FAST ADAPTIVE CHANNEL ESTIMATION ALGORITHMS FOR CDMA SYSTEMS

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ABSTRACT

The paper presents adaptive blind channel estimation and tracking algorithms for CDMA systems. Using only the spreading code of the user of interest, three blind estimation algorithms are proposed to estimate the channel response from the received data sequence. The idea is based on Minimum Variance (MV) receivers. Simulation results show that the proposed algorithms perform better than the previously proposed algorithms, also they are less complex and have a fast convergence rate.

1. INTRODUCTION

In Direct-Sequence CDMA communication systems, channel propagation is considered as one of the major effects that limit system performance. These effects can result in different signal distortion such frequency selective or non-selective fading [1]. Compensation of channel fading due to multipath propagation is possible through use of correlators such as RAKE receivers that coherently process multipath components. Each correlator or finger requires an estimate of its path delay and attenuation.

There have been a number of channel estimation algorithms such as training based algorithms, blind or semi-blind algorithms. In order to reduce complexity, more attention is being paid to blind and semi-bind channel estimation techniques over the training based ones. Also the presence of multipath delays destroys the assumed orthogonality between users' spreading codes. As a result, the accuracy of training based estimators is severely limited by the cross interference between data and pilot symbols.

The goal of the paper is to derive some fast blind adaptive channel estimation algorithms. Then to use these algorithms to implement low complexity receivers such as RAKE receivers for multiuser CDMA systems. This will be achieved by employing constrained optimization techniques based on MV receivers [2, 3]. We recursively minimize the output variance of the received signal subject to some constraints which are also jointly updated. Three different algorithms are proposed in this paper and compared with previously proposed algorithms [2, 3, 4, 5, 6, 7].

The paper is organized as follows. In Sec. 2, we introduce the model and assumptions. In Sec. 3, we discuss the MV receivers. In Sec. 4 we review previously proposed blind channel estimation algorithms. In Sec. 5 we present the new proposed algorithms. In Sec. 6 we present simulation results and discussion before we conclude in Sec. 7.

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2. DATA MODEL

Consider the base-band down-link received signal of a K users in a BPSK DS-CDMA system,

$$y(n) = \sum_{k=1}^{K} y_k(n) + v(n), \tag{1}$$

where,

$$y_k(n) = \sum_{l=-\infty}^{\infty} b_k(l) s_k(n - \tau_k - lT_s) \tag{2}$$

is the k'th user received signal $y_k(n)$, $b_k(l)$ is the transmitted symbol, T_s is the symbol duration, τ_k is the delay, and $s_k(n)$ is given by

$$s_k(n) = \sum_{m=-\infty}^{\infty} g_k(m) c_k(n - mT_c), \tag{3}$$

where $g_k(n)$ is the channel response, $c_k(n)$ is the k'th user unit energy spreading code of length N and $T_c = \frac{T_s}{N}$ is the chip duration. Let user k be the user of interest. Then,

$$\mathbf{y}(n) = \mathbf{s}_k b_k(n) + \sum_{k'=1, k' \neq k}^{K} \mathbf{s}_{k'} b_{k'}(n) + \mathbf{v}(n), \tag{4}$$

where for q different multipaths,
$\mathbf{y}(n) = [y(1)\ y(2)\ \cdots\ y(N+q-1)]^T$, $\mathbf{s}_k = \mathbf{C}_k \mathbf{g}_k$

$$\mathbf{C}_k = \begin{bmatrix} c_k(1) & & \mathbf{0} \\ \vdots & \ddots & c_k(1) \\ c_k(N) & & \vdots \\ \mathbf{0} & \ddots & c_k(N) \end{bmatrix}_{(N+q-1)\times q}, \quad \mathbf{g}_k = \begin{bmatrix} g_k(1) \\ g_k(2) \\ \vdots \\ g_k(q) \end{bmatrix}_{(q\times 1)}.$$

