ENHANCED BLIND SUBSPACE-BASED SIGNATURE WAVEFORM ESTIMATION IN CDMA SYSTEMS WITH CIRCULAR NOISE

Keyvan Zarifi[†] *Alex B. Gershman*[*]

[†]Dept. of Comm. Systems, University of Duisburg-Essen, Germany
[*]Dept. of ECE, McMaster University, Hamilton, Ontario, Canada

ABSTRACT

A new subspace-based blind algorithm for signature waveform estimation in direct-sequence code division multiple-access (DS-CDMA) systems is proposed. Our technique represents a generalization of the popular technique by Liu and Xu and additionally exploits non-circularity of the transmitted signals and circularity of noise to increase the dimension of the observation space twice while keeping the dimension of the signal subspace unchanged. This leads to a substantially improved performance of the proposed algorithm and enables to apply it to scenarios with larger numbers of active users and lengthier user channels as compared to the original algorithm by Liu and Xu.

1. INTRODUCTION

One of the most challenging problems in multiuser detection in DS-CDMA systems is the problem of unknown signature waveform mismatch. Such a mismatch may be caused by a distortion of the signature waveform of the user-of-interest by frequency-selective fading effects. As a mismatched signature waveform can lead to a severe degradation of the performance of multiuser detectors [1]-[2], signature waveform estimation at the receiver side is an important prerequisite of any multiuser detection procedure. Signature waveform estimation techniques can be classified into *training-based* and *blind* groups of methods. Because of a limited bandwidth efficiency of the training-based techniques [1], blind algorithms have attracted recently a considerable attention. Among latter algorithms, subspace-based methods [1], [3]-[5] represent a promising trend. Although blind subspace-based signature waveform estimation techniques have an obvious potential to provide excellent estimation accuracy, certain practical hurdles may make them inapplicable to real-world environments. For instance, these techniques are only applicable to underloaded systems, i.e., the systems where the number of active users is less than the dimension of the observation space [3]-[4]. Another limitation emerges from the fact that usually the problem of signature waveform estimation boils down to a channel identification problem [1], [3]-[5]. Since all of blind subspace-based methods implicitly [1], [3] or explicitly [4]-[5] pose some restrictions on the length of the channel, their reliability in a long-delay multipath environment is questionable. The main contribution of this paper is to propose an enhanced version of one popular subspace-based signature waveform estimation technique which, as compared to its conventional counterpart, is capable to identify lengthier channels in more heavily loaded environments.

While the idea of our technique can be easily adapted to modify any other subspace-based method, we develop it based on the popular algorithm by Liu and Xu [4] which is hereafter called the LX algorithm. To develop our approach, it is assumed that all transmitted symbols are drawn from the BPSK constellation and the ambient noise is a white and circular process [6]-[7]. Borrowing the idea which has been earlier successfully applied in array processing [7], we exploit the noise circularity property jointly with the non-circular property of the transmitted signals to double the dimension of the observation space without affecting the dimension of the signal subspace. Using the so-obtained extended observation space, the proposed algorithm is shown to facilitate identification of lengthier channels in more heavily loaded environments than the LX algorithm. As the developed extended model provides more equations to estimate the sampled channel impulse response, the proposed technique is shown to substantially outperform the LX algorithm when both algorithms are implemented using the least-squares (LS) method. We also provide a sufficient condition for channel identifiability of the proposed algorithm and show that any channel which could be identified using the LX algorithm is also identifiable by our technique.

2. SIGNAL MODEL

Let us consider a K-user synchronous DS-CDMA system[1]. The continuous-time baseband received signal can be modeled as

$$x(t) = \sum_{m=-\infty}^{\infty} \sum_{k=1}^{K} A_k b_k(m) w_k(t - mT_s) + v(t) \tag{1}$$

where T_s is the symbol period, A_k is the amplitude of the received signal of the kth user, $b_k(m) \in \{-1, +1\}$ is the mth transmitted data symbol of the kth user, $w_k(t)$ is the signature waveform of the kth user, and $v(t)$ is additive zero-mean circular white noise with variance σ^2.

