EFFICIENT ADAPTIVE BLIND MAI SUPPRESSION IN DS/CDMA BY EMBEDDED CONSTRAINT PARALLEL PROJECTION TECHNIQUES

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ABSTRACT

This paper presents two novel blind set-theoretic adaptive filtering algorithms for Multiple Access Interference (MAI) suppression in DS/CDMA systems. We naturally formulate the problem of MAI suppression as minimizing asymptotically a sequence of cost functions under some linear constraint defined by desired user's signature. The proposed algorithms embed the constraint in the direction of adaptation, and thus the adaptive filter moves toward the optimal filter without stepping away from the constraint set. In addition, using parallel processors, the proposed algorithms attain good performance behavior with low computational complexity. Geometric interpretation clarifies an advantage of the proposed methods over some conventional methods. Simulation results demonstrate that the proposed algorithms achieve much faster convergence than the conventional methods with a moderate number of concurrent processors.

1. INTRODUCTION

The aim of this paper is to develop a computationally efficient algorithm with high speed of convergence in DS/CDMA systems of high throughput. In order to realize high throughput systems, blind methods for Multiple Access Interference (MAI) suppression, which do not require a training sequence, have been in great demand [1–7]. A blind adaptive multiuser detection method was proposed in [2] and a simple set-theoretic blind method called *Space Alternating Generalized Projection (SAGP)* was proposed in [4]. The SAGP shows better performance in the steady state at the cost of poorer convergence rate than the method in [2].

The *Constrained Normalized Least Mean Square (CNLMS) algorithm* was proposed in [8], which embeds a constraint, defined by desired user's signature, in the direction of adaptation. Hence, the algorithm shows faster convergence than the projected NLMS algorithm (see [9] and references therein). Unfortunately, the CNLMS still does not show sufficient speed of convergence because it takes just one datum into account at each iteration. On the other hand, a novel blind MAI suppression method, which we call *Blind Projected Parallel Projection (BPPP) algorithm*, has been recently established [7]. It is reported that the BPPP exhibits better initial convergence behavior than some blind and non-blind methods, while keeping good performance in the steady state. Adaptation of a filter in the BPPP is constructed by two steps at each iteration (cf. Remark 1): (i) shift the filter in a steepest decent direction of a sequence of cost functions and (ii) enforce it in the constraint set.

This paper presents two embedded constraint algorithms, derived from a set-theoretic adaptive filtering scheme named *Adaptive Projected Subgradient Method (APSM)* [10–13], for blind MAI suppression. The proposed algorithms generalize the idea of the CNLMS with the object of taking into account more than one datum with several parallel processors at each iteration. The first stage of the algorithms estimates the amplitude of the transmitted signal and the transmitted bits. The second stage adapts a filter in order to minimize a new sequence of cost functions, defined by using some *stochastic property sets* [see (7) in Sec. 3], for robustness, based on the estimates. The adaptation in the second stage is constructed by one step, since the constraint is embedded in the direction of adaptation due to the new sequence of cost functions, which implies that the adaptive filter does not step away from the constraint set. Geometric interpretation clarifies an advantage of the proposed algorithms over the CNLMS, the SAGP and the BPPP algorithms (see Remark 1). Simulation results exemplify significant improvement expected by the geometric interpretation.

2. PRELIMINARIES

A. System Model

A binary phase-shift keying (BPSK) short-code DS/CDMA system is depicted below. The received data process $(\boldsymbol{r}[i])_{i\in\mathbb{N}} \subset \mathbb{R}^N$ (N: the length of the signature) is

$$\boldsymbol{r}[i] = A_1 b_1[i]\boldsymbol{s}_1 + \sum_{l=2}^{L} A_l \bar{b}_l[i]\bar{\boldsymbol{s}}_l + \boldsymbol{n}[i],\ \forall i \in \mathbb{N}, \tag{1}$$

where

$A_1 \in \mathbb{R}$: amplitude of the 1-st (desired) user
$b_1[i] \in \{-1, 1\}$: i-th transmitted bit of the desired user
$\boldsymbol{s}_1 \in \{-1, 1\}^N$: signature of the desired user
$\boldsymbol{n}[i] \in \mathbb{R}^N$: i-th noise vector.

