# NOVEL ADAPTIVE METHODS FOR NARROWBAND INTERFERENCE CANCELLATION IN CDMA MULTI-USER DETECTION

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#### ABSTRACT

When overlaying spread spectrum (SS) transmission over a narrowband system, the performance will be significantly degraded by the narrowband signal. This paper presents an adaptive predictor approach for narrowband interference suppression in SS codedivision multiple-access (CDMA) systems. Two adaptive methods are derived, one uses linear predictor and the other employs nonlinear predictor. They both use a predictor to estimate the interference which is then subtracted from the received signal to improve performance. The proposed adaptive methods are blind in the sense that they do not require training data. The proposed methods not only provide faster convergence speed than the case without using a predictor, but also give better *BER* performance. At a *BER* of  $10^{-4}$  and *SINR* = -20dB, the proposed methods yield 1.4dB and 2.4dB improvement in *SNR*( $E_b/N_0$ ) respectively.

#### **1. INTRODUCTION**

Spread Spectrum Systems have many attractive characteristics, including multiple accessing capability, multi-path fading resistance, privacy and low probability of intercept transmission [1]. They also have the ability to operate well in the presence of narrowband interference (NBI). If the NBI is strong, however, the performance will still suffer a severe degradation. It has been shown in [2] that by employing some active interference cancellation scheme, the performance can be improved substantially. Hence there is a strong need to develop powerful NBI suppression schemes.

NBI suppression techniques can be classified into two categories [3]: estimator/subtracter approach and transform-domain methods. For estimator/subtracter approach, NBI is first predicted and then subtracted from the received signal. On the other hand, a transform-domain NBI suppression method notches out the NBI by using a mask in the frequency domain.

In this paper, we will concentrate on the adaptive estimator/ subtracter approach for NBI removal. Adaptive techniques for NBI suppression in spread spectrum system have been investigated over the past few years. Masreliez [4] proposed approximate conditional mean (ACM) filter, which is a modification of the Kalman filter that deals with non-Gaussian measurement noise. In [5], Vijayan and Poor applied the ACM filter to suppress NBI in spread spetrum system. Krishnamurthy and Logothetis [6] proposed an adaptive algorithm which combines a recursive hidden Markov model (HMM) estimator and Kalman filter. These systems do not make use of the CDMA code. They work well at the expense of computational complexity.

This paper proposes the computationally efficient and effective adaptive techniques for NBI suppression in DS-CDMA system: Adaptive Linear Predictor Algorithm and Adaptive Non-Linear Predictor Algorithm. Unlike previous methods, both techniques utilize the known CDMA code to improve performance. The first method obtains the interference reduced signal by subtracting the NBI estimated through an adaptive predictor, and then passes the interference reduced signal through an adaptive filter to produce the user bit estimates. The second technique improves interference estimation and hence achieves better user bit estimates by removing an estimate of the CDMA data from the received signal before it is utilized to obtain an interference estimate. The proposed algorithms are blind in the sense that no training data is required.

The paper is organized as follows. Section 2 provides the background and the current adaptive algorithm. The proposed adaptive linear and non-linear algorithms are derived and analyzed in Section 3. Section 4 presents simulation results for the proposed methods, and Section 5 is the conclusions.

## 2. PRELIMINARY

#### 2.1. Problem Formulation

In the presence of NBI and after sampling at chip rate, the received signal vector within a symbol period is:

$$\mathbf{r}(n) = \mathbf{y}(n) + \mathbf{i}(n) + \boldsymbol{\epsilon}(n) \tag{1}$$

where *n* is the symbol time index,  $\mathbf{y}(n)$  is the  $N \times 1$  CDMA signal vector,  $\mathbf{i}(n)$  is the  $N \times 1$  NBI vector, and  $\epsilon(n)$  is the  $N \times 1$  additive white gaussian noise(AWGN) vector of power  $\sigma_n^2$ , and *N* is the number of chips in a symbol period. To simplify our study, we assume no channel distortion in the received signal. The proposed technique can be modified to account for channel distortion.