Assuming that the spreading code is short (i.e., the chip sequence period is the same as the symbol period) and that the user channel impulse responses are qiasi-static (i.e., fixed during the observation period), the signature waveform of the kth user can be written as [4]

$$w_k(t) = \sum_{l=1}^{L_c} c_k[l] h_k\big(t - lT_c\big) \tag{2}$$

where $\mathbf{c}_k \triangleq \big[c_k[1], c_k[2], \ldots, c_k[L_c]\big]^T$ is the spreading code vector of the kth user, $h_k(t)$ is the channel impulse response of the kth user, L_c is the spreading factor, $T_c = T_s/L_c$ is the chip period, and $(\cdot)^T$ stands for the transpose.

[1]The extension to the asynchronous case is direct [8].

Let $h_k(t)$ have a finite support of $[0, \alpha_k T_c]$, where $L-1 \leq \max\{\alpha_1, \ldots, \alpha_K\} < L$ and L is a positive integer. We assume that the maximum duration of the channel impulse response is shorter than the symbol period T_s, so that at least some part of the received signal is not contaminated by intersymbol interference (ISI)[2]. Sampling (1) at the interval corresponding to the nth transmitted symbol with the period of T_c and ignoring the first $L-1$ samples contaminated by ISI, the $(L_c - L+1) \times 1$ ISI-free received sampled data vector can be expressed as [4]

$$\mathbf{x}(n) = \sum_{k=1}^{K} A_k b_k(n) \mathbf{w}_k + \mathbf{v}(n) \tag{3}$$

where $\mathbf{x}(n) \triangleq [x(nT_s + LT_c), x(nT_s + (L+1)T_c), \ldots, x(nT_s + L_c T_c)]^T$, $\mathbf{w}_k \triangleq [w_k(LT_c), w_k((L+1)T_c), \ldots, w_k(L_c T_c)]^T$, and $\mathbf{v}(n) \triangleq [v(nT_s + LT_c), v(nT_s + (L+1)T_c), \ldots, v(nT_s + L_c T_c)]^T$. Using (2), the vector $\mathbf{w}_k$ can be written as

$$\mathbf{w}_k = \begin{bmatrix} c_k[L] & \ldots & c_k[1] \\ c_k[L+1] & \ldots & c_k[2] \\ \vdots & \ddots & \vdots \\ c_k[L_c] & \ldots & c_k[L_c - L + 1] \end{bmatrix} \mathbf{h}_k \triangleq \mathbf{C}_k \mathbf{h}_k \tag{4}$$

where $\mathbf{h}_k \triangleq [h_k(0), h_k(T_c), \cdots, h_k((L-1)T_c)]^T$. As the spreading code of the desired user is known at the receiver, if the channel vector $\mathbf{h}_k$ is estimated, then $\mathbf{w}_k$ can be directly obtained from (4). Thus, throughout this paper we consider the problem of channel vector rather than signature waveform vector estimation. Without any loss of generality, let us assume that $\|\mathbf{h}_k\| = 1$, i.e., the norm of $\mathbf{h}_k$ is absorbed in the corresponding amplitude A_k. Then, we can rewrite (3) in a compact form as

$$\mathbf{x}(n) = \mathbf{W}\mathbf{b}(n) + \mathbf{v}(n) \tag{5}$$

where $\mathbf{W} = [A_1 \mathbf{w}_1, A_2 \mathbf{w}_2, \ldots, A_K \mathbf{w}_K]$, and $\mathbf{b}(n) = [b_1(n), b_2(n), \ldots, b_K(n)]^T$.

3. BLIND SIGNATURE WAVEFORM ESTIMATION

First, let us briefly overview the LX algorithm of [4]. From (5), we have that the received data correlation matrix $\mathbf{R}$ is given by

$$\mathbf{R} \triangleq \mathrm{E}\{\mathbf{x}(n)\mathbf{x}(n)^H\} = \mathbf{W}\mathbf{W}^H + \sigma^2 \mathbf{I}. \tag{6}$$

The matrix (6) can be eigendecomposed as

$$\mathbf{R} = \mathbf{U}_s \mathbf{\Lambda}_s \mathbf{U}_s^H + \sigma^2 \mathbf{U}_n \mathbf{U}_n^H \tag{7}$$

