Subtracting $\overline{y}_n$ from the estimate of $\langle y(t)\rangle$ yields an estimate of the mean noise-free electrical current $\overline{y}_s$.

We can also estimate the total noise spectral density $2\sigma^2$, which can be written as:

$$2\sigma^2 = \frac{2\overline{y}_n}{\sum_{k=1}^{2N}\lambda_k}. \tag{4}$$

Since λ_k only depends on the optical and electrical filters, the denominator of (4) is deterministic for a given receiver structure. Therefore, an estimate for $2\sigma^2$ can be obtained from the estimate of $\overline{y}_n$ described above.

The information of the optical phase and polarization state of the signal is lost when the photodetector converts the optical signal into an electrical current. Consequently, it is impossible to uniquely determine the vectors $\mathbf{u}_x$ and $\mathbf{u}_y$ in (1) from the current $\overline{y}_s$. However, a key observation in our development is that the calculation of the pdf of the electrical current is insensitive to the optical phase provided that the optical filter bandwidth is wide compared to the signal bandwidth [13].

When the optical filter bandwidth is wide enough, we can construct an equivalent noise-free optical signal $S'_x(t)$ and $S'_y(t)$ with $S'_y(t) = 0$ from the estimated filtered noise-free electrical current as follows: (I) Estimate the filtered noise-free electrical current as described above and compute its Fourier transform; (II) Calculate the noise-free electrical current after the photodiode by dividing the Fourier transform of the estimated filtered noise-free electrical

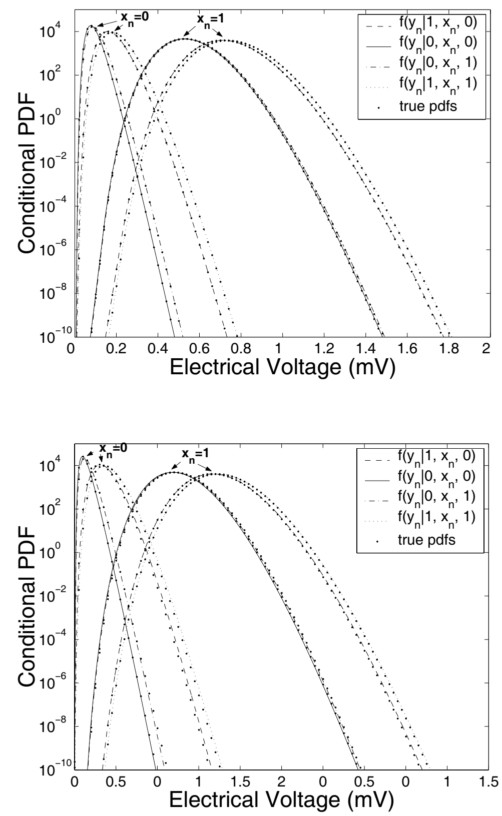

Fig. 1. Comparison between the estimated and the true conditional pdfs of the electrical current. The optical filter bandwidth is (a) 80 GHz; (b) 40 GHz.

current by the transfer function of the electrical filter; (III) Construct an equivalent real optical signal by finding the square root of the time domain electrical signal calculated from step II; and (IV) Calculate an equivalent noise-free optical signal $S'_x(t)$ before the optical filter by dividing the Fourier transform of the equivalent real optical signal from step III by the transfer function of the optical filter.

As our simulation results show, the approximation to the optical equivalent signals using the procedure described is quite reliable for practical optical filter bandwidths. As an example, we estimate the electrical pdf conditioned on a three-bit sequence. We use y_n to denote the sampled electrical current $y(nt_0)$ in the n-th bit slot after clock recovery, and x_n to denote the corresponding transmitted information bit. In implementation, the conditional pdf $f_y(y_n|x_{n-1}, x_n, x_{n+1})$ can be obtained by initially transmitting a known bit sequence, which can then be used to estimate the total noise spectral density, and the noise-free signal for all eight bit patterns of interest—since we assume three bit interactions—using the method described above. In Fig. 1, we compare the estimated conditional pdfs of the electric current obtained using the method described above with the true pdfs for different optical filter bandwidths. As we can see, the estimated conditional pdfs agree with the true pdfs very well. These results suggest that the electrical pdfs are not very sensitive to the bandwidth of the optical filter, even for relatively narrow band filters. The estimated conditional pdf can accurately evaluate the tails of the pdf of the current, a task that can be difficult in experiments.

3. INTEGRATED CODING AND EQUALIZATION

In section 2, we develop a practical method to accurately estimate the conditional pdfs of a received electrical current given bit sequences. The estimated conditional pdfs can be used for maximum-likelihood (ML) based equalization techniques, such as MLSE and MAP detection, in PMD compensation [5]. Meanwhile, they can also be used for ML-based soft decoding methods, such as Viterbi algorithm or other sequential estimation methods. In this paper, we propose an effective integrated coding and equalization (ICE) method using the estimated conditional pdfs for optical communications systems. The proposed ICE scheme is different from not only the ML-based joint coding and equalization (JCE) methods based on [10] (see e.g. [7]), but also from those not based on ML estimation (see e.g. [6]).

