DIFFERENTIAL CORRELATION FOR GALILEO/GPS RECEIVERS

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ABSTRACT

Current enhanced sensitivity GPS receivers developed for moderate indoor reception increase the observation period with *noncoherent integration* of the envelopes of coherently integrated predetection samples. This paper analyzes a novel method, *differential correlation*, where each current coherently integrated predetection sample is multiplied with the complex conjugate of the previous predetection sample. The products are then accumulated. The deterministic signal and stochastic noise components with their probability density functions and moments are derived algebraically for differential correlation and conventional noncoherent integration, while also considering signal and implementation nonidealities. The paper shows that *differential correlation* can offer an average sensitivity gain over conventional *noncoherent integration* of around 1.5 dB.

1. INTRODUCTION

Enhanced sensitivity GPS and future Galileo receivers need to extend the observation period for spreading code synchronization up to one second or more. The data bit stream is thereby provided through assistance information on a mobile communications channel. The target is to synchronize the local reference codes with the received spreading codes of several satellites in order to calculate the propagation delays for position determination. The satellite signals are very weak with just -158 dBW in outdoor line-of-sight propagation. The maximal coherent integration period is limited by the coherence time of the propagation channel, oscillator accuracy, and size of the frequency search bins. The conventional approach is therefore to continue with noncoherent integration of squared predetection envelopes. The novel alternative approach presented in this paper is to proceed with differentially coherent integration instead.

2. COHERENT PREDETECTION

The received Galileo/GPS signal can be expressed in its complex-valued, low-pass equivalent form as

$$r_{\mathrm{lp}}(t) = \sqrt{2C}d(t)c(t)\mathrm{e}^{\mathrm{j}\phi(t)} + n(t), \tag{1}$$

where C denotes the carrier power, $d(t)$ the data modulation, $c(t)$ the received spreading code, $\phi(t)$ the signal phase, and $n(t)$ complex-valued, zero-mean, white Gaussian noise with variance

$$\sigma_n^2 = 2\mathrm{E}\left\{\Re\{n\}^2\right\} = 2\mathrm{E}\left\{\Im\{n\}^2\right\} = 2N_0BF. \tag{2}$$

$N_0 = 1.38 \cdot 10^{-23}\mathrm{J}/^\circ\mathrm{K} \cdot 290^\circ\mathrm{K}$ denotes the thermal noise power spectral density, F the receiver noise figure, and $B = 1/T_{\mathrm{s}}$ the bandwidth of the anti-aliasing filter for the sample period T_{s}.

Despreading with the local PRN reference code $c_{\mathrm{r},\nu}$ and coherent integration of $L = T_{\mathrm{i}}/T_{\mathrm{s}}$ chips, with T_{i} the coherent integration time and N the code sequence period, yields

$$s_\mu = \sqrt{2C}\sum_{\nu=1}^{L}\left[d_\nu c_\nu c^*_{\mathrm{r},\nu+\tau \bmod N}\mathrm{e}^{\mathrm{j}\phi_\nu} + c^*_{\mathrm{r},\nu+\tau \bmod N}\right]. \tag{3}$$

For sufficiently small average frequency deviations Δf_μ during an interval $[(\mu-1)T_{\mathrm{i}},\ \mu T_{\mathrm{i}}]$, the approximation

$$\begin{aligned}\mathrm{e}^{\mathrm{j}\phi_\nu} &\approx \frac{1}{T_{\mathrm{i}}}\int_0^{T_{\mathrm{i}}} \mathrm{e}^{\mathrm{j}(2\pi\Delta f_\mu t + \varphi_\mu)}\ \mathrm{d}t \\ &= \mathrm{e}^{\mathrm{j}(\pi\Delta f_\mu T_{\mathrm{i}} + \varphi_\mu)}\mathrm{sinc}\left(\Delta f_\mu T_{\mathrm{i}}\right),\end{aligned} \tag{4}$$

$$\varphi_\mu = \varphi_{\mu-1} + 2\pi\Delta f_{\mu-1}T_{\mathrm{i}}, \tag{5}$$

and constant d_ν during $[(\mu-1)T_{\mathrm{i}},\ \mu T_{\mathrm{i}}]$ results in [4]

$$s_\mu \approx y_\mu + w_\mu, \tag{6}$$

$$y_\mu = \sqrt{2C}d_\mu R_{rc}(\tau)\mathrm{sinc}\left(\Delta f_\mu T_{\mathrm{i}}\right)\mathrm{e}^{\mathrm{j}(\pi\Delta f_\mu T_{\mathrm{i}} + \varphi_\mu)}, \tag{7}$$

$$R_{rc}(\tau) = \sum_{\nu=1}^{L} c_\nu c^*_{\mathrm{r},\nu+\tau \bmod N}. \tag{8}$$