Moreover, A_l ($2 \le l \le L$) is the amplitude of the l-th interferer, $\bar{b}_l[i]$ and $\bar{\boldsymbol{s}}_l$ are respectively the i-th interfering symbol bits and interfering vectors generated by interfering users' parameters such as associated data bits and signature. In the case of K users, $L-1$, the number of interferers, can range from $K-1$ to $2(K-1)$, due to relative delays of the $K-1$ interfering users [1].

The goal of this paper is to efficiently suppress MAI, $\sum_{l=2}^{L} A_l \bar{b}_l[i]\bar{\boldsymbol{s}}_l$ in (1), with a linear filter $\boldsymbol{h} \in \mathbb{R}^N$ without amplifying the noise $\boldsymbol{n}[i]$.

B. Adaptive Projected Subgradient Method [10, 11, 13]

Let $C \subset \mathbb{R}^N$ be a nonempty closed convex set[1]. Then, the *projection operator* $P_C : \mathbb{R}^N \to C$ maps a vector $\boldsymbol{x} \in \mathbb{R}^N$ to the unique vector $P_C(\boldsymbol{x}) \in C$ s.t. $\mathrm{d}(\boldsymbol{x}, C) := \|\boldsymbol{x} - P_C(\boldsymbol{x})\| = \min_{\boldsymbol{y}\in C} \|\boldsymbol{x}-\boldsymbol{y}\|$, where $\|\boldsymbol{x}\| := \langle \boldsymbol{x}, \boldsymbol{x}\rangle^{1/2}$, $\forall \boldsymbol{x} \in \mathbb{R}^N$ ($\langle \boldsymbol{x}, \boldsymbol{y}\rangle := \boldsymbol{x}^T\boldsymbol{y}$, $\forall \boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^N$, and the superscript T stands for transposition).

[1] A set $K \subset \mathbb{R}^N$ is *convex* provided that $\forall \boldsymbol{x}, \boldsymbol{y} \in K$, $\forall \nu \in (0, 1)$, $\nu\boldsymbol{x} + (1-\nu)\boldsymbol{y} \in K$.

Also let $\Theta_n : \mathbb{R}^N \to [0, \infty)$ be a continuous *convex* function[2] and $\partial\Theta(\boldsymbol{y})$ the *subdifferential*[3] of Θ at $\boldsymbol{y}$. Then, the following scheme asymptotically minimizes $(\Theta_n)_{n\in\mathbb{N}}$ over C.

Scheme 1 (Adaptive Projected Subgradient Method (APSM) [10, 11, 13]) *Generate a sequence* $(\boldsymbol{h}_n)_{n\in\mathbb{N}}$ *by*

$$\boldsymbol{h}_{n+1} := \begin{cases} P_C\left(\boldsymbol{h}_n - \lambda_n \frac{\Theta_n(\boldsymbol{h}_n)}{\|\Theta'_n(\boldsymbol{h}_n)\|^2}\Theta'_n(\boldsymbol{h}_n)\right), \\ \quad \text{if } \Theta'_n(\boldsymbol{h}_n) \neq \boldsymbol{0}, \\ \boldsymbol{h}_n, \quad \text{otherwise,} \end{cases} \tag{2}$$

where $\boldsymbol{h}_0 \in \mathbb{R}^N$, $\Theta'_n(\boldsymbol{h}_n) \in \partial\Theta_n(\boldsymbol{h}_n)$ *and* $\lambda_n \in [0, 2]$ *is the relaxation parameter.*

Basic properties of Scheme 1 are given in APPENDIX.