If there are K users in the CDMA system and their bit values are  $b_k(n), k = 1, 2, ..., K$ , then  $\mathbf{y}(n)$  can be expressed as

$$\mathbf{y}(n) = \sum_{k=1}^{K} b_k(n) \mathbf{s}_k = \mathbf{Sb}(n)$$
(2)

where  $\mathbf{s}_k$  is the  $N \times 1$  spreading code vector of user k,  $\mathbf{S} = \begin{bmatrix} \mathbf{s}_1 & \dots & \mathbf{s}_K \end{bmatrix}$  is the  $N \times K$  CDMA code matrix and  $\mathbf{b}(n) = \begin{bmatrix} b_1(n) & b_2(n) & \dots & b_K(n) \end{bmatrix}^T$  is the user bit vector, and  $b_k(n)$  is equal to 1 or -1 with equal probability.

We adopt the Autoregressive modelling with order M for NBI from [2]:

$$i(m) = \sum_{j=1}^{M} a_j i(m-j) + e(m)$$
(3)

where *m* is the sampling time index that is related to symbol index *n* and chip index l, l = 0, 1, ..., N - 1 by m = nN + l, e(m) is the unpredictable component and has a power of  $\sigma_e^2$ .

Given the received data vector  $\mathbf{r}(n)$ , we wish to find  $b_k(n)$  adaptively. In the following a conventional non-predictor algorithm for NBI cancellation is presented.

## 2.2. Adaptive Multiuser Detection without Predictor

Fig. 1 is the block diagram to extract the bits for user k by using an adaptive filter  $\mathbf{h}_k$ . The user bit estimate is  $\hat{b}_k(n) = \mathbf{r}^T(n)\mathbf{h}_k$ .



**Fig. 1**. adaptive multi-user detection using a single filter  $h_k$ 

The error in estimating the user bit is,

$$e_k(n) = b_k(n) - \mathbf{h}_k^T \mathbf{r}(n). \tag{4}$$

We minimize  $E[e_k^2(n)]$  to determine  $\mathbf{h}_k$  through the LMS algorithm [7]:

$$\mathbf{h}_k(n+1) = \mathbf{h}_k(n) + 2\mu e_k(n)\mathbf{r}(n).$$
(5)

The above update requires the true user bit  $b_k(n)$ , which is not known. After using (4), (5) can be approximated by:

$$\mathbf{h}_{k}(n+1) = \mathbf{h}_{k}(n) + 2\mu(E[b_{k}(n)\mathbf{r}(n)] - \mathbf{h}_{k}^{T}(n)\mathbf{r}(n)\mathbf{r}(n))$$
(6)

where we have replaced  $b_k(n)\mathbf{r}(n)$  by  $E[b_k(n)\mathbf{r}(n)]$ .

The CDMA user bits are independent of each other, and are independent with the NBI and AWGN. Hence substituting (1) simplifies  $E[b_k(n)\mathbf{r}(n)]$  to  $E[b_k(n)\mathbf{y}(n)] = \mathbf{s}_k$ , the CDMA code for user k. As a result, (6) becomes

$$\mathbf{h}_{k}(n+1) = \mathbf{h}_{k}(n) + 2\mu(\mathbf{S}_{k} - \mathbf{h}_{k}^{T}(n)\mathbf{r}(n)\mathbf{r}(n)).$$
(7)

An advantage of (7) is that it does not require any training symbols.

The system in Fig. 1 only extracts the data bits for user k. It can be used to extract the data bits for other users by changing  $h_k$ .

### 3. PROPOSED METHODS

## 3.1. Adaptive Linear Predictor Algorithm

The amount of predictability of an NBI is much higher than that of the CDMA signal, and AWGN has a flat spectum and therefore unpredictable. We can first pass the received signal through an adaptive predictor to reduce the NBI, and after that, to obtain better estimate for CDMA user bits by the LMS adaptive algorithm.