$w_\mu = w_{\mathrm{Q},\mu} + \mathrm{j}w_{\mathrm{I},\mu}$ denotes complex-valued, zero-mean, white Gaussian noise with variance

$$\sigma_w^2 = 2\mathrm{E}\left\{w_{\mathrm{I}}^2\right\} = 2\mathrm{E}\left\{w_{\mathrm{Q}}^2\right\} = \frac{L^2}{T_{\mathrm{i}}}2N_0F. \tag{9}$$

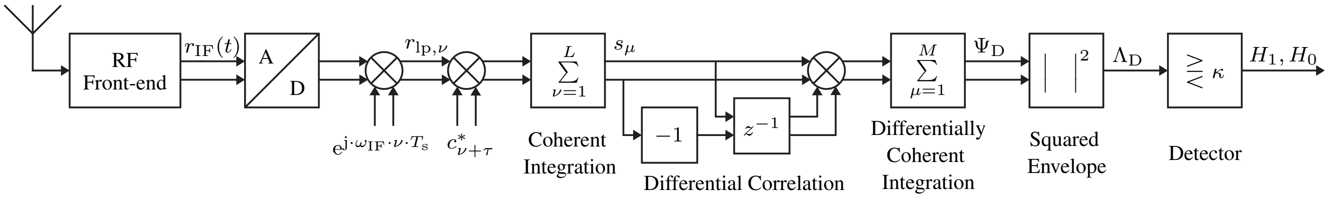

Fig. 1. Galileo/GPS receiver channel deploying differential correlation.

3. DIFFERENTIAL CORRELATION

Differential correlation multiplies each current predetection sample with the complex conjugate of the previous predetection sample, accumulates these products, and takes the squared envelope at the very end, leading to the test statistic

$$\Lambda_{\mathrm{D}} = \left|\sum_{\mu=1}^{M} s_\mu s^*_{\mu-1}\right|^2 \triangleq \left|\sum_{\mu=1}^{M}(a_\mu + g_\mu + h_\mu + v_\mu)\right|^2, \tag{10}$$

$$a_\mu = 2Cd_\mu d_{\mu-1}R^2_{rc}(\tau)\mathrm{sinc}\left(\Delta f_\mu T_{\mathrm{c}}\right) \cdot \mathrm{sinc}\left(\Delta f_{\mu-1}T_{\mathrm{i}}\right)\mathrm{e}^{\mathrm{j}[\pi(\Delta f_\mu - \Delta f_{\mu-1})T_{\mathrm{c}} + \varphi_\mu - \varphi_{\mu-1}]}, \tag{11}$$

$$g_\mu = y_\mu(w_{\mathrm{I},\mu-1} - \mathrm{j}w_{\mathrm{Q},\mu-1}), \tag{12}$$

$$h_\mu = y^*_{\mu-1}(w_{\mathrm{I},\mu} + \mathrm{j}w_{\mathrm{Q},\mu}), \tag{13}$$

$$v_\mu = w_{\mathrm{I},\mu}w_{\mathrm{I},\mu-1} + w_{\mathrm{Q},\mu}w_{\mathrm{Q},\mu-1} + \mathrm{j}(w_{\mathrm{Q},\mu}w_{\mathrm{I},\mu-1} - w_{\mathrm{I},\mu}w_{\mathrm{Q},\mu-1}). \tag{14}$$

