ANALYSIS AND OPTIMIZATION OF INTERLEAVE-DIVISION MULTIPLE-ACCESS COMMUNICATION SYSTEMS

Kai Li, Xiaodong Wang

Columbia University
Department of Electrical Engineering
New York, NY 10027, USA

Ping Li

City University of Hong Kong
Department of Electronic Engineering
Hong Kong, China

ABSTRACT

In this paper we focus on the analysis and optimization of the interleave-division multiple-access (IDMA) system. The spectral efficiencies of the coded IDMA system are analyzed. Optimal power allocation among users in IDMA to maximize the spectral efficiency with finite-alphabet constellation is also considered. Differential evolution is adopted to solve the power profile optimization problem.

1. INTRODUCTION

In this paper, we consider analysis and optimization of IDMA systems introduced in [1]. With the extrinsic information transfer (EXIT) chart technique [2], the spectral efficiencies of coded IDMA in both single-cell and multi-cell scenarios are analyzed. With binary codebooks, the capacity of an IDMA system is bounded by 1 bit/sec/Hz with equal rate and equal power configuration. Recent results show that, with rate or power control and stripping decoding, the optimal spectral efficiency of a multiple-access channel can be approached even with finite input constellations, e.g., BPSK or QPSK constellations [3]. Here we formulate the power allocation problem for IDMA systems. Similar to the degree profile design of the irregular LDPC codes, the differential evolution technique [4] is utilized to solve the power profile optimization problem. It is shown that with optimized power profiles, the turbo-Hadamard coded IDMA system can approach the optimal spectral efficiency with finite input constellations.

The reminder of this paper is organized as follows. Section 2 describes the IDMA system and the iterative chip-level multiuser detection. Section 3 describes the low-rate coded IDMA system and analyzes the spectral efficiencies of the single-cell and multi-cell IDMA systems with equal power configuration. The optimal power allocation for IDMA systems is also addressed. Section 4 contains the conclusions.

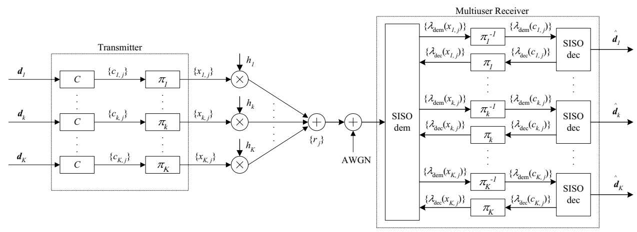


Fig. 1. IDMA transceiver.

2. INTERLEAVE-DIVISION MULTIPLE-ACCESS

For simplicity, here we consider only a synchronous uncoded BPSK modulated spread spectrum system with K users over a time-invariant single-path channel. As shown in Fig. 1, at the transmitter end, the nth information bit of the kth user $d_{k,n} \in \{+1, -1\}$ in the input data stream $\boldsymbol{d}_k$ is spread by a length-S spreading sequence $\boldsymbol{s}_k$ in the form $d_{k,n} \rightarrow d_{k,n}\boldsymbol{s}_k$. A chip-level interleaver π_k of length P is then applied to produce the transmitted signals $\{x_{k,j}\}$. Concentrating on the kth user, in the single-cell scenario, the received signal at chip instant j can be written as

$$r_j = A_k h_{k,j}\, x_{k,j} + \zeta_{k,j},\ j = 1, ..., P \tag{1}$$

where $\zeta_{k,j} \triangleq \sum_{k' \neq k} A_{k'} h_{k',j}\, x_{k',j} + n_j$ is the sum of the multiuser interference and the additive noise with respect to the kth user; $x_{k,j} \in \{+1, -1\}$ and $h_{k,j}$ denote the chip and the complex channel coefficient at chip instant j, respectively; A_k is the amplitude, and n_j is the zero-mean, complex, additive white Gaussian noise (AWGN) with variance $\sigma_{ch}^2 = N_0/2$ per dimension.