where the $\mathbf{\Lambda}_s = \mathrm{diag}(\lambda_1 + \sigma^2, \ldots, \lambda_K + \sigma^2)$ contains the K largest (signal-subspace) eigenvalues of $\mathbf{R}$, $\mathbf{U}_s$ contains the corresponding eigenvectors, and the $(L_c - L+1) \times (L_c - L - K+1)$ matrix $\mathbf{U}_n$ contains the noise-subspace eigenvectors of $\mathbf{R}$ associated with its minimal eigenvalue σ^2. Without any loss of generality, let us assume that the channel vector that has to be estimated is $\mathbf{h}_1$. As $\mathrm{range}(\mathbf{W}) = \mathrm{range}(\mathbf{U}_s)$, we have $\mathbf{U}_n^H \mathbf{w}_1 = \mathbf{U}_n^H \mathbf{C}_1 \mathbf{h}_1 = \mathbf{0}$. Hence, $\mathbf{h}_1$ is a nontrivial solution of the following equation:

$$\mathbf{U}_n^H \mathbf{C}_1 \mathbf{h} = \mathbf{0}. \tag{8}$$

[2] Maximal admissible duration of the channel impulse response will be discussed in Section 3.

It has been shown in [4] that if a certain identifiability condition is satisfied, then this solution is unique up to a scaling factor. Therefore, in such case $\mathbf{h}_1$ can be uniquely identified from (8). In practice, only the sample correlation matrix $\hat{\mathbf{R}}$ is available, and (8) should be solved in the LS sense. Details of implementation of the LX algorithm can be found in [4] and, hence, are omitted here.

As (8) is a set of $L_c - L - K + 1$ complex equations and L complex unknowns, uniqueness of its nontrivial solution requires that

$$2L + K \leq L_c + 1. \tag{9}$$

It can be observed that (9) severely restricts both the maximum channel order L and the number of active users K. To circumvent this impairment, let us take into account that all the transmitted symbols are drawn from the BPSK constellation, and, therefore, they are non-circular, whereas the noise is assumed to be circular. Using the latter fact, we have [6]

$$\mathrm{E}\{\mathbf{v}(n)\mathbf{v}(n)^T\} = \mathbf{0}. \tag{10}$$

Introducing the $2(L_c - L+1) \times 1$ extended received data vectors $\bar{\mathbf{x}}(n) \triangleq [\mathbf{x}^T(n) \mathbf{x}^H(n)]^T$, we have

$$\bar{\mathbf{x}}(n) = \begin{bmatrix} \mathbf{W} \\ \mathbf{W}^* \end{bmatrix} \mathbf{b}(n) + \begin{bmatrix} \mathbf{v}(n) \\ \mathbf{v}^*(n) \end{bmatrix} \tag{11}$$

where $(\cdot)^*$ stands for the complex conjugate. Using (10) along with (11), we obtain

$$\bar{\mathbf{R}} = \mathrm{E}\{\bar{\mathbf{x}}(n)\bar{\mathbf{x}}^H(n)\} = \begin{bmatrix} \mathbf{W} \\ \mathbf{W}^* \end{bmatrix} \begin{bmatrix} \mathbf{W} \\ \mathbf{W}^* \end{bmatrix}^H + \sigma^2 \mathbf{I}. \tag{12}$$

The eigendecomposition of $\bar{\mathbf{R}}$ can be written as

$$\bar{\mathbf{R}} = \bar{\mathbf{U}}_s \bar{\mathbf{\Lambda}}_s \bar{\mathbf{U}}_s^H + \sigma^2 \bar{\mathbf{U}}_n \bar{\mathbf{U}}_n^H \tag{13}$$

where $\bar{\mathbf{\Lambda}}_s = \mathrm{diag}(\bar{\lambda}_1 + \sigma^2, \ldots, \bar{\lambda}_K + \sigma^2)$ contains the K largest (signal-subspace) eigenvalues of $\bar{\mathbf{R}}$, the $2(L_c - L+1) \times K$ matrix $\bar{\mathbf{U}}_s$ contains the corresponding eigenvectors, and the $2(L_c - L + 1) \times (2(L_c - L+1) - K)$ matrix $\bar{\mathbf{U}}_n$ contains the noise-subspace eigenvectors of $\bar{\mathbf{R}}$ associated with its minimal eigenvalue σ^2.