The accumulation of the zero-mean complex Gaussian noise

$$\sum_{\mu=1}^{M}(g_\mu + h_\mu) \triangleq \theta = \sum_{\mu=0}^{M}\left(\Re\{y_{\mu+1} + y_{\mu-1}\}\, w_{\mathrm{I},\mu} + \Im\{y_{\mu+1} + y_{\mu-1}\}\, w_{\mathrm{Q},\mu}\right) + \mathrm{j}\sum_{\mu=0}^{M}\left(\Im\{y_{\mu+1} - y_{\mu-1}\}\, w_{\mathrm{I},\mu} + \Re\{y_{\mu-1} - y_{\mu+1}\}\, w_{\mathrm{Q},\mu}\right) \tag{15}$$

yields another zero-mean complex Gaussian variable θ with

$$\mathrm{E}\left\{\Re\left\{\theta\right\}^2\right\} = \frac{\sigma_w^2}{2}\sum_{\mu=0}^{M}\left|y_{\mu+1} + y_{\mu-1}\right|^2, \tag{16}$$

$$\mathrm{E}\left\{\Im\left\{\theta\right\}^2\right\} = \frac{\sigma_w^2}{2}\sum_{\mu=0}^{M}\left|y_{\mu+1} - y_{\mu-1}\right|^2, \tag{17}$$

$$y_{-1} = y_{M+1} = 0. \tag{18}$$

The product of two zero-mean, statistically independent, normally distributed random variables

$$w_{\{\mathrm{I\ or\ Q}\},\mu}w_{\{\mathrm{I\ or\ Q}\},\mu-1} \triangleq u_\mu \tag{19}$$

obeys the normal product distribution [6]

$$p_u(u) = \frac{\mathrm{K}_0\left(\frac{|u|}{\sigma_w^2}\right)}{\pi\sigma_w^2}, \tag{20}$$

where $\mathrm{K}_n(x)$ is the modified Bessel function of second kind and order n. By applying [1]

$$\int_0^\infty t^\mu \mathrm{K}_\nu(t)\ \ dt = 2^{\mu-1}\Gamma\left(\frac{\mu+\nu+1}{2}\right)\Gamma\left(\frac{\mu-\nu+1}{2}\right), \tag{21}$$

where $\Gamma(x)$ denotes the Gamma function, the variance of the resulting zero-mean normal product distribution is

$$\sigma_u^2 = \mathrm{E}\{u^2\} = \int_{-\infty}^{\infty} u^2 p_u(u)\ \ du = 2\int_0^\infty \frac{\sigma_w^2}{2\pi}t^2\mathrm{K}_0(t)\frac{\sigma_w^2}{2}\ \ dt = \frac{\sigma_w^4}{\pi}\Gamma^2\left(\frac{3}{2}\right) = \frac{\sigma_w^4}{4}. \tag{22}$$

The accumulation of the complex-valued, normal product distributed noise v_μ can be rewritten as

$$\sum_{\mu=1}^{M} v_\mu = \sum_{\mu=1}^{M/2}\Big[w_{\mathrm{I},2\mu-1}w_{\mathrm{I},2\mu-2} + w_{\mathrm{Q},2\mu-1}w_{\mathrm{Q},2\mu-2} + \mathrm{j}(w_{\mathrm{Q},2\mu-1}w_{\mathrm{I},2\mu-2} - w_{\mathrm{I},2\mu-1}w_{\mathrm{Q},2\mu-2})\Big] + \sum_{\mu=1}^{M/2}\Big[w_{\mathrm{I},2\mu}w_{\mathrm{I},2\mu-1} + w_{\mathrm{Q},2\mu}w_{\mathrm{Q},2\mu-1} + \mathrm{j}(w_{\mathrm{Q},2\mu}w_{\mathrm{I},2\mu-1} - w_{\mathrm{I},2\mu}w_{\mathrm{Q},2\mu-1})\Big] \triangleq \vartheta\ , \tag{23}$$

such that there are eight accumulations of statistically independent variables. With the central limit theorem, for sufficiently large M, all eight accumulations converge to uncorrelated, zero-mean, Gaussian distributed variables with

$$\mathrm{E}\left\{\left(\sum_{\mu=1}^{M/2} u_{2\mu}\right)^2\right\} = \mathrm{E}\left\{\left(\sum_{\mu=1}^{M/2} u_{2\mu-1}\right)^2\right\} = \frac{M}{8}\sigma_w^4\,. \tag{24}$$