Soft chip demodulator: As shown in Fig. 1, the iterative chip-by-chip receiver consists of a soft-input soft-output (SISO) chip demodulator and a bank of K single-user SISO decoder working in a turbo manner. The chip demodulator performs a chip-by-chip demodulation based on the channel input and the prior information provided by the decoders. Each $x_{k,j}$ is a random variable with mean $\mathbb{E}\{x_{k,j}\}$ and variance $\text{Var}\{x_{k,j}\}$ (initialized to 0 and 1 respectively). From (1), we have $\mathbb{E}\{r_j\} = \sum_{k=1}^{K} A_k h_{k,j}\,\mathbb{E}\{x_{k,j}\}$ and $\text{Var}\{r_j\} = \sum_{k=1}^{K} |A_k h_{k,j}|^2\,\text{Var}\{x_{k,j}\} + 2\sigma_{ch}^2$. When K is large, with the central limit theorem, $\zeta_{k,j}$ in (1) can be approximated by a Gaussian random variable with mean and variance given respectively by $\mathbb{E}\{\zeta_{k,j}\} = \mathbb{E}\{r_j\} - A_k h_{k,j}\,\mathbb{E}\{x_{k,j}\}$ and $\text{Var}\{\zeta_{k,j}\} = \text{Var}\{r_j\} - |A_k h_{k,j}|^2 \cdot \text{Var}\{x_{k,j}\}$. Then the chip demodulator provides the log-likelihood ratios (LLRs) of $\{x_{k,j}\}$ as

$$\lambda_{\text{dem}}(x_{k,j}) = \frac{4A_k \cdot \Re\left(h_{k,j}^* \cdot (r_j - \mathbb{E}\{\zeta_{k,j}\})\right)}{\text{Var}\{\zeta_{k,j}\}}. \qquad (2)$$

The demodulator outputs $\{\lambda_{\text{dem}}(x_{k,j})\}_{j=1}^{P}$ are de-interleaved to form $\{\lambda_{\text{dem}}(c_{k,j})\}_{j=1}^{P}$ and then fed to the single-user decoder/ de-spreader as the *a priori* information.

Soft de-spreader/repetition decoder: Focusing on the chips related to $d_{k,1}$, the first bit of user k. Recall that $d_{k,1}$ is spread into the chip sequence $d_{k,1}\boldsymbol{s}_k = \{c_{k,j}\}$, where $\boldsymbol{s}_k = \{s_{k,j}\}$ is the binary signature sequence (over $\{+1,-1\}$) for user k. Due to the interleaver, $\{\lambda_{\text{dem}}(c_{k,j})\}$ are assumed uncorrelated. Denote the interleaving operation for user k as $\pi_k(j) = j'$, i.e., $c_{k,j} = x_{k,j'}$. Then the *a posteriori* LLR output of the repetition decoder for $d_{k,1}$ can be computed from $\{\lambda_{\text{dem}}(c_{k,j})\}$ as

$$\Lambda_{\text{rep}}(d_{k,1}) = \textstyle\sum_{j=1}^{S} s_{k,j}\,\lambda_{\text{dem}}(c_{k,j}). \qquad (3)$$

The extrinsic for the chip $c_{k,j}$ within $d_{k,1}\boldsymbol{s}_k$ is then given by $\lambda_{\text{rep}}(c_{k,j}) = \ln\left(\frac{\Pr(c_{k,j}=+1|\boldsymbol{r})}{\Pr(c_{k,j}=-1|\boldsymbol{r})}\right) - \lambda_{\text{dem}}(c_{k,j})$. Since $c_{k,j} = +1$ if $s_j(k) = d_{k,1}$ and $c_{k,j} = -1$ otherwise, then

$$\lambda_{\text{rep}}(c_{k,j}) = s_{k,j}\,\Lambda_{\text{rep}}(d_{k,1}) - \lambda_{\text{dem}}(c_{k,j}). \qquad (4)$$