Using $\bar{\mathbf{x}}$ instead of $\mathbf{x}$, the dimension of the observation space is increased twice, while the dimension of the signal subspace remains unchanged. Since $\mathrm{range}(\bar{\mathbf{U}}_s) = \mathrm{range}\left(\begin{bmatrix} \mathbf{W} \\ \mathbf{W}^* \end{bmatrix}\right)$ and $\bar{\mathbf{U}}_n$ is the orthogonal complement of $\bar{\mathbf{U}}_s$, we have

$$\bar{\mathbf{U}}_n^H \begin{bmatrix} \mathbf{w}_1 \\ \mathbf{w}_1^* \end{bmatrix} = \bar{\mathbf{U}}_n^H \begin{bmatrix} \mathbf{C}_1 \mathbf{h}_1 \\ \mathbf{C}_1^* \mathbf{h}_1^* \end{bmatrix} = \mathbf{0}. \tag{14}$$

Denoting the matrix which contains the first $L_c - L + 1$ rows of $\bar{\mathbf{U}}_n$ as $\bar{\mathbf{U}}_{n_1}$, and the matrix which contains the remaining last $L_c - L + 1$ rows of $\bar{\mathbf{U}}_n$ as $\bar{\mathbf{U}}_{n_2}$, we find from (14) that

$$\bar{\mathbf{U}}_{n_1}^H \mathbf{C}_1 \mathbf{h}_1 + \bar{\mathbf{U}}_{n_2}^H \mathbf{C}_1^* \mathbf{h}_1^* = \mathbf{0}. \tag{15}$$

To solve (15) for $\mathbf{h}_1$, let $\mathbf{T}_1 \triangleq \bar{\mathbf{U}}_{n_1}^H \mathbf{C}_1$ and $\mathbf{T}_2 \triangleq \bar{\mathbf{U}}_{n_2}^H \mathbf{C}_1^*$. Then, (15) can be rewritten in the following equivalent form

$$\begin{bmatrix} \mathrm{Re}(\mathbf{T}_1) + \mathrm{Re}(\mathbf{T}_2) & \mathrm{Im}(\mathbf{T}_2) - \mathrm{Im}(\mathbf{T}_1) \\ \mathrm{Im}(\mathbf{T}_1) + \mathrm{Im}(\mathbf{T}_2) & \mathrm{Re}(\mathbf{T}_1) - \mathrm{Re}(\mathbf{T}_2) \end{bmatrix} \begin{bmatrix} \mathrm{Re}(\mathbf{h}_1) \\ \mathrm{Im}(\mathbf{h}_1) \end{bmatrix} = \mathbf{0} \tag{16}$$

where $\mathrm{Re}(\cdot)$ and $\mathrm{Im}(\cdot)$ stand for the real and the imaginary parts, respectively. As an alternative of using the LX algorithm, we propose to use (16) for signature waveform estimation. In the finite sample case, our algorithm can be summarized as the follows:

1. Find the sample estimate $\hat{\bar{\mathbf{R}}} = \frac{1}{N}\sum_{k=1}^{N}\bar{\mathbf{x}}(k)\bar{\mathbf{x}}^H(k)$ of the correlation matrix $\bar{\mathbf{R}}$.
2. Compute the eigendecomposition of $\hat{\bar{\mathbf{R}}}$ in the form

$$\hat{\bar{\mathbf{R}}} = \hat{\bar{\mathbf{U}}}_s\hat{\bar{\mathbf{\Lambda}}}_s\hat{\bar{\mathbf{U}}}_s^H + \hat{\bar{\mathbf{U}}}_n\hat{\bar{\mathbf{\Lambda}}}_n\hat{\bar{\mathbf{U}}}_n^H \tag{17}$$

   where the matrices $\hat{\bar{\mathbf{U}}}_s$, $\hat{\bar{\mathbf{\Lambda}}}_s$ and $\hat{\bar{\mathbf{U}}}_n$ are the finite-sample estimates of the matrices $\bar{\mathbf{U}}_s$, $\bar{\mathbf{\Lambda}}_s$ and $\bar{\mathbf{U}}_n$, respectively, and $\hat{\bar{\mathbf{\Lambda}}}_n$ is the $(2(L_c - L + 1) - K) \times (2(L_c - L + 1) - K)$ matrix of the noise-subspace eigenvalues of $\hat{\bar{\mathbf{R}}}$.
3. Using the first and the last $(L_c - L + 1)$ rows of $\hat{\bar{\mathbf{U}}}_n$, obtain the matrices $\hat{\bar{\mathbf{U}}}_{n_1}$ and $\hat{\bar{\mathbf{U}}}_{n_2}$ which are the finite-sample estimates of the matrices $\bar{\mathbf{U}}_{n_1}$ and $\bar{\mathbf{U}}_{n_2}$, respectively. Compute $\hat{\mathbf{T}}_1 = \hat{\bar{\mathbf{U}}}_{n_1}^H\mathbf{C}_1$ and $\hat{\mathbf{T}}_2 = \hat{\bar{\mathbf{U}}}_{n_2}^H\mathbf{C}_1^*$.
4. Compute the matrix