Combining the eight independent variables leads, for sufficiently large M, to complex, zero-mean, white Gaussian noise ϑ with variance

$$\sigma_\vartheta^2 = 2\mathrm{E}\left\{\Re\{\vartheta\}^2\right\} = 2\mathrm{E}\left\{\Im\{\vartheta\}^2\right\} = M\sigma_w^4. \tag{25}$$

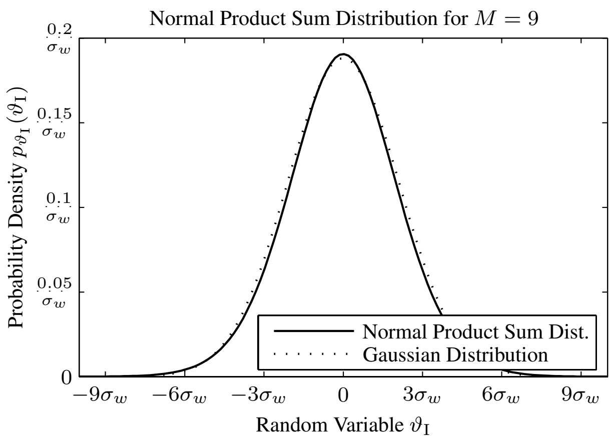

Fig. 2. Comparison of the normal product sum distribution $p_{\vartheta_\mathrm{I}}(\vartheta_\mathrm{I})$ versus the corresponding Gaussian distribution for a small $M = 9$.

Simulations, such as the one presented in Fig. 2, have shown that for $M = 9$ the accumulated variable already converges to a Gaussian distribution with a high degree of accuracy.

4. DIFFERENTIAL CORRELATION RESULT

Combining all results for the differential correlation

$$\Lambda_\mathrm{D} = \left| \sum_{\mu=1}^{M} s_\mu s_{\mu-1}^* \right|^2 \triangleq |\Psi_\mathrm{D}|^2 \tag{26}$$

leads, for sufficiently large M, to a complex Gaussian variable Ψ_D with mean value

$$\begin{aligned} m_\Psi =& \mathrm{E}\left\{\Psi_\mathrm{D}\right\} = 2CR_{rc}^2(\tau) \sum_{\mu=1}^{M} \Big[d_\mu d_{\mu-1} \mathrm{sinc}\left(\Delta f_\mu T_\mathrm{c}\right) \\ & \cdot \mathrm{sinc}\left(\Delta f_{\mu-1} T_\mathrm{i}\right) \mathrm{e}^{\mathrm{j}\left[\pi(\Delta f_\mu - \Delta f_{\mu-1}) T_\mathrm{c} + \varphi_\mu - \varphi_{\mu-1}\right]} \Big] \end{aligned} \tag{27}$$

and variances

$$\begin{aligned} \sigma_{\Psi,\mathrm{I}}^2 &= \mathrm{E}\left\{\Re\{\Psi_\mathrm{D} - m_\Psi\}^2\right\} \\ &= M \frac{\sigma_w^4}{2} + \frac{\sigma_w^2}{2} \sum_{\mu=0}^{M} \left|y_{\mu+1} + y_{\mu-1}\right|^2, \end{aligned} \tag{28}$$

$$\begin{aligned} \sigma_{\Psi,\mathrm{Q}}^2 &= \mathrm{E}\left\{\Im\{\Psi_\mathrm{D} - m_\Psi\}^2\right\} \\ &= M \frac{\sigma_w^4}{2} + \frac{\sigma_w^2}{2} \sum_{\mu=0}^{M} \left|y_{\mu+1} - y_{\mu-1}\right|^2, \end{aligned} \tag{29}$$

where y_μ and σ_w^2 are defined in (7) and (9) respectively. For a stable frequency deviation, the differential correlation delivers signal components that are all in phase to each other

$$\varphi_\mu - \varphi_{\mu-1} \Big|_{\Delta f = \mathrm{const.}} = 2\pi \Delta f T_\mathrm{i} = \mathrm{const.}, \tag{30}$$

$$\begin{aligned} m_\Psi \Big|_{\Delta f = \mathrm{const.}} &= 2CR_{rc}^2(\tau) \mathrm{sinc}^2\left(\Delta f T_\mathrm{i}\right) \\ & \cdot \mathrm{e}^{\mathrm{j}2\pi\Delta f T_\mathrm{i}} \sum_{\mu=1}^{M} d_\mu d_{\mu-1}. \end{aligned} \tag{31}$$