3. MINIMUM VARIANCE RECEIVERS

The idea of estimating the transmitted symbol $b_k(n)$ is to find a vector $\mathbf{f}$ such that $(N+q-1)$

$$\hat{b}_k(n) = \mathbf{f}^H \mathbf{y}(n). \tag{5}$$

For MV receivers [2, 3], it has been shown that the vector $\mathbf{f}$ can be found by minimizing the variance of the zero mean output symbols $\hat{b}_k(n)$, i.e.,

$$\zeta = E\{||\hat{b}_k(n)||^2\} = \mathbf{f}^H \mathbf{R}_y \mathbf{f}\ ,\ \mathbf{R}_y = E\{\mathbf{y}(n)\mathbf{y}^H(n)\}. \tag{6}$$

The minimization of ζ is subject to the constraint that the response of the user of interest is constant, i.e.

$$\mathbf{f}^H \mathbf{C}_k \mathbf{g}_k = 1. \tag{7}$$

Since the solution of Eqn. (7) includes a scaling factor and a phase ambiguity in $\mathbf{g}$, assuming that $\mathbf{g}^H\mathbf{g} = 1$, Eqn. (7) becomes

$$\mathbf{C}_k^H \mathbf{f} = \mathbf{g}_k. \tag{8}$$

3.1. Optimum Solution

Using Lagrange multipliers, it has been shown in [2] that for a given channel response $\mathbf{g}_k = \mathbf{g}$ which is unknown with Toeplitz spreading matrix $\mathbf{C}_k = \mathbf{C}$, the optimum solution to the vector $\mathbf{f}$ is

$$\mathbf{f}_{opt} = \mathbf{R}_y^{-1}\mathbf{C}(\mathbf{C}^H\mathbf{R}_y^{-1}\mathbf{C})^{-1}\mathbf{g}, \tag{9}$$

which leads to the minimum output variance,

$$\zeta_{opt} = \mathbf{f}_{opt}^H\mathbf{R}_y\mathbf{f}_{opt} = \mathbf{g}^H(\mathbf{C}^H\mathbf{R}_y^{-1}\mathbf{C})^{-1}\mathbf{g}. \tag{10}$$

4. EXISTING ALGORITHMS

4.1. Adaptive LMS Algorithms

In [3, 4, 5], three very similar adaptive LMS algorithms are proposed for the estimation of $\mathbf{f}$ and $\mathbf{g}$. The first LMS algorithm is based on the following cost function,

$$\zeta_1 = \mathbf{f}^H\mathbf{R}_y\mathbf{f} + \boldsymbol{\lambda}^H(\mathbf{C}^H\mathbf{f} - \mathbf{g}) + \boldsymbol{\lambda}(\mathbf{f}^H\mathbf{C} - \mathbf{g}^H) + \rho(\mathbf{g}^H\mathbf{g} - 1) \tag{11}$$

The idea of estimating $\mathbf{f}$ and $\mathbf{g}$ using Eqn. (11) is to minimize ζ_1 w.r.t $\mathbf{f}$ and maximize it w.r.t $\mathbf{g}$. First one initializes $\mathbf{f}$ and $\mathbf{g}$ with certain values, then when a new symbol arrives at $(n+1)$, their values are updated according to the LMS algorithm,

$$\mathbf{f}_{n+1} = \mathbf{f}_n - \mu_{\mathbf{f}}\nabla_{\zeta/\mathbf{f}} \tag{12}$$

$$\mathbf{g}_{n+1} = \mathbf{g}_n + \mu_{\mathbf{g}}\nabla_{\zeta/\mathbf{g}} \tag{13}$$

where $\nabla_{\zeta/\mathbf{x}}$ is the partial gradient of ζ w.r.t the vector $\mathbf{x}$, i.e. $\nabla_{\zeta/\mathbf{x}} = \frac{\partial\zeta}{\partial\mathbf{x}}$