Fig. 2 is the block diagram of the first proposed method. The method uses an adaptive predictor  $\hat{\mathbf{a}}$  to estimate the NBI by utilizing the received data, which is then subtracted from r(m) to form the interference reduced signal vector  $\tilde{\mathbf{r}}(n)$ .  $\tilde{\mathbf{r}}(n)$  will pass through a filter  $\mathbf{p}_k$  to produce the estimate of  $b_k(n)$ . Both  $\hat{\mathbf{a}}$  and  $\mathbf{p}_k$  are found adaptively.



Fig. 2. adaptive linear predictor method structure.

The interference estimate is  $\mathbf{u}^T(m)\hat{\mathbf{a}}$ , where  $\mathbf{u}(m) = [r(m-1), r(m-2), \dots, r(m-M)]^T$  and  $\hat{\mathbf{a}} = [\hat{a}_1, \hat{a}_2, \dots, \hat{a}_M]^T$ . After subtracting the NBI estimate, the resulted signal is:

$$\tilde{r}(m) = r(m) - \sum_{j=1}^{M} \hat{a}_j r(m-j) = r(m) - \mathbf{u}^T(m) \mathbf{\hat{a}}.$$
 (8)

We minimize  $E[\tilde{r}^2(m)]$  to determine  $\hat{a}$  by the LMS algorithm [7]:

$$\hat{\mathbf{a}}(m+1) = \hat{\mathbf{a}}(m) + 2\mu \tilde{r}(m)\mathbf{u}(m).$$
(9)

Assuming the true value of the user bit is available, the error in estimating the users bit is,

$$e_k(n) = b_k(n) - \mathbf{p}_k^T \tilde{\mathbf{r}}(n) \tag{10}$$

where  $\tilde{\mathbf{r}}(n) = [\tilde{r}(nN+N-1), \tilde{r}(nN+N-2), \dots, \tilde{r}(nN)]^T$ . Similarly, we can also minimize  $E[e_k^2(n)]$  to determine  $\mathbf{p}_k$  by LMS [7]:

$$\mathbf{p}_k(n+1) = \mathbf{p}_k(n) + 2\mu e_k(n)\tilde{\mathbf{r}}(n).$$
(11)

The true user bit is not available in practice. However, when putting (10) into (11), and replacing  $b_k(n)\tilde{\mathbf{r}}(n)$  by  $E[b_k(n)\tilde{\mathbf{r}}(n)]$ , the above update equation can be approximated by:

$$\mathbf{p}_{k}(n+1) = \mathbf{p}_{k}(n) + 2\mu(E[b_{k}(n)\mathbf{\tilde{r}}(n)] - \mathbf{p}_{k}^{T}(n)\mathbf{\tilde{r}}(n)\mathbf{\tilde{r}}(n)\mathbf{\tilde{r}}(n)).$$
(12)

From (1) and (8) and using the fact that  $E[\mathbf{b}(n)\mathbf{b}(n)^T] = \mathbf{I}$ and  $E[\mathbf{b}(n)\mathbf{b}(n-1)^T] = \mathbf{O}$ ,  $E[b_k(n)\mathbf{\tilde{r}}(n)]$  can be simplified to  $\mathbf{A}\begin{bmatrix}\mathbf{s}_k\\\mathbf{o}\end{bmatrix}$ , where

$$\mathbf{A} = \begin{bmatrix} 1 & -\hat{a}_1 & \dots & -\hat{a}_M & 0 & \dots & 0\\ 0 & 1 & -\hat{a}_1 & \dots & -\hat{a}_M & \dots & 0\\ \vdots & & & & & \\ 0 & \dots & 0 & 1 & -\hat{a}_1 & \dots & -\hat{a}_M \end{bmatrix} .$$
(13)

Hence (12) can be expressed by the known signal parameters as:

$$\mathbf{p}_{k}(n+1) = \mathbf{p}_{k}(n) + 2\mu(\mathbf{A}\begin{bmatrix}\mathbf{s}_{k}\\\mathbf{0}\end{bmatrix} - \mathbf{p}_{k}^{T}(n)\mathbf{\tilde{r}}(n)\mathbf{\tilde{r}}(n)) \quad (14)$$

where  $s_k$  is the spreading code vector for user k. Note that (14) does not require the true user bit to form the error for adaptation.