3. PROPOSED BLIND EMBEDDED CONSTRAINT ADAPTIVE ALGORITHMS

This section provides two set-theoretic algorithms for a blind adaptive receiver. All data that can be utilized for adaptation are assumed to be the sequence of received vectors $(\boldsymbol{r}[i])_{i\in\mathbb{N}}$ and desired user's signature $\boldsymbol{s}_1$. A minimum mean square error (MMSE) filter is given as follows [5]:

$$\boldsymbol{h}^* \in \operatorname*{argmin}_{\boldsymbol{h}\in C_s} E\left\{(\langle \boldsymbol{h}, \boldsymbol{r}[i]\rangle - A_1 b_1[i])^2\right\},$$

where $E\{\cdot\}$ denotes the expectation and

$$C_s := \{\boldsymbol{h} \in \mathbb{R}^N : \langle \boldsymbol{h}, \boldsymbol{s}_1\rangle = 1\} \tag{3}$$

is the constraint set. Since A_1 and $b_1[i]$ are not available, we use the following estimates [4]:

$$\widehat{A}_{1,n+1} = \widehat{A}_{1,n} + \gamma\left(|\langle \boldsymbol{h}_n, \boldsymbol{r}[n]\rangle| - \widehat{A}_{1,n}\right), \tag{4}$$

$$\widehat{b}_{1,n}[i] = \operatorname{sgn}\langle \boldsymbol{h}_n, \boldsymbol{r}[i]\rangle, \tag{5}$$

where $\widehat{A}_{1,0} = 0$ and $\gamma \in (0, 1]$. With these estimates, the problem is reformulated as finding a point in

$$\operatorname*{argmin}_{\boldsymbol{h}\in C_s} E\left\{\left(\langle \boldsymbol{h}, \boldsymbol{r}[i]\rangle - \widehat{A}_{1,n+1}\widehat{b}_{1,n}[i]\right)^2\right\}. \tag{6}$$

Here, the constraint set C_s avoids the *self-nulling*[4] (i.e., cancelling desired user's signal). Instead of the expectation in (6), we newly introduce the following stochastic property sets (cf. [14]):

$$C_\rho^{(n)}[i] := \left\{\boldsymbol{h} \in \mathbb{R}^N : \left(\langle \boldsymbol{h}, \boldsymbol{r}[i]\rangle - \widehat{A}_{1,n+1}\widehat{b}_{1,n}[i]\right)^2 \leq \rho\right\}, \tag{7}$$

$\forall n \in \mathbb{N}$, $\forall i \in \mathcal{I}_n := \{n, n-1, \cdots, n-q+1\}$, where $\rho \geq 0$ is a parameter to determine the reliability[5] of the set to contain $\boldsymbol{h}^*$ (NOTE: ρ can be time-varying). Intuitively, increase of ρ inflates the set $C_\rho^{(n)}[i]$, and thus we call ρ *the inflation parameter*.

[2]A function $\Theta : \mathbb{R}^N \to \mathbb{R}$ is said to be *convex* if $\Theta(\nu\boldsymbol{x} + (1-\nu)\boldsymbol{y}) \leq \nu\Theta(\boldsymbol{x}) + (1-\nu)\Theta(\boldsymbol{y})$, $\forall \boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^N$ and $\forall \nu \in (0, 1)$.

[3]The *subdifferential* of Θ at $\boldsymbol{y}$ is the set of all the *subgradients* of Θ at $\boldsymbol{y}$; $\partial\Theta(\boldsymbol{y}) := \{\boldsymbol{a} \in \mathbb{R}^N : \langle \boldsymbol{x} - \boldsymbol{y}, \boldsymbol{a}\rangle + \Theta(\boldsymbol{y}) \leq \Theta(\boldsymbol{x}), \forall \boldsymbol{x} \in \mathbb{R}^N\}$.

[4]In the case that the amplitude of some interferer is greater than that of a desired user, the filter may track not the desired user but the interferer. In this case, desired user's signal is suppressed. The set C_s can avoid such a situation.

[5]A strategic way to design ρ is currently under investigation.

Since C_s is completely reliable to contain $\boldsymbol{h}^*$, our strategy is to use C_s as a hard (absolute) constraint set and $\{C_\rho^{(n)}[i]\}_{i\in\mathcal{I}_n}$ as a collection of sets to which the distance should be reduced.