4.2. Adaptive RLS Algorithm

In [3, 8] an RLS algorithm is also used. The channel parameters $\mathbf{g}$ is found as the eigenvector which corresponds to the minimum eigenvalue of the quadratic function $\mathbf{C}^H\mathbf{R}_y^{-1}\mathbf{C}$. The main problem with this method is the calculation of $\mathbf{R}_y^{-1}$ where, a Kalman RLS recursive algorithm is used. After initializing $\mathbf{R}_y^{-1}(n-1)$, the algorithm proceeds as follows,

$$\mathbf{k}(n) = \frac{\hat{\mathbf{R}}_y^{-1}(n-1)\mathbf{y}(n)}{v + \mathbf{y}^H(n)\hat{\mathbf{R}}_y^{-1}(n-1)\mathbf{y}(n)} \tag{14}$$

$$\hat{\mathbf{R}}_y^{-1}(n) = \frac{1}{v}\hat{\mathbf{R}}_y^{-1}(n-1)[\mathbf{I} - \mathbf{k}(n)\mathbf{y}^H(n)] \tag{15}$$

$$\mathbf{g}_{opt} = \arg\min_{||\mathbf{g}||=1} \mathbf{g}^H\mathbf{C}^H\mathbf{R}_y^{-1}(n)\mathbf{C}\mathbf{g} \tag{16}$$

It has been suggested that if q, the length of $\mathbf{g}$ is small enough then SVD can be applied such that $\mathbf{g}_{opt}$ is considered as the eigenvector which corresponds to the minimum eigenvalue of $\left[\mathbf{C}^H\mathbf{R}_y^{-1}(n)\mathbf{C}\right]_{(q\times q)}$

4.3. Subspace-Based Algorithm

In [4, 5, 6, 7], the optimum channel response $\mathbf{g}_{opt}$ are found by

$$\mathbf{g}_{opt} = \arg\min_{||\mathbf{g}||=1} \mathbf{g}^H\mathbf{C}^H\mathbf{R}_y^{-m}\mathbf{C}\mathbf{g}. \tag{17}$$

In this algorithm one can see that $\mathbf{R}_y^{-m}$ is used instead of $\mathbf{R}_y^{-1}$. First The covariance matrix $\mathbf{R}_y$ is decomposed by SVD as

$$\mathbf{R}_y = [\mathbf{U}_s\ \mathbf{U}_n]\begin{bmatrix} \boldsymbol{\Lambda}_s + \sigma_v^2\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \sigma_v^2\mathbf{I} \end{bmatrix}\begin{bmatrix} \mathbf{U}_s^H \\ \mathbf{U}_n^H \end{bmatrix}, \tag{18}$$

where $\boldsymbol{\Lambda}_s = \text{diag}\{\lambda_1^2, \ldots, \lambda_q^2\}$, $\mathbf{U}_s$ and $\mathbf{U}_n$ represent the signal and noise subspaces, respectively. $\mathbf{R}_y^{-m}$ is determined using the noise subspace as

$$\sigma_v^{2m}\mathbf{R}_y^{-m} = \mathbf{U}_n\mathbf{U}_n^H + \mathbf{U}_s\ \text{diag}\left\{\left(\frac{\sigma_v^2}{\lambda_i + \sigma_v^2}\right)^m\right\}\mathbf{U}_s^H. \tag{19}$$

Since $\left(\frac{\sigma_v^2}{\lambda_i+\sigma_v^2}\right)^m < 1$ then

$$\lim_{m\to\infty} \sigma_v^{2m}\mathbf{R}_y^{-m} = \mathbf{U}_n\mathbf{U}_n^H. \tag{20}$$

Once $\mathbf{U}_n$ is found, $\mathbf{g}_{opt}$ can be estimated as the eigenvector which corresponds to the minimum eigenvalue of the matrix $\mathbf{C}^H\mathbf{U}_n\mathbf{U}_n^H\mathbf{C}$.