$$\mathbf{\Psi} = \begin{bmatrix} \mathrm{Re}(\hat{\mathbf{T}}_1) + \mathrm{Re}(\hat{\mathbf{T}}_2) & \mathrm{Im}(\hat{\mathbf{T}}_2) - \mathrm{Im}(\hat{\mathbf{T}}_1) \\ \mathrm{Im}(\hat{\mathbf{T}}_1) + \mathrm{Im}(\hat{\mathbf{T}}_2) & \mathrm{Re}(\hat{\mathbf{T}}_1) - \mathrm{Re}(\hat{\mathbf{T}}_2) \end{bmatrix}. \tag{18}$$

   and find the $2L \times 1$ *minor* eigenvector of $\mathbf{\Psi}^H\mathbf{\Psi}$, i.e., the eigenvector associated with the smallest eigenvalue of this matrix. Denote this eigenvector as $\mathbf{s}$.
5. Estimate the channel vector $\mathbf{h}_1$ as

$$\hat{\mathbf{h}}_1 = \mathbf{s}_1 + j\mathbf{s}_2 \tag{19}$$

   where $\mathbf{s}_1$ and $\mathbf{s}_2$ are the $L \times 1$ subvectors of $\mathbf{s}$ which contain its first and last L entries, respectively.

The linear system in (16) contains $4(L_c - L + 1) - 2K$ real equations and $2L$ real unknowns. To have a unique nontrivial solution for (16), it is necessary that the number of unknowns is less than or equal to the number of equations, that is,

$$3L + K \leq 2L_c + 1. \tag{20}$$

Comparing (20) with (9) and taking to account that L_c is typically much larger than L, it can be observed that the proposed algorithm has a substantially more relaxed condition on the maximal identifiable channel length L and the number of active users K. Also, from the fact that the number of real equations is increased in the proposed algorithm from $2(L_c - L + 1) - 2K$ to $4(L_c - L + 1) - 2K$ (while keeping the number of unknowns unchanged), it may be expected that the channel estimation accuracy of our algorithm will be better than that of the LX technique.

4. SUFFICIENT IDENTIFIABILITY CONDITION

In this section, we prove that the proposed algorithm has a sufficient identifiability condition which is identical to the necessary and sufficient condition established in [4] for the LX algorithm.

Lemma 1: Assume that $\mathbf{C}_1$ is a full column-rank matrix. If

$$\mathrm{rank}[\mathrm{range}(\mathbf{C}_1) \cap \mathrm{range}(\mathbf{W})] = 1 \tag{21}$$

then, up to an arbitrary scaling factor, the following equation has its unique nontrivial solution equal to $\mathbf{h}_1$:

$$\bar{\mathbf{U}}_{n_1}^H\mathbf{C}_1\mathbf{h} + \bar{\mathbf{U}}_{n_2}^H\mathbf{C}_1^*\mathbf{h}^* = \mathbf{0}. \tag{22}$$

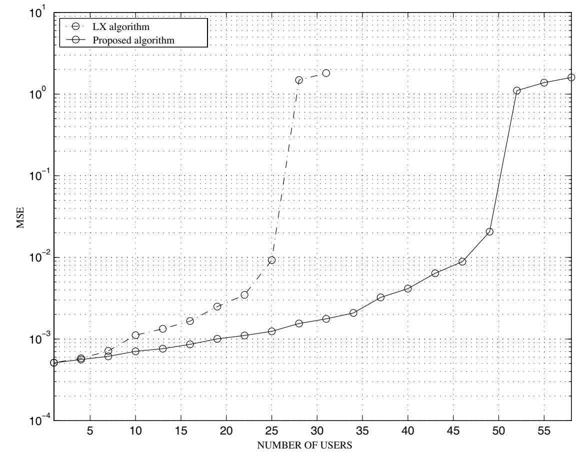

Figure 1: MSEs of the methods tested versus the number of users K.