In the following, we compute the ideal solution for  $\mathbf{p}_k$ . For general purpose, the filters for different users are collected as a matrix  $\mathbf{P} = [\mathbf{p}_1 \dots \mathbf{p}_K]$ . The ideal solution for  $\mathbf{P}$  is  $\mathbf{Q}$ , which minimizes the MSE of the user bit estimate:  $J = E[(\mathbf{b}(n) - \mathbf{Q}\tilde{\mathbf{r}}(n))^T(\mathbf{b}(n) - \mathbf{Q}\tilde{\mathbf{r}}(n))]$ . After the minimization process,  $\mathbf{Q}$  is equal to:

$$\mathbf{Q} = [\mathbf{q}_1 \quad \dots \quad \mathbf{q}_K ] = E[\mathbf{b}(n)\tilde{\mathbf{r}}(n)^T]E[\tilde{\mathbf{r}}(n)\tilde{\mathbf{r}}(n)^T]^{-1}.$$
(15)

Upon using (1) and (8) and some algebraic manipulations,  $\mathbf{Q}$  can be expressed as:

$$\mathbf{Q} = \begin{bmatrix} \mathbf{S}^T & \mathbf{O} \end{bmatrix} \mathbf{A}^T \{ \mathbf{A} \begin{bmatrix} \mathbf{S}\mathbf{S}^T & \mathbf{O} \\ \mathbf{O} & \mathbf{S}_M \mathbf{S}_M^T \end{bmatrix} \mathbf{A}^T + \sigma_n^2 \mathbf{A} \mathbf{A}^T + \mathbf{A} \mathbf{R}_i \mathbf{A}^T \}^{-1}$$
(16)

where  $\mathbf{R}_i$  is the theoretical autocorrelation matrix of the interference and  $\mathbf{R}_i$  can be expressed in terms of the AR coefficients [8].

### 3.2. Adaptive Non-Linear Predictor Algorithm

In terms of the interference, the CDMA signal acts as noise. As a result, if we first subtract the CDMA signal from the received data before applying linear predictor to estimate the interference, the results would be even better. Fig. 3 is the block diagram of our second method. It estimates  $b_k(n)$  by using an adaptive nonlinear predictor. This method starts with the user bit solution from the linear predictor  $\tilde{\mathbf{b}}(n) = \mathbf{P}^T \tilde{\mathbf{r}}(n)$  to form the CDMA signal estimate:

$$\hat{\mathbf{y}}(n) = \mathbf{Sb}(n). \tag{17}$$





 $\hat{\mathbf{y}}(n)$  is subtracted from the received signal to remove most of the CDMA signal,

$$\hat{\mathbf{r}}(n) = \mathbf{r}(n) - \hat{\mathbf{y}}(n) = \mathbf{y}(n) - \hat{\mathbf{y}}(n) + \mathbf{i}(n) + \boldsymbol{\epsilon}(n).$$
(18)

It is then processed by an adaptive predictor  $\mathbf{g} = [g_1, g_2, \dots, g_M]^T$  to obtain NBI estimate. The interference removed signal is

$$\bar{r}(m) = r(m) - \mathbf{g}^T \hat{\mathbf{u}}(m) \tag{19}$$

where  $\hat{\mathbf{u}}(m) = [\hat{r}(m-1), \hat{r}(m-2), \dots, \hat{r}(m-M)]^T$ , and the sampling time index m is related to symbol index n and chip index l,  $l = 0, 1, \dots, N-1$  by m = nN + l. The interference removed signal vector  $\overline{\mathbf{r}}(n)$  will then go through a filter  $\mathbf{f}_k$  to produce the estimate of the user bits  $b_k(n)$ . Both  $\mathbf{g}$  and  $\mathbf{f}_k$  will be found adaptively.