5. NEW PROPOSED ALGORITHMS

In CDMA systems, since the spreading code of each user, $\mathbf{c}_k$, is known to both, the transmitter and the receiver. Then in order to reduce the system complexity and enhance the accuracy of the estimated channel parameters, we consider the RAKE receiver shown in Fig. 1, where the vector $\mathbf{f}$ in Eqn. (5) is replaced by,

$$f(n) = c(n) * h(n) \qquad \text{or} \qquad \mathbf{f} = \mathbf{C}\mathbf{h}, \tag{21}$$

where $\mathbf{C}$ is the same Toeplitz spreading matrix as before and the vector $\mathbf{h}$ contains the RAKE finger taps for user k. Since $\mathbf{C}$ is known,

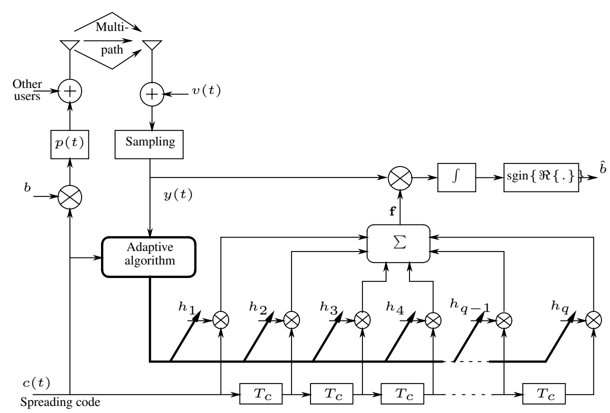

Fig. 1. A Blind Adaptive Based RAKE Receiver

then one can see that instead of estimating an $(N+q-1)$ parameters, $\mathbf{f}$ as in [3], we only need to estimate a q parameters, $\mathbf{h}$. Clearly this will reduce the system complexity and give better estimates. With this definition Eqn. (7) becomes,

$$\mathbf{h}^H\mathbf{C}^H\mathbf{C}\mathbf{g} = 1\ ,\ \ \mathbf{g}^H\mathbf{g} = 1 \tag{22}$$

The interesting thing about Eqn. (22) is that the term $\mathbf{C}^H\mathbf{C} \approx \mathbf{I}_{q\times q}$. Under this assumption, Eqn. (22) then becomes $\mathbf{h}^H\mathbf{g} \approx 1$ with $\mathbf{g}^H\mathbf{g} = 1$ it leads to $\mathbf{h} \approx \mathbf{g}$. Since $\mathbf{g}^H\mathbf{g} = 1$ we have $\mathbf{h}^H\mathbf{h} = 1$.
Due to this assumption one can estimate the channel response using a more efficient cost function that includes the previous constraints,

$$\zeta = \mathbf{h}^H(\mathbf{C}^H\mathbf{R}_y\mathbf{C})\mathbf{h} \tag{23}$$

and the optimum channel response $\mathbf{h}_{opt}$ which is $\mathbf{g}_{opt}$ can then be found from

$$\mathbf{h}_{opt} = \arg\max_{||\mathbf{h}||=1} \mathbf{h}^H(\mathbf{C}^H\mathbf{R}_y\mathbf{C})\mathbf{h}. \tag{24}$$

In [3], channel estimation requires minimizing the cost function w.r.t $\mathbf{f}$ and maximize w.r.t $\mathbf{g}$. In this approach it is less complex, it does

not require estimation of the two vectors, it requires only one vector, $\mathbf{h}_{opt}$. Also in Eqn. (24), one can see that finding $\mathbf{h}_{opt}$ does not require the estimation of $\mathbf{R}_y^{-1}$ or $\mathbf{R}_y^{-m}$ compared to the solution proposed by [3, 4, 5, 6, 7].
Estimation of $\mathbf{h}_{opt}$ in this work will be determined by,