Now, let us derive the proposed algorithms from Scheme 1 with the sets defined above. Given $q \in \mathbb{N}\setminus\{0\}$, let $\{\omega_\iota^{(n)}\}_{\iota\in\mathcal{I}_n} \subset (0, 1]$ satisfying $\sum_{\iota\in\mathcal{I}_n}\omega_\iota^{(n)} = 1$, $\forall n \in \mathbb{N}$, be the weights for parallel projection. Application of $C = \mathbb{R}^N$ and

$$\Theta_n(\boldsymbol{h}) := \begin{cases} \displaystyle\sum_{\iota\in\mathcal{I}_n} \frac{\omega_\iota^{(n)}}{L_n^{(1)}} \mathrm{d}(\boldsymbol{h}_n, C_\rho^{(n)}[\iota] \cap C_s)\mathrm{d}(\boldsymbol{h}, C_\rho^{(n)}[\iota] \cap C_s), \\ \text{if} \quad L_n^{(1)} := \sum_{\iota\in\mathcal{I}_n}\omega_\iota^{(n)}\mathrm{d}(\boldsymbol{h}_n, C_\rho^{(n)}[\iota] \cap C_s) \neq 0, \\ 0, \quad \text{otherwise,} \end{cases}$$

to Scheme 1 yields the following algorithm.

Algorithm 1 (Blind Parallel Constrained Projection Algorithm)

Requirements: *the control sequence* $\mathcal{I}_n$, *the weights* $\omega_\iota^{(n)} \geq 0$ *s.t.* $\sum_{\iota\in\mathcal{I}_n}\omega_\iota^{(n)} = 1$, *the signature* $\boldsymbol{s}_1$, *the projection matrix* $\boldsymbol{Q} := I - \boldsymbol{s}_1\boldsymbol{s}_1^T$ *(I: the identity matrix), the inflation parameter* $\rho \geq 0$, *the step size* $\lambda_n \in [0, 2]$ *and the forgetting factor* $\gamma \in [0, 1)$.

Initialization: $A_{1,0} = 0$, $\boldsymbol{h}_0 = \boldsymbol{s}_1 \in C_s$

Algorithm:

1) Estimation of A_1 *and* $b_1[\iota]$

$$\widehat{A}_{1,n+1} = \widehat{A}_{1,n} + \gamma\left(|\langle \boldsymbol{h}_n, \boldsymbol{r}[n]\rangle| - \widehat{A}_{1,n}\right)$$

$$\widehat{b}_{1,n}[\iota] = \mathit{sgn}\langle \boldsymbol{h}_n, \boldsymbol{r}[\iota]\rangle, \quad \iota \in \mathcal{I}_n$$

2) Adaptation of filter

$$\boldsymbol{h}_{n+1} = \boldsymbol{h}_n + \lambda_n\mathcal{M}_n^{(1)}\left(\sum_{\iota\in\mathcal{I}_n}\omega_\iota^{(n)}P_{C_\rho^{(n)}[\iota]\cap C_s}(\boldsymbol{h}_n) - \boldsymbol{h}_n\right), \tag{8}$$

where

$$P_{C_\rho^{(n)}[\iota]\cap C_s}(\boldsymbol{h}) = \begin{cases} \boldsymbol{h} - \dfrac{\langle \boldsymbol{h}, \boldsymbol{r}[\iota]\rangle - \widehat{A}_{1,n+1}\widehat{b}_{1,n}[\iota] - \sqrt{\rho}}{\boldsymbol{r}[\iota]^T\boldsymbol{Q}\boldsymbol{r}[\iota]}\boldsymbol{Q}\boldsymbol{r}[\iota], \\ \quad \text{if } \langle \boldsymbol{h}, \boldsymbol{r}[\iota]\rangle - \widehat{A}_{1,n+1}\widehat{b}_{1,n}[\iota] > \sqrt{\rho}, \\ \boldsymbol{h} - \dfrac{\langle \boldsymbol{h}, \boldsymbol{r}[\iota]\rangle - \widehat{A}_{1,n+1}\widehat{b}_{1,n}[\iota] + \sqrt{\rho}}{\boldsymbol{r}[\iota]^T\boldsymbol{Q}\boldsymbol{r}[\iota]}\boldsymbol{Q}\boldsymbol{r}[\iota], \\ \quad \text{if } \langle \boldsymbol{h}, \boldsymbol{r}[\iota]\rangle - \widehat{A}_{1,n+1}\widehat{b}_{1,n}[\iota] < -\sqrt{\rho}, \\ \boldsymbol{h}, \quad \text{otherwise.} \end{cases} \tag{9}$$