Proof: Proof is by contradiction. First, (22) can be rewritten as

$$\bar{\mathbf{U}}_n^H \begin{bmatrix} \mathbf{C}_1\mathbf{h} \\ \mathbf{C}_1^*\mathbf{h}^* \end{bmatrix} = \mathbf{0}. \tag{23}$$

Using (14), we notice that (23) has at least one nontrivial solution $\mathbf{h} = \mathbf{h}_1$. Let us assume that (23) has another solution $\mathbf{g} \neq \alpha\mathbf{h}_1$, where α is an arbitrary scaling factor. This assumption yields

$$\begin{bmatrix} \mathbf{C}_1\mathbf{g} \\ \mathbf{C}_1^*\mathbf{g}^* \end{bmatrix} \in \mathrm{range}\left(\begin{bmatrix} \mathbf{W} \\ \mathbf{W}^* \end{bmatrix}\right). \tag{24}$$

From (24), it follows that there is some vector $\mathbf{f}$ such that

$$\begin{bmatrix} \mathbf{C}_1\mathbf{g} \\ \mathbf{C}_1^*\mathbf{g}^* \end{bmatrix} = \begin{bmatrix} \mathbf{W} \\ \mathbf{W}^* \end{bmatrix}\mathbf{f}. \tag{25}$$

Hence, we have $\mathbf{C}_1\mathbf{g} = \mathbf{W}\mathbf{f}$. On the other hand, we know that $\mathbf{C}_1\mathbf{h}_1 = \mathbf{w}_1 = \mathbf{W}\mathbf{e}_1/A_1$ where $\mathbf{e}_1 \triangleq [1, 0, \ldots, 0]^T$. Therefore, we have

$$\mathbf{C}_1[\mathbf{g}\ \mathbf{h}_1] = \mathbf{W}[\mathbf{f}\ \mathbf{e}_1/A_1]. \tag{26}$$

Since $\mathbf{g}$ and $\mathbf{h}_1$ are linearly independent and $\mathbf{C}_1$ is a full column-rank matrix, (26) implies that $\mathrm{rank}[\mathrm{range}(\mathbf{C}_1) \cap \mathrm{range}(\mathbf{W})]$ is at least equal to 2, which contradicts to (21). This completes the proof of Lemma 1. $\square$

It is noteworthy that the sufficient identifiability condition (21) coincides with the *necessary* and *sufficient* identifiability condition of the LX algorithm [4]. Therefore, any channel vector which could be identified by the LX algorithm is also identifiable using the proposed technique. Note that the reverse statement does not necessarily hold true.

5. SIMULATION RESULTS

In all numerical examples, $L_c = 40$ and the spreading sequence associated with each user has been randomly drawn from the binary set of $\{-1, +1\}$ and then fixed throughout all examples. Similarly, the entries of the channel vectors have been randomly and independently drawn from a zero-mean complex Gaussian distribution and then have been normalized so that $\|\mathbf{h}_k\| = 1$ ($k = 1, \ldots, K$) and fixed throughout all examples. Throughout the simulations, we assume that all users have identical powers and $N =$

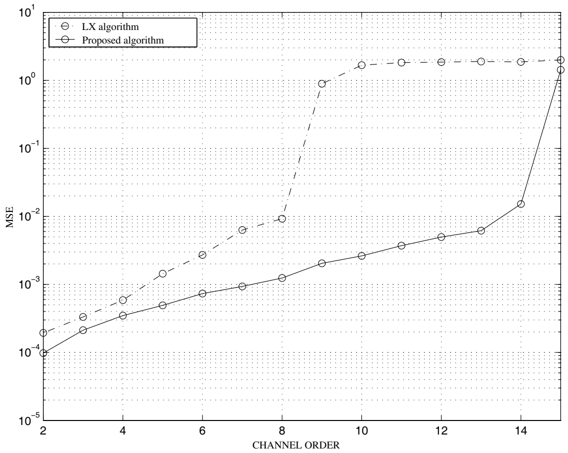

Figure 2: MSEs of the methods tested versus the channel order L.

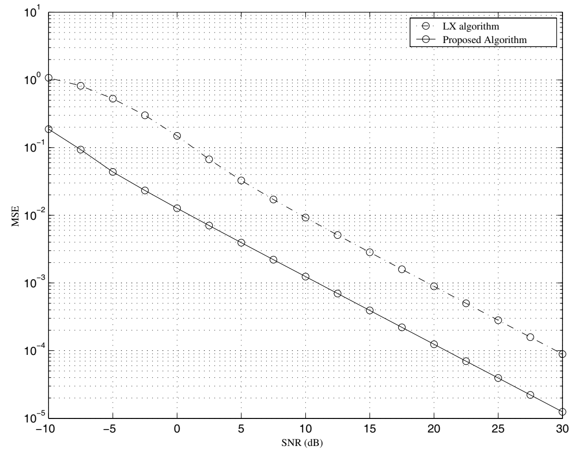

Figure 3: MSEs of the methods tested versus the SNR.