1. A fast Adaptive LMS Algorithm
2. Low Cost SVD Algorithm
3. The Maximum eigenvalue Power Based Algorithm

The three methods are given in Table 1 and summarized as follows,

5.1. Proposed Adaptive LMS Algorithm

In this LMS algorithm the estimation of the optimum channel response, $\mathbf{h}_{opt}$ is less in complexity compared to the LMS algorithm in [3]. According to Eqn. (24), first we define the quadratic cost function,

$$\zeta = \mathbf{h}^H(\mathbf{C}^H\mathbf{R}_y\mathbf{C})\mathbf{h}. \tag{25}$$

Then $\mathbf{h}_{opt}$ can be found adaptively by maximizing ζ w.r.t $\mathbf{h}$ for each incoming data $\mathbf{y}(n)$, i.e. we initialize $\mathbf{h}_n$ and $\mathbf{R}_y(n-1)$ with certain values, then we update them according to the LMS algorithm

$$\mathbf{h}_{n+1} = \mathbf{h}_n + \mu_{\mathbf{h}}\nabla_{\zeta/\mathbf{h}} \tag{26}$$

where,

$$\nabla_{\zeta/\mathbf{h}} = 2(\mathbf{C}^H\mathbf{R}_y(n)\mathbf{C})\mathbf{h}_n, \tag{27}$$

and $\mathbf{R}_y(n)$ is estimated recursively by

$$\mathbf{R}_y(n) = \beta\mathbf{R}_y(n-1) + \mathbf{y}(n)\mathbf{y}^H(n),\ 0 < \beta < 1 \tag{28}$$

This is followed by normalizing the vector $\mathbf{h}_{n+1}$ by its energy. The algorithm is given in Table 1 part 2-a).

5.2. Proposed SVD Algorithm

In this method we also used SVD to estimate the channel response, but with less calculations than the methods given in [3, 4], as the proposed method does not involve the estimation of $\mathbf{R}_y^{-1}$ using a Kalman filter or $\mathbf{R}_y^{-m}$ in [5, 6, 7] that use two successive SVD operations, the first is to find the noise subspace of $\mathbf{R}_y$, $\mathbf{U}_n$ and the second is to find the eigenvalues and eigenvectors of $\mathbf{C}^H\mathbf{U}_n\mathbf{U}_n^H\mathbf{C}$.
In this method, we use the same assumption as before that $\mathbf{C}^H\mathbf{C} \approx \mathbf{I}_{q\times q}$ and the solution for $\mathbf{h}_{opt}$ in Eqn. (24). Using SVD the channel response at the n'th received symbol, $\mathbf{h}_n$ is found as

$$\mathbf{h}_n = \text{eigenvector}\{\mathbf{C}^H\mathbf{R}_y(n)\mathbf{C}\}, \tag{29}$$

which corresponds to the maximum eigenvalue, $\mathbf{R}_y(n)$ is the same as in Eqn. (28) . The algorithm is given in Table 1 part 2-b).

5.3. Proposed Power Algorithm

The eigenvectors of a matrix $\mathbf{A}_{L\times L}$ are the set of L-orthornomal vectors, $\mathbf{x} = \mathbf{u}_i$, $i = 1, \cdots, L$, $\mathbf{u}^H\mathbf{u} = 1$ that are the non-trivial solutions of

$$\mathbf{A}\mathbf{x} = \lambda\mathbf{x} \qquad i.e. \qquad \mathbf{A}\mathbf{u}_i = \lambda_i\mathbf{u}_i \tag{30}$$

Any vector $\mathbf{x}$ can be expressed in terms of these sets as

$$\mathbf{x} = a_1\mathbf{u}_1 + a_1\mathbf{u}_1 + \cdots + a_L\mathbf{u}_L \tag{31}$$