$$\mathcal{M}_n^{(1)} := \begin{cases} \dfrac{\sum_{\iota\in\mathcal{I}_n}\omega_\iota^{(n)}\left\|P_{C_\rho^{(n)}[\iota]\cap C_s}(\boldsymbol{h}_n) - \boldsymbol{h}_n\right\|^2}{\left\|\sum_{\iota\in\mathcal{I}_n}\omega_\iota^{(n)}P_{C_\rho^{(n)}[\iota]\cap C_s}(\boldsymbol{h}_n) - \boldsymbol{h}_n\right\|^2}, \\ \quad \text{if } \boldsymbol{h}_n \notin \bigcap_{\iota\in\mathcal{I}_n}C_\rho^{(n)}[\iota] \cap C_s, \\ 1, \quad \text{otherwise.} \end{cases}$$

Note that (9) holds under the condition $\boldsymbol{h} \in C_s$. Fortunately, however, $\boldsymbol{h}_n \in C_s$, $\forall n \in \mathbb{N}$, because (i) $\boldsymbol{h}_0 \in C_s$ and (ii) $\boldsymbol{h}_n \in C_s \Rightarrow \boldsymbol{h}_{n+1} \in C_s$ from (3) and (8).

On the other hand, application of $C = \mathbb{R}^N$ and $\Theta_n(\boldsymbol{h}) := \Phi_n(P_{C_s}(\boldsymbol{h}))$, where

$$\Phi_n(\boldsymbol{h}) := \begin{cases} \displaystyle\sum_{\iota\in\mathcal{I}_n}\frac{\omega_\iota^{(n)}}{L_n^{(2)}}\mathrm{d}(\boldsymbol{h}_n, C_\rho^{(n)}[\iota])\mathrm{d}(\boldsymbol{h}, C_\rho^{(n)}[\iota]), \\ \quad \text{if} \quad L_n^{(2)} := \sum_{\iota\in\mathcal{I}_n}\omega_\iota^{(n)}\mathrm{d}(\boldsymbol{h}_n, C_\rho^{(n)}[\iota]) \neq 0, \\ 0, \quad \text{otherwise,} \end{cases}$$

to Scheme 1 yields the following algorithm.

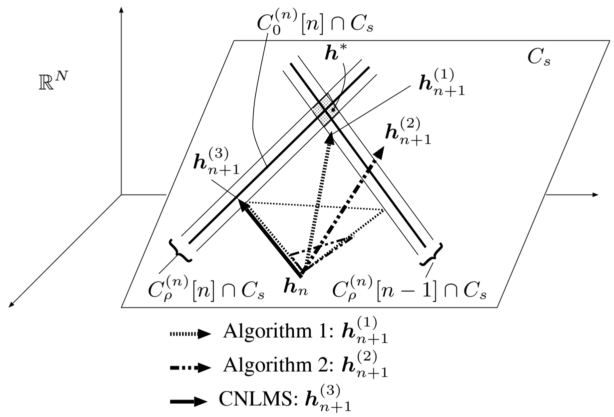

Fig. 1. Geometric interpretation of embedded constraint methods: the proposed algorithms and the CNLMS algorithm. The dotted area shows $C_\rho^{(n)}[n] \cap C_\rho^{(n)}[n-1] \cap C_s$.

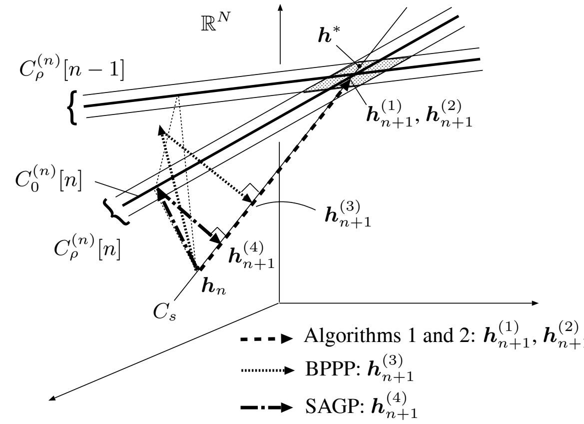

Fig. 2. Geometric interpretation of non-embedded constraint methods (the SAGP and the BPPP) and the proposed algorithms. The dotted area shows $C_\rho^{(n)}[n] \cap C_\rho^{(n)}[n-1]$.