The test statistic for differential correlation, $\Lambda_\mathrm{D} = |\Psi_\mathrm{D}|^2$ is the sum of two statistically independent non-central Chi-squared distributions with different variances, leading to the probability density [5]

$$\begin{aligned} p_{\Lambda_\mathrm{D}}(\Lambda) =& \frac{1}{2\sigma_{\Psi,\mathrm{Q}}^2} \sqrt{\frac{\Lambda}{m_\mathrm{D}}} \exp\left(-\frac{\Re\{m_\Psi\}}{2\sigma_{\Psi,\mathrm{I}}^2} - \frac{\Im\{m_\Psi\}}{2\sigma_{\Psi,\mathrm{Q}}^2} \right) \\ & \cdot \exp\left(-\frac{\Lambda}{2\sigma_{\Psi,\mathrm{I}}^2} \right) \sum_{k=0}^{\infty} \sum_{m=0}^{\infty} \Bigg[\frac{\Gamma(1+k+m)}{k! m! \Gamma(1+m)} \\ & \cdot \left(\frac{\sqrt{\Lambda} \Im\{m_\Psi\}^2 \sigma_{\Psi,\mathrm{I}}^2}{2\Re\{m_\Psi\} \sigma_{\Psi,\mathrm{Q}}^4} \right)^m \left(\frac{\sqrt{\Lambda}(\sigma_{\Psi,\mathrm{Q}}^2 - \sigma_{\Psi,\mathrm{I}}^2)}{\Re\{m_\Psi\} \sigma_{\Psi,\mathrm{Q}}^2} \right)^k \\ & \cdot \mathrm{I}_{1+k+m} \left(\frac{\sqrt{\Lambda} \Re\{m_\Psi\}}{\{\sigma_{\Psi,\mathrm{I}}^2\}} \right) \Bigg], \end{aligned} \tag{32}$$

where $\Gamma(x)$ denotes the Gamma function and $\mathrm{I}_n(x)$ the modified Bessel function of first kind and order n.

5. CONVENTIONAL NONCOHERENT INTEGRATION RESULT

Current enhanced sensitivity GPS receivers usually take the squared envelope of the predetection samples s_μ after coherent integration and accumulate the envelopes

$$\Lambda_\mathrm{S} = \sum_{\mu=0}^{M} |s_\mu|^2. \tag{33}$$

The probability density of Λ_S is therefore a non-central Chi-squared distribution [2]

$$p_{\Lambda_\mathrm{S}}(\Lambda) = \frac{1}{\sigma_w^2} \left(\frac{\Lambda}{\gamma_\mathrm{S}^2} \right)^{\frac{M}{2}} \exp\left(-\frac{\Lambda + \gamma_\mathrm{S}^2}{\sigma_w^2} \right) \mathrm{I}_M \left(\frac{2\sqrt{\Lambda \gamma_\mathrm{S}^2}}{\sigma_w^2} \right) \tag{34}$$

with $2M + 2$ degrees of freedom and the noncentrality parameter

$$\gamma_\mathrm{S}^2 = \sum_{\mu=0}^{M} |m_{s_\mu}|^2 = 2CR_{rc}^2(\tau) \sum_{\mu=0}^{M} \mathrm{sinc}^2\big(\Delta f_\mu T_\mathrm{i}\big). \tag{35}$$

A detailed analysis of conventional noncoherent integration for Galileo and GPS, including multipath fading scenarios is provided in [4].