We minimize  $E[(\hat{r}(m) - \mathbf{g}^T(m)\hat{\mathbf{u}}(m))^2]$  to determine  $\mathbf{g}$  through the LMS algorithm [7]:

$$\mathbf{g}(m+1) = \mathbf{g}(m) + 2\mu[\hat{r}(m) - \mathbf{g}^{T}(m)\hat{\mathbf{u}}(m)]\hat{\mathbf{u}}(m) . \quad (20)$$

Similarly, we can minimize  $E[e_k^2(n)]$  to determine  $\mathbf{f}_k$  by LMS [7]:

$$\mathbf{f}_k(n+1) = \mathbf{f}_k(n) + 2\mu e_k(n)\overline{\mathbf{r}}(n), \qquad (21)$$

where  $\mathbf{\bar{r}}(n) = [\bar{r}(nN+N-1), \bar{r}(nN+N-2), \dots, \bar{r}(nN)]^T$  and  $e_k(n) = b_k(n) - \mathbf{f}_k^T \mathbf{\bar{r}}(n)$ . Substituting it into (21) and replacing  $b_k(n)\mathbf{\bar{r}}(n)$  by  $E[b_k(n)\mathbf{\bar{r}}(n)]$ , (21) can be approximated by,

$$\mathbf{f}_k(n+1) = \mathbf{f}_k(n) + 2\mu(E[b_k(n)\overline{\mathbf{r}}(n)] - \mathbf{f}_k^T(n)\overline{\mathbf{r}}(n)\overline{\mathbf{r}}(n)\overline{\mathbf{r}}(n)).$$
(22)

Upon using (18), (19), and the fact that  $E[\mathbf{b}(n)\mathbf{b}(n)^T] = \mathbf{I}$  and  $E[\mathbf{b}(n)\mathbf{b}(n-1)^T] = \mathbf{O}$ , we can simplify  $E[b_k(n)\mathbf{\bar{r}}(n)]$  so that (22) can be expressed in terms of the known signal parameters as,

$$\mathbf{f}_{k}(n+1) = \mathbf{f}_{k}(n) + 2\mu(\mathbf{G}\begin{bmatrix}\mathbf{s}_{k}\\\mathbf{0}\end{bmatrix} + \hat{\mathbf{G}}\begin{bmatrix}\mathbf{s}_{k}\\\mathbf{0}\end{bmatrix}$$
$$\mathbf{p}_{k}^{T}\mathbf{A}\begin{bmatrix}\mathbf{s}_{k}\\\mathbf{0}\end{bmatrix} - \mathbf{f}_{k}^{T}(n)\bar{\mathbf{r}}(n)\bar{\mathbf{r}}(n))$$
(23)

where

$$\mathbf{G} = \begin{bmatrix} 1 & -g_1 & \dots & -g_M & 0 & \dots & 0 \\ 0 & 1 & -g_1 & \dots & -g_M & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & \dots & \dots & \dots & -g_M \end{bmatrix}$$
(24)  
and  
$$\hat{\mathbf{G}} = \begin{bmatrix} \mathbf{I} & \mathbf{O} \end{bmatrix} - \mathbf{G}.$$
(25)

## 4. SIMULATIONS

In this section, we will examine the performance of the proposed techniques. Gold Code [9] of length N = 7 is employed to form the CDMA code matrix **S**,

where the number of users is K = 4. The NBI is an AR process of order M = 2. The number of ensemble runs is  $10^5$ , and we use the last half of the symbols to compute *BER*. Two types of NBI are considered with different amount of predictability. Type I NBI has poles at 0.96  $e^{\pm j0}$ , and type II has poles at 0.99  $e^{\pm j\frac{\pi}{10}}$ . Type I NBI is more predictable than Type II NBI. For the two proposed algorithms, the initial adaptation step size for  $\hat{a}$  and g is 0.002, which is divided by 2 for every 60,000 data samples. The adaptation step size for  $\mathbf{p}_k$  and  $\mathbf{f}_k$  is 0.0002.