Algorithm 2 (Blind Constrained Parallel Projection Algorithm)

Requirements & Initialization: *the same as Algorithm 1*

Algorithm:

1) Estimation of A_1 and $b_1[\iota]$: the same as Algorithm 1
2) Adaptation of filter

$$\boldsymbol{h}_{n+1} = \boldsymbol{h}_n + \lambda_n \mathcal{M}_n^{(2)} P_{\widetilde{C}_s}\left(\sum_{\iota \in \mathcal{I}_n} \omega_\iota^{(n)} P_{C_\rho^{(n)}[\iota]}(\boldsymbol{h}_n) - \boldsymbol{h}_n \right), \quad (10)$$

where $\widetilde{C}_s := \{\boldsymbol{h} \in \mathbb{R}^N : \langle \boldsymbol{h}, \boldsymbol{s}_1 \rangle = 0\}$ *and*

$$P_{\widetilde{C}_s}(\boldsymbol{h}) = \boldsymbol{Q}\boldsymbol{h},$$

$$P_{C_\rho^{(n)}[\iota]}(\boldsymbol{h}) = \begin{cases} \boldsymbol{h} - \dfrac{\langle \boldsymbol{h}, \boldsymbol{r}[\iota] \rangle - \widehat{A}_{1,n+1}\widehat{b}_{1,n}[\iota] - \sqrt{\rho}}{\|\boldsymbol{r}[\iota]\|^2}\boldsymbol{r}[\iota], \\ \qquad \text{if } \langle \boldsymbol{h}, \boldsymbol{r}[\iota] \rangle - \widehat{A}_{1,n+1}\widehat{b}_{1,n}[\iota] > \sqrt{\rho}, \\ \boldsymbol{h} - \dfrac{\langle \boldsymbol{h}, \boldsymbol{r}[\iota] \rangle - \widehat{A}_{1,n+1}\widehat{b}_{1,n}[\iota] + \sqrt{\rho}}{\|\boldsymbol{r}[\iota]\|^2}\boldsymbol{r}[\iota], \\ \qquad \text{if } \langle \boldsymbol{h}, \boldsymbol{r}[\iota] \rangle - \widehat{A}_{1,n+1}\widehat{b}_{1,n}[\iota] < -\sqrt{\rho}, \\ \boldsymbol{h}, \qquad \text{otherwise}, \end{cases}$$

$$\mathcal{M}_n^{(2)} := \begin{cases} \dfrac{\sum_{\iota \in \mathcal{I}_n} \omega_\iota^{(n)} \left\| P_{C_\rho^{(n)}[\iota]}(\boldsymbol{h}_n) - \boldsymbol{h}_n \right\|^2}{\left\| P_{\widetilde{C}_s}\left(\sum_{\iota \in \mathcal{I}_n} \omega_\iota^{(n)} P_{C_\rho^{(n)}[\iota]}(\boldsymbol{h}_n) - \boldsymbol{h}_n \right) \right\|^2}, \\ \qquad \text{if } \sum_{\iota \in \mathcal{I}_n} \omega_\iota^{(n)} P_{C_\rho^{(n)}[\iota]}(\boldsymbol{h}_n) - \boldsymbol{h}_n \notin \widetilde{C}_s^{\perp}, \\ 1, \qquad \text{otherwise}. \end{cases}$$