80 received data samples are available. All simulation curves are averaged over 500 realizations of the transmitted symbols and noise. In all figures, the Mean-Square-Error (MSE) of the channel estimate associated with the first user is displayed.

Figure 1 shows the MSEs of LX algorithm and the proposed technique versus the number of users K. In this figure, the channel length $L = 8$ is chosen. The signal-to-noise ratio (SNR) of all users is assumed to be equal to 10 dB. Substituting the selected values of L_c and L in (9), it can be concluded that the LX algorithm cannot properly operate if the number of users is larger than 25. This theoretical observation is fully validated by Figure 1. Note that, if $K \geq L_c - L + 1 = 33$, then the dimension of the noise subspace reduces to zero and it becomes impossible to use the LX-algorithm. At the same time, as can be observed from this figure, the proposed technique is able to identify the desired channel vector in a system with up to 50 active users. Also, even within the operating range of the LX algorithm ($K \leq 25$), this technique is outperformed by the proposed algorithm. As mentioned before, these performance improvements are due to the fact that the proposed technique exploits more equations to estimate the channel vector than the LX algorithm.

Figure 2 shows the MSEs of the both algorithms tested versus the channel length L. In this figure, the number of users $K = 25$ is fixed and all user SNRs are equal to 10 dB. From (9), we conclude that using the LX algorithm, it is not possible to identify the channel with $L > 8$. This theoretical observation is fully validated by Figure 2. From this figure, we also see that the proposed algorithm provides reliable estimates of the channel vector with the length up to $L = 14$. Moreover, this algorithm outperforms the LX technique for all values of L tested.

Figure 3 shows the MSEs of the both algorithms tested versus the user SNR for $L = 8$ and $K = 25$. Similarly to the previous two figures, substantial performance improvements of the proposed algorithm relative to the LX algorithm can be observed.

6. CONCLUSIONS

A new blind subspace-based algorithm for signature waveform estimation in DS-CDMA systems has been proposed. Our algorithm uses the key idea of the popular LX technique [4], but additionally exploits non-circularity of the transmitted signals and circularity of noise to increase the dimension of the observation space twice with respect to the LX algorithm while keeping the dimension of the signal subspace unchanged. As a result, the proposed technique has a substantially enhanced performance as compared to the LX approach. Moreover, the proposed algorithm is applicable to scenarios with an increased number of active users that have lengthier channels as compared to the scenarios which can be treated by the LX technique.

7. REFERENCES

[1] X. Wang and H. V. Poor, "Blind multiuser detection: a subspace approach," *IEEE Trans. Information Theory*, vol. 44, pp. 677-690, Mar. 1998.

[2] K. Zarifi, S. Shahbazpanahi, A. B. Gershman, and Z.-Q. Luo, "Robust blind multiuser detection based on the worst-case performance optimization of the MMSE receiver," to appear in *IEEE Trans. Signal Processing*.

[3] S. E. Bensley and B. Aazhang, "Subspace-based channel estimation for code-devision multiple-access communication systems," *IEEE Trans. Communications*, vol. 44, pp. 1009-1020, Aug. 1996.

[4] H. Liu and G. Xu, "A subspace method for signature waveform estimation in synchronous CDMA systems," *IEEE Trans. Communications*, vol. 44, pp. 1346-1354, Oct. 1996.

[5] X. Wang and H. V. Poor, "Blind equalization and multiuser detection in dispersive CDMA channels," *IEEE Trans. Communications*, vol. 46, pp. 91-103, Jan. 1998.

[6] S. Buzzi and H. V. Poor, "A single-antenna blind receiver for multiuser detection in unknown correlated noise," *IEEE Trans. Vehicular Technology*, vol. 51, pp. 209-215, Jan. 2002.

[7] M. Haardt and F. Römer, "Enhancement of unitary ESPRIT for non-circular sources," *Proc. ICASSP'04*, May 2004, Montreal, Canada.

[8] U. Madhow and M. Honig, "MMSE interference suppression for direct-sequence spread-spectrum CDMA," *IEEE Trans. Communications*, vol. 42, pp. 3178-3188, Dec. 1994.