The ML-based JCE method has its theoretical foundation given in [10]. It has a MLSE receiver structure consisting of a whitened matched filter followed by a Viterbi decoder for Gaussian channels with inter-symbol interference (ISI). In optical communications channels, however, we cannot directly use the general concept of the ML based JCE. Due to the presence of square-law detection in the receiver for an optical communications system, the output electrical current consists of three parts: signal-signal beat, signal-noise beat, and noise-noise beat. We cannot find a whitening filter in the electrical domain so that the filtered output noise after sampling is independent and identically distributed for a signal-dependent noise [11]. To calculate the statistics of the filtered electrical current after the receiver, it is necessary to characterize both the optical channel and the receiver, as explained in section 2.

We develop the ICE scheme based on the conditional pdf calculation. The decoding process is integrated with MAP detection (equalization) by using the conditional pdfs directly in the soft decoding process. Since the conditional pdfs are estimated in the presence of both ASE noise and PMD-induced ISI, the soft decoding and MAP detection are jointly optimized to reduce the BER. ICE approach is applicable to several cases of practical interest including turbo product codes. For simplicity, here, we illustrate the application of ICE using a convolutional code as an example.

A rate k/n convolutional encoder takes k input bits and generates n output bits with each shift of its internal registers. Suppose that we have an input sequence composed of L k-bit blocks, the output sequence will consist of L n-bit blocks (one for each input block) as well as M additional blocks, where M is the length of the longest shift register in the convolutional encoder. After optical fiber transmission and receiver detection, a distorted vector $\mathbf{y}$ of the received codeword is outputted, and the convolutional decoder generates a maximum likelihood estimate $\hat{\mathbf{x}}$ as the corresponding transmitted codeword.

If a communications channel is memoryless then the noise process affecting a given bit in the received word is independent of the noise process affecting all of the other received bits. However, in optical communications systems, due to PMD and chromatic dispersion, the optical communications channel cannot be assumed to be memoryless. Moreover, due to the complicated interactions between square-law detection and the effects of the optical and the electrical filters in the receiver, the noise distribution is no longer Gaussian, but is a generalized chi-square distribution [12]. These two fundamental changes require the convolutional codes decoding algorithms, such as the Viterbi algorithm, to be modified accordingly.

In a convolutional decoder trellis, a single block of $\mathbf{y}$ and $\hat{\mathbf{x}}$ correspond to a single branch in the trellis, for example, the received and the estimated codewords corresponding to a single trellis branch in the ith block for $\mathbf{y}$ and $\hat{\mathbf{x}}$ are defined as $\mathbf{y}_i = (y_i^{(0)}, y_i^{(1)}, \ldots, y_i^{(n-1)})$ and $\hat{\mathbf{x}}_i = (\hat{x}_i^{(0)}, \hat{x}_i^{(1)}, \ldots, \hat{x}_i^{(n-1)})$, respectively. Here, the subscript i corresponds to the block number while the superscript corresponds to the bit within the block. Extended-block is introduced at the beginning and the end of $\hat{\mathbf{x}}_i$ to account for the ISI effects from other blocks such that

$$\begin{cases} \hat{x}_i^{(m)} = \hat{x}_{i-1}^{(n+m)} & -K-1 < m < 0 \\ \hat{x}_i^{(m)} = \hat{x}_{i+1}^{(m-n+1)} & K+n > m > n. \end{cases} \tag{5}$$

Here, the optical channel has a memory length of $2K+1$ and $n \gg K$.

Since the conditional pdfs have a generalized chi-square distribution, path-metrics based on the distance measure can not be formed to simplify the likelihood function as in channels with Gaussian statistics. Instead, one needs to use directly the conditional probabilities such that the maximum-likelihood based ICE algorithm is written as

$$p(\mathbf{y}|\hat{\mathbf{x}}) = \prod_{i=1}^{L+M-1} \left(\prod_{j=0}^{n-1} p(y_i^{(j)}|\hat{x}_i^{(j-K)} \cdots \hat{x}_i^{(j-1)}, \hat{x}_i^{(j)}, \hat{x}_i^{(j+1)} \cdots \hat{x}_i^{(j+K)}) \right), \tag{6}$$

where the jth bit $\hat{x}_i^{(j)}$ is located at the center of the bit sequence block $\left(\hat{x}_i^{(j-K)} \cdots \hat{x}_i^{(j-1)}, \hat{x}_i^{(j)}, \hat{x}_i^{(j+1)} \cdots \hat{x}_i^{(j+K)}\right)$. Based on Eqn. (6), soft output Viterbi algorithm with estimated conditional pdfs can be established for ICE.