If we assume there is a unique (only one) largest eigenvector say $\lambda_1 > \lambda_1 > \ldots > \lambda_L$ then, we can find λ_1 and its corresponding eigenvector $\mathbf{u}_1$ of the matrix $\mathbf{A}$ by the Power method [10], as described by the following iterative equations,

$$\begin{aligned} \lambda_1 &= \lim_{n\to\infty} \frac{||\mathbf{x}_{n+1}||}{||\mathbf{x}_n||} \\ \mathbf{u}_1 &= \lim_{n\to\infty} \frac{\mathbf{x}_{n+1}}{||\mathbf{x}_{n+1}||} \end{aligned} \tag{32}$$

where

$$\mathbf{x}_{n+1} = \mathbf{A}\mathbf{x}_n \tag{33}$$

This is valid for an initial $\mathbf{x}_n$ which is not orthogonal to the matrix $\mathbf{A}$. Since $\mathbf{h}_{opt}$ is the eigenvector which corresponds to the maximum eigenvalue of $\left(\mathbf{C}^H\mathbf{R}_y\mathbf{C}\right)$, then the previous Power method can be used to estimate the channel response, see Table 1 part 2-c).

Table 1. Proposed LMS, SVD and Power Algorithms

Step 1: Initialize $\mathbf{h}_n$ and choose a step size $0 < \mu_h < 1$
Step 2: Let $\mathbf{R}_y(n-1) = 0$ and choose the correlation matrix updating factor $0.8 \leq \beta < 1$
Step 3: For $n = 1, 2, \ldots$

1. $\mathbf{R}_y(n) = \beta\mathbf{R}_y(n-1) + \mathbf{y}(n)\mathbf{y}^H(n)$
2. For:
   2-a) LMS Algorithm: $\nabla_{\zeta/\mathbf{h}} = 2\left(\mathbf{C}^H\mathbf{R}_y(n)\mathbf{C}\right)\mathbf{h}_n$ then Update $\mathbf{h}_{n+1} = \mathbf{h}_n + \mu_{\mathbf{h}}\nabla_{\zeta/\mathbf{h}}$
   2-b) SVD Algorithm: Find $\mathbf{h}_{n+1} = \text{eigenvector}\{\mathbf{C}^H\mathbf{R}_y(n)\mathbf{C}\}$ which corresponds to the maximum eigenvalue
   2-c) Power Algorithm: Find $\mathbf{h}_{n+1} = \mathbf{C}^H\mathbf{R}_y(n)\mathbf{C}\mathbf{h}_n$
3. Normalize $\mathbf{h}_{n+1}$ so that $\mathbf{h}_{n+1} = \frac{\mathbf{h}_{n+1}}{||\mathbf{h}_{n+1}||}$ is a unity norm vector
4. Continue until convergence occurs

5.4. Remarks on the Case Where $\mathbf{C}^H\mathbf{C} \neq \mathbf{I}$

In some spreading code systems $\mathbf{C}^H\mathbf{C} \neq \mathbf{I}$ as before. By returning back to the constraint in Eqn. (22) where, $\mathbf{h}^H\mathbf{C}^H\mathbf{C}\mathbf{g} = 1$, the resultant estimated parameters of the previous algorithms will no longer be $\mathbf{h}$. These parameters, $\mathbf{h}_b$, will then be biased by the term $\mathbf{C}^H\mathbf{C}$. To validate Eqn. (22) and the minimization constraint that $\mathbf{h}_b^H\mathbf{h}_b = 1$, the unbiased parameters $\mathbf{h}_{ub}$ that represent an estimate of $\mathbf{g}$ are then determined by

$$\mathbf{h}_{ub} = \left(\mathbf{C}^H\mathbf{C}\right)^{-1}\mathbf{h}_b \tag{34}$$

Clearly we can still use the previous algorithms to estimate $\mathbf{h}_b$. Then, we calculate the term $\mathbf{C}^H\mathbf{C}$ once and fix $\mathbf{h}_b$ at each iteration.