Algorithm 2 belongs to the family of *Embedded Constraint Adaptive Projected Subgradient Method (EC-APSM)* [10, 11, 13]. We remark that $\boldsymbol{Q}\boldsymbol{a} = \boldsymbol{a} - \boldsymbol{s}_1(\boldsymbol{s}_1^T\boldsymbol{a})$, $\forall \boldsymbol{a} \in \mathbb{R}^N$, requires $2N$ multiplications to compute. Moreover, $\forall \iota \in \mathcal{I}_n \setminus \{n\}$, "$\boldsymbol{r}[\iota]^T\boldsymbol{Q}\boldsymbol{r}[\iota]$ and $\boldsymbol{Q}\boldsymbol{r}[\iota]$ in Algorithm 1" and "$\|\boldsymbol{r}[\iota]\|^2$ in Algorithm 2" are computed at the previous iterations. By these remarks, we see that both Algorithms 1 and 2 require $(4q+5)N$ multiplications at each iteration. With q concurrent processors, furthermore, since each term in the summation in (8) [or (10)] can be computed in parallel, the number of multiplications imposed on each processor is reduced to no more than $9N$; i.e., the complexity order imposed on each processor is linear. This implies that the proposed algorithms are suitable for real-time implementation.

Figures 1 and 2 illustrate geometric interpretations of filter adaptation in Algorithms 1 and 2 compared with a simple embedded scheme (the CNLMS), and non-embedded schemes (the SAGP and the BPPP), respectively. The number of parallel processors q is set to 2, and the uniform weights, $\omega_\iota^{(n)} = 1/2$ $(\forall \iota = 1, 2)$, are employed. For the BPPP, the step size is set to $\mathcal{M}_n$. For the other methods, the step sizes are set to 1. The optimal filter $\boldsymbol{h}^*$ is assumed to belong to the intersection $C_\rho^{(n)}[n] \cap C_\rho^{(n)}[n-1] \cap C_s$. A remark on geometric comparisons is given below.

Remark 1 (Geometric Comparisons)
From Fig. 1, we see that the proposed algorithms generate closer points to the optimal filter $\boldsymbol{h}^*$ than the CNLMS due to its parallel structure; i.e., the proposed algorithms utilize multiple data simultaneously. As also seen in the figure, Algorithm 1 takes an averaged direction of exact projections onto $\{C_\rho^{(n)}[\iota] \cap C_s\}_{\iota \in \mathcal{I}_n}$, while Algorithm 2 takes an averaged direction of relaxed projections. The relaxation depends on the angle between $\boldsymbol{s}_1$ (the normal vector of the hyperplane C_s) and $\boldsymbol{r}[\iota]$ (that of the boundary hyperplane of $C_\rho^{(n)}[\iota]$).

From Fig. 2, we see that the BPPP generates a closer point to $\boldsymbol{h}^*$ than the SAGP, and the proposed algorithms generate even closer points than the BPPP and the SAGP, due to its embedded constraint structure. We also see that the SAGP and the BPPP are constructed by two steps; the second step is to enforce the filter in the constraint set. On the other hand, the CNLMS and the proposed algorithms adapt the filter along the constraint set, hence they are constructed by one step.

The following section shows some numerical comparison exemplifying the discussion in Remark 1.

4. SIMULATION RESULTS

Computer simulations examine the speed of convergence of the proposed blind algorithms (Algorithms 1 and 2) under signal-to-noise ratio (SNR) =15 dB (This situation is the same as or even worse than many other reports; e.g., [4, 6]). We compare the proposed methods with the following ones: the OPM-GP [2, 4], the SAGP [4], the blind CNLMS [8] and the BPPP [7]. The performance characteristic is shown by the ensemble-averaged output signal to interference-plus-noise ratio (SINR) [1, 2, 4]. The number of interfering users is $(K-1) = 5$, and all users have amplitude 10 times greater than the amplitude of the desired signal

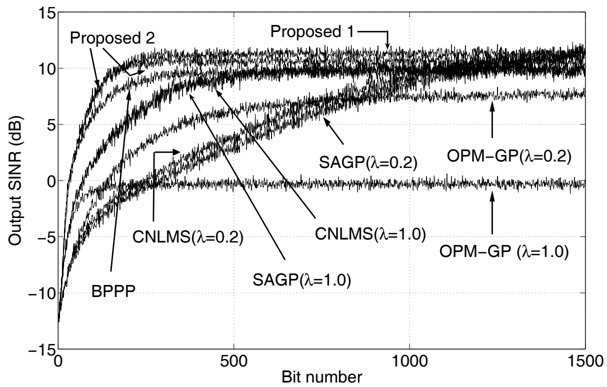

Fig. 3. Output SINR curves for SNR = 15 dB.