4. SIMULATION RESULTS

Our numerical simulations are based on a 10 Gb/s return-to-zero (RZ) 1000 km transmission system using Gaussian pulses with full width at half maximum (FWHM) of 50 ps and peak power of 1 mW. To include the effects of ISI due to all-order PMD, we use the coarse-step method with 800 sections evenly distributed over the 1000 km transmission line, as described in [14]. We do not impose any relationship between the principal states of the fiber and the input polarization state of the light. ASE noise is added in the optical domain. After the fiber propagation and optical amplification, the distorted optical signal—in two polarization states—is filtered by a Gaussian optical filter with a FWHM bandwidth of 80 GHz, and passes through a photodetector and then a 5th-order electrical Bessel filter with a 3 dB bandwidth of 8 GHz. The electrical current is sampled by a recovered clock. The estimated conditional pdfs shown in Fig. 1 are used to generate the conditional cumulative density functions (cdfs) stored in a look-up table to be used in the ICE scheme.

We compare the proposed ICE scheme to the following JCE schemes: 1) Adaptive thresholding (AT) [4] concatenated with Viterbi hard decoding, 2) MAP detector [4] concatenated with Viterbi hard decoding, and 3) MAP soft probabilities concatenated with soft output Viterbi algorithm (SOVA) [15]. We use a 3-bit MAP detector (hard and soft) and rate $1/2$ convolutional code with

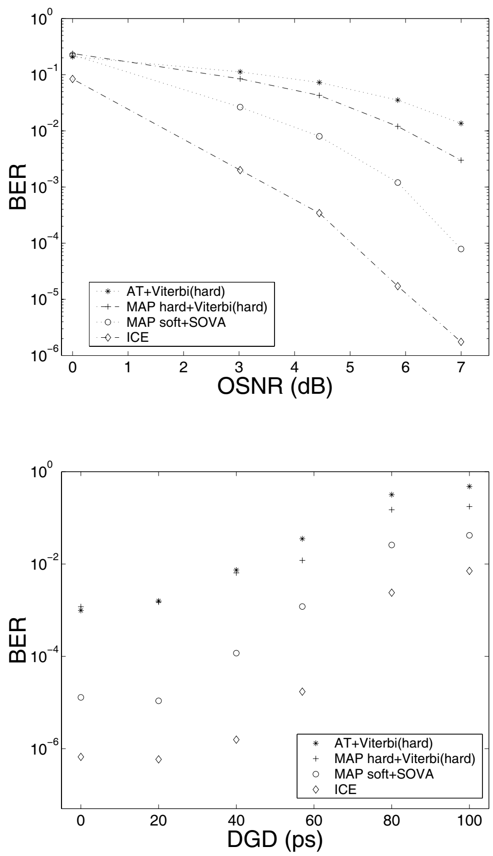

Fig. 2. Comparison of four methods: AT concatenated with Viterbi hard decoder, MAP detector concatenated with Viterbi hard decoder, MAP soft probabilities concatenated with SOVA, and ICE. (a) BER vs OSNR; (b) BER vs DGD.

its code polynomial generators defined as $f_1(D) = 1 + D + D^2$ and $f_2(D) = 1 + D^2$. We compare these four structures for different differential group delays (DGDs) and optical signal-to-noise ratios (OSNRs). The results shown in Fig. 2a are for a fixed fiber realization with a DGD of 57 ps and Fig. 2b is for a fixed OSNR around 6 dB. DGDs are chosen near the mean DGDs of fiber realizations and Monte Carlo simulations are performed for a 16-bit pseudo-random bit sequence.

As shown in Fig. 2a, ICE provides significant improvement over the other methods as OSNR increases from 0 to 7 dB. When OSNR is 7 dB, ICE provides almost two orders of magnitude gain with respect to MAP soft probabilities concatenated with SOVA, and more than three orders of magnitude gain over the other hard-decision methods.

To study the ICE for all-order PMD compensation with different mean DGDs, we use six fiber realizations with mean DGD values of zero ps (no PMD), 20 ps, 40 ps, 57 ps, 80 ps, and 100 ps. We choose the fiber realization such that its mean DGD approximately equals the DGD at the center frequency of the optical channel. The OSNR at the receiver is 6 dB. In Fig. 2b, we compare the ICE performance with the other JCE methods. These results show that the ICE has the lowest BER for different DGD values. When the DGD increases, the BER gap among ICE and the JCEs tends to decrease. This is to be expected since as the DGD increases, the ISI produced by PMD will gradually spread beyond the immediate neighboring bits, hence violating the assumption that the ISI is well contained in a three bit pattern. To implement ICE for large DGD, the conditional pdf needs to be estimated with larger memory length.

5. SUMMARY

We demonstrate an effective method for estimating the conditional pdf of the filtered electrical current. We propose an integrated coding and equalization method for optical communications systems using the conditional pdf. The proposed integrated method offers significant performance improvement over approaches based on concatenation of coding and equalization. More powerful codes, such as the turbo product codes, can be investigated to develop more effective and practical schemes, which also have the potential to be integrated with MAP-based equalization..

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