6. SIMULATION RESULTS

In the following examples, we compare the performance of the new proposed algorithms with some existing algorithms. For comparisons, the optimum solution $\mathbf{g}_o$ and $\mathbf{h}_o$ are estimated using the SVD technique calculated using an estimate of the correlation matrix $\mathbf{R}_y$ over the full signal length. $\mathbf{g}$ and $\mathbf{h}$ in each case are normalized by their first parameter. In all examples the MLBS spreading codes are used except when mentioned, MC represents the number of Monte Carlo runs.
Example 1:
In this example we compare the MSE between a channel which is estimated using the adaptive RLS algorithm in Sec. 4.1 as the one which has the best performance in [3, 4, 6] and the proposed LMS, SVD and Power algorithms given in Sec. 5. The example input settings are given in Table 2 and the results are shown in Fig. 2.

Table 2. Example 1: Input Settings

Method	K	N	q	μ_f	μ_g	μ_h	SNR	MC
Ref. [3, 4]	16	32	4	0.0009	0.02	-	6 dB	50
Sec. 5	16	32	4	-	-	0.01	6 dB	50

Discussion of Example 1:
Simulation results in Fig. 2 show that the proposed new algorithms performed much better and converged faster than the proposed RLS algorithms in [3, 6] that use Kalman filters for the estimation of $\mathbf{R}_y^{-1}$ or $\mathbf{R}_y^{-m}$. Obviously the complexity of the proposed algorithms are also much cheaper.

Example 2:
In this example we show the performance of the SVD algorithm proposed in Sec. 5.2 using the assumption that $\mathbf{C}^H\mathbf{C} \approx \mathbf{I}$. The algorithm is applied to a system with $K = 16$ users, $N = 32$ chips, $q = 8$ paths. Results in Fig. 3 compare the MSE between:
- the estimated and the optimum channels, $\mathbf{h}$ and $\mathbf{g}_o$
- the estimated and the true channels, $\mathbf{h}$ and $\mathbf{g}$

Discussion of Example 2:
Using the assumption that the term $\mathbf{C}^H\mathbf{C} \approx \mathbf{I}$, simulation results in Fig. 3 show that the proposed adaptive SVD algorithm converges very fast to the optimum solution, also the MSE between the estimated channel, $\mathbf{h}$ and the true channel, $\mathbf{g}$ is small compared with MSE lower band in [3, 5, 7]. The algorithm is low complex since it does not require the estimation of $\mathbf{R}_y^{-1}$ or $\mathbf{R}_y^{-m}$ as in [3, 5, 7] or the noise subspace as in [4, 5, 6, 7].

Example 3:
In this example we use the proposed Power algorithm given in Sec. 5.3 to a system where $\mathbf{C}^H\mathbf{C} \neq \mathbf{I}$ as in Sec. 5.4. The algorithm is applied to a system with $K = 16$ users, $N = 32$ chips, $q = 8$ paths. The MSE is compared as shown in Fig. 4 between:
- the biased optimum and the biased estimated channels, $\mathbf{g}_o$ and $\mathbf{h}_b$
- the unbiased optimum and the unbiased estimated channels, $\left(\mathbf{C}^H\mathbf{C}\right)^{-1}\mathbf{g}_o$ and $\left(\mathbf{C}^H\mathbf{C}\right)^{-1}\mathbf{h}_b$
- the true and the biased estimated channels, $\mathbf{g}$ and $\mathbf{h}_b$
- the true and the unbiased estimated channels $\mathbf{g}$ and $\left(\mathbf{C}^H\mathbf{C}\right)^{-1}\mathbf{h}_b$

Discussion of Example 3:
Simulation results in Fig. 4 where $\mathbf{C}^H\mathbf{C} \neq \mathbf{I}$ show that the MSE between the biased and unbiased, optimum and estimated channels are very much the same and that support Eqn. (34), also the MSE between the true channel response, $\mathbf{g}$ and the biased estimated response, $\mathbf{h}_b$ is high. After fixing $\mathbf{h}_b$ using, $\mathbf{h}_{ub} = \left(\mathbf{C}^H\mathbf{C}\right)^{-1}\mathbf{h}_b$ using the method in Sec. 5.4, the MSE between $\mathbf{g}$ and $\mathbf{h}_{ub}$ become very small, i.e. the algorithm succeeded to estimate the channel response.