$A_1 = 1$. Signals are modulated by 31-length gold sequences, which are chosen randomly.

We initialize $\boldsymbol{h}_0 = \boldsymbol{s}_1$ for all algorithms, and $\boldsymbol{x}_0 = \boldsymbol{0}$ for OPM-GP. For the proposed algorithms and the BPPP, common parameters are employed: $\boldsymbol{r}[i] = \boldsymbol{r}[1]$ for $i \leq 1$, $q = 16$, $\rho = 0$, $\gamma = 0.01$ and $\omega_\iota^{(n)} = \frac{1}{q}$, $\forall \iota \in \mathcal{I}_n$. For the BPPP, step size is set to $0.2\mathcal{M}_n$. For Algorithms 1, 2, the step sizes are set to $\lambda_n = 0.2$. For the OPM-GP, the SAGP and the CNLMS, two different step sizes are used: 0.2 and 1.0.

As expected from Remark 1, we observe that the proposed algorithms outperform all other methods in terms of speed of convergence, while attaining good SINR in the steady state. Moreover, the additional computational complexity imposed by the proposed algorithms can be somehow alleviated by using processors in parallel (see under Algorithm 2).

5. CONCLUDING REMARKS

This paper has presented two blind adaptive filtering algorithms for MAI suppression in DS/CDMA systems. Since the proposed algorithms are based on parallel projection with embedded constraint, they achieve closer points to the optimal filter than some conventional methods at each iteration. Simulation results have shown that the proposed algorithms exhibit excellent performance. Finally, we remark that Algorithm 1 can be generalized by using an arbitrary linear variety[6] instead of C_s and an arbitrary collection of closed convex sets instead of $\{C_\rho^{(n)}[\iota]\}_{\iota \in \mathcal{I}_n}$.

APPENDIX

Scheme 1 has the following properties [10, 11, 13].

(a) (Monotonicity)

$$\left\| \boldsymbol{h}_{n+1} - \boldsymbol{h}^{*(n)} \right\| \leq \left\| \boldsymbol{h}_n - \boldsymbol{h}^{*(n)} \right\|,$$

$$\forall \boldsymbol{h}^{*(n)} \in \Omega_n := \{\boldsymbol{h} \in C : \Theta_n(\boldsymbol{h}) = \inf_{\boldsymbol{x} \in C} \Theta_n(\boldsymbol{x})\},\ \forall n \in \mathbb{N}.$$

(b) (Asymptotic minimization)

Suppose $(\Theta'_n(\boldsymbol{h}_n))_{n\in\mathbb{N}}$ is bounded and $\exists N_0$ s.t. (i) $\inf_{\boldsymbol{x}\in C} \Theta_n(\boldsymbol{x}) = 0, \forall n \geq N_0$ and (ii) $\Omega := \bigcap_{n \geq N_0} \Omega_n \neq \emptyset$. Then, we have

$$\lim_{n\to\infty} \Theta_n(\boldsymbol{h}_n) = 0.$$

[6]Given $\boldsymbol{v} \in \mathbb{R}^N$ and a closed subspace $M \subset \mathbb{R}^N$, the translation of M by $\boldsymbol{v}$ defines the *linear variety* $V := \boldsymbol{v} + M := \{\boldsymbol{v} + \boldsymbol{m} : \boldsymbol{m} \in M\}$. If $\dim(M^\perp) = 1$, V is called *hyperplane*, which can be expressed as $V = \{\boldsymbol{x} \in \mathbb{R}^N : \langle \boldsymbol{a}, \boldsymbol{x} \rangle = c\}$ for some $(\boldsymbol{0} \neq)\boldsymbol{a} \in \mathbb{R}^N$ and $c \in \mathbb{R}$.

Note that Θ'_n used to derive Algorithm 1 (or Algorithm 2) in Sec. 3 is automatically bounded [11].

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