Fig. 4. Average BER of the proposed methods at -20dB SINR.

Fig.4 gives the average bit error rate of the 4 users of the proposed methods, where the SINR is fixed at -20dB. As we can see from the Fig.4, for type I NBI and at  $BER = 10^{-4}$ , the required SNR of the proposed adaptive non-linear predictor method



Fig. 5. Worst case BER of the proposed methods at -20dB SINR.



Fig. 6. MSDs of the no predictor and linear predictor cases.

is only about 16.6dB, and that of the linear predictor method is about 17.6dB, whereas for the no predictor case, it needs SNR = 19.0dB in order to achieve the same BER.

Fig.5 shows the the worst BER among the 4 users under the same condition of Fig.4. Comparing Fig.4 and Fig.5, the conclusion is that the non-linear predictor method maintains the BER performances for the worst users as in the average BER case, while the performance degrades for linear predictor and no predictor methods. For type I NBI and at  $BER = 10^{-4}$ , the required SNR of the adaptive non-linear predictor method is 2dB lower than that of the linear predictor case.

Fig.6 gives the convergence behavior by plotting the normalized mean square deviation (MSD) of the 4 users:  $(\sum_{k=1}^{4} ||(\mathbf{h}_k - \mathbf{h}_k^*)||^2)/(\sum_{k=1}^{4} ||(\mathbf{h}_k^*)||^2)$  for the case without using a predictor (Fig.1) and  $(\sum_{k=1}^{4} ||(\mathbf{p}_k - \mathbf{q}_k)||^2)/(\sum_{k=1}^{4} ||(\mathbf{q}_k)||^2)$  for the case of using an adaptive linear predictor (Fig.2), where  $\mathbf{h}^*$  is the Wiener filter solution of  $\mathbf{h}$ ,  $E_b/N_0$  is 20dB and the results are based on type I NBI. The step size for the no predictor case is the biggest under the condition that  $\mathbf{h}_k$  will converge.

It is clear that the convergence speed of the proposed linear predictor is much faster than that of the no predictor case, furthermore, simulation indicates that the steady state misadjustment of the case without a predictor is around 0.0031 and that of the case with a linear predictor is around 0.0015. The steady state misadjustment with a linear predictor not only removes the NBI better, but also contributes to a faster convergence speed to produce better user bit estimates. The reason for faster convergence is that the eigenvalue spread of the autocorrelation matrix in the NBI removed signal is much smaller than that of the received signal without removing the NBI.

Fig.7 demonstrates the adaptation ability of the proposed techniques. Initially, the system have only 3 users. At n = 100,000, the number of users increases to 4. As shown from Fig.7, proposed algorithm tracks the system variation much better than the conventional approach.

## 5. CONCLUSIONS

We have proposed linear predictor and non-linear predictor adaptive methods for NBI cancellation in CDMA Multi-User Detec-



tion. The methods use an adaptive predictor to estimate the NBI which is then subtracted from the received signal to obtain better user bit estimates. The proposed methods not only converge faster than the case without using a predictor, but also yield better *BER* at steady state. The proposed algorithms are blind and do not require any training data. At a *BER* of  $10^{-4}$  and *SINR* =-20dB, the linear and non-linear predictor methods yield 1.4dB and 2.4dB in  $E_b/N_0$  improvement respectively.

## 6. REFERENCES

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