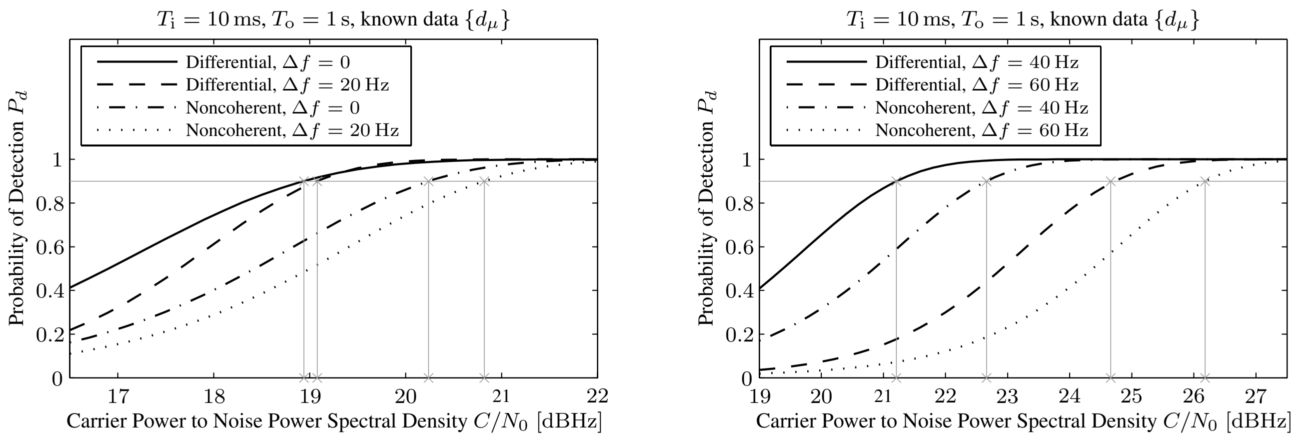

Fig. 3. Probabilities of detection for differential correlation and conventional noncoherent integration, where $M = 99$, $P_f = 10^{-3}$, $R_{rc}(0) = L$, $R_{rc}(\tau \neq 0) = L \cdot 10^{-1.08}$ (according to [3]), $C/N_0|_\mathrm{estimated} = 2C/N_0|_\mathrm{real}$, and $F = 3\,\mathrm{dB}$.

6. SYNCHRONIZATION DETECTION

The detector is based on the Neyman-Pearson criterion, which maximizes the probability of detection P_d for a given probability of false alarm P_f. The threshold κ is therefore calculated for a fixed probability of false alarm

$$P_\mathrm{f} = \int_\kappa^\infty p_{\Lambda|H_0}(\Lambda|\, H_0) \quad d\Lambda \tag{36}$$

using hypothesis H_0. Cross-correlation $R_{rc}(\tau \neq 0)$ plus noise w is present for H_0, whereas the correlation peak $R_{rc}(0)$ plus noise w is present for hypothesis H_1. κ is then utilized to test Λ_D and Λ_S for synchronization of the local reference code $c_{\mathrm{r},\nu}$ with the received spreading code c_ν

$$\Lambda \underset{H_0}{\overset{H_1}{\gtrless}} \kappa. \tag{37}$$

The resulting probabilities of detection

$$P_\mathrm{d} = \int_\kappa^\infty p_{\Lambda|H_1}(\Lambda|\, H_1) \quad d\Lambda \tag{38}$$

for hypothesis H_1 are presented in Fig. 3, where it can be observed that the acquisition threshold for differential correlation is in average around 1.5 dB better than the one for conventional noncoherent integration. The data bit stream d_μ is provided through assistance data on a mobile communications channel and therefore known a-priori. Enhanced sensitivity reception requires an implicit estimation of C/N_0, for which an estimation error of 3 dB is assumed, such that $C/N_0|_\mathrm{estimated} = 2C/N_0|_\mathrm{real}$.

7. CONCLUSION

Differential correlation can provide a sensitivity gain over conventional *noncoherent integration* that depends on the respective scenario and averages around 1.5 dB. This was confirmed through simulations with a wide variety of parameter sets, including time-variant frequency deviations.

All deterministic and stochastic signal components with their probability density functions and moments have been derived algebraically after (a) down-conversion to baseband, (b) despreading and coherent integration, (c) differential correlation, (d) differentially coherent accumulation, (e) squared envelope of differentially coherent accumulation, and (f) conventional noncoherent integration. Nonidealities in the receiver and signal structure are incorporated.

8. REFERENCES

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