7. CONCLUSION

The problem of blind channel estimation in CDMA systems is considered. Using only the spreading code of the user of interest and the received data sequence, three low complexity algorithms are proposed to estimate and track the impulse response of the channel. While existing blind methods suffer from high computational complexity, due to large SVDs and matrix inversions, the proposed methods overcome both problems. The computation complexity of the new algorithms are compared to that of the existing approaches. Simulation results show that the proposed algorithms convergence rate is fast and the estimation accuracy is also desirable.

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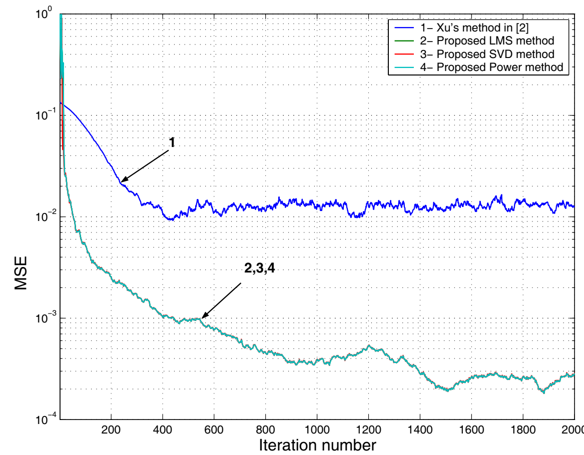

Fig. 2. MSE in case of 16 users

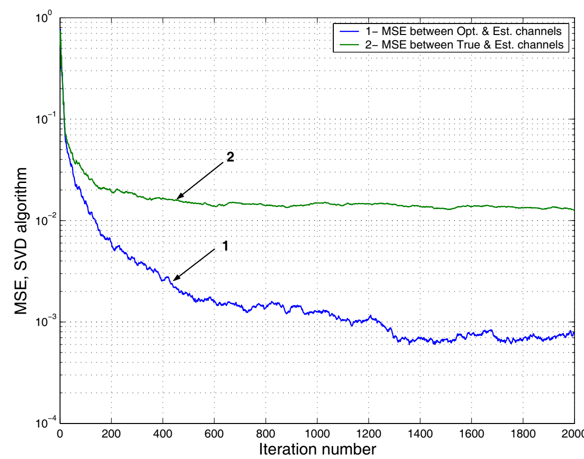

Fig. 3. MSE for the case $(\mathbf{C}^H\mathbf{C})^{-1} \approx \mathbf{I}$, 16 users

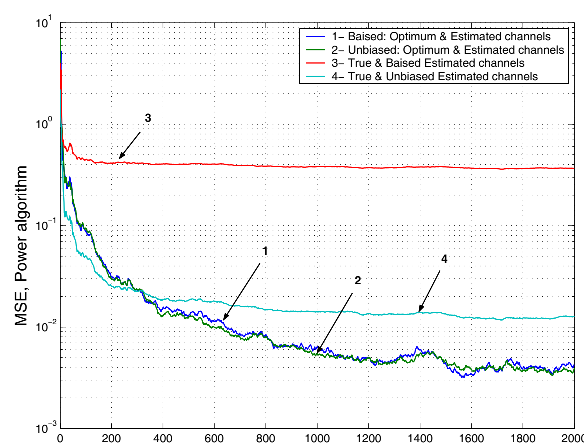

Fig. 4. EX3: MSE for the case $(\mathbf{C}^H\mathbf{C})^{-1} \neq \mathbf{I}$, 16 users