The extrinsic LLRs $\{\lambda_{\text{rep}}(c_{k,j})\}$ are then interleaved and fed back to the chip demodulator as the *a priori* information to update $\{\mathbb{E}\{x_{k,j}\}\}$ and $\{\text{Var}\{x_{k,j}\}\}$ as follows $\mathbb{E}\{x_{k,j}\} = \tanh\left(\frac{1}{2}\lambda_{\text{rep}}(x_{k,j})\right)$ and $\text{Var}\{x_{k,j}\} = 1 - \mathbb{E}\{x_{k,j}\}^2$, which in turn are used by the chip demodulator to refine its outputs for the next iteration. The above procedure is repeated for a certain number of iterations. In the last iteration, the repetition decoder produces hard decisions $\hat{\boldsymbol{d}}_k$ for the information bits $\boldsymbol{d}_k$ based on the *a posteriori* LLRs given by (3).

3. LOW-RATE CODED IDMA

In CDMA systems, spreading can be achieved by low-rate coding, which leads to the code-spread systems. Inspired by this, an alternative coded IDMA system can be obtained from uncoded IDMA by replacing the spreading/de-spreading with a low-rate channel encoder/decoder, such as the low-rate turbo-Hadamard code [5]. Here we analyze the spectral efficiencies of turbo-Hadamard coded IDMA systems.

3.1. Spectral Efficiency Under Equal Power Allocation

In the case that all users employ an identical coding rate R_c (bits per symbol), the spectral efficiency of a CDMA system is given by $\nu = \frac{K}{S} R_c$, with $\Omega = S/R_c$ as the total bandwidth expansion factor. For turbo-Hadamard coded IDMA, we have $S = 1$ and $R_c = 1/\Omega$. As in [2], for a certain coding rate R_c, we use the largest number of users K^* that can transmit their bits reliably on the channel as a measurement of the system capacity. Then the maximum spectral efficiency in bits/sec/Hz is given by

$$\nu_{\text{IDMA}}^*(\Omega, \text{SNR}) = \frac{K^*(\Omega, \text{SNR})}{\Omega}. \qquad (5)$$

In the case that all users have equal power, and parallel iterative processing is employed, $K^*(\Omega, \text{SNR})$ can be accurately obtained by the EXIT chart analysis as shown in [2]. Specifically, for a given SNR and Ω, we first obtain the EXIT transfer characteristics of the chip modulator with Monte Carlo method for different K, and the *asymptotic user number threshold* K^* [2] can be obtained by finding the maximum K with which a detection tunnel between the EXIT curves of chip modulator and the code can still exist.

3.2. Optimal Power Allocation

As shown in [6], the maximum spectral efficiency of random CDMA with optimal decoding is achieved when the system load $\beta \to \infty$ and it coincides with the AWGN single-user capacity ν^*, implicitly given by $\frac{4^{\nu^*}-1}{\nu^*} = \frac{E_b}{N_0}$. With binary codebooks and equal rate equal power configuration, the system capacity is bounded by 1 bit/sec/Hz. Recent result shows that, with rate or power control along with the stripping decoding scheme, the optimal spectral efficiency can be approached even with finite input constellations, e.g., BPSK or QPSK [3]. Here we consider the application of the similar principle to low-rate coded IDMA systems. For simplicity, we consider the equal rate, unequal power case.

3.2.1. Problem Formulation

Consider a low-rate coded IDMA system with K users which are grouped into M classes. The number of users in the mth class is K_m, and the *class load* $\beta_m \triangleq K_m/S = K_m$ (for low-rate coded IDMA, $S = 1$). Thus, the total system load is $\beta = \sum_{m=1}^{M} \beta_m$ users per chip. Users in class m have the same received SNR, denoted by γ_m. Without loss of generality, we assume $\gamma_1 \leq \cdots \leq \gamma_M$. The total system spectral efficiency is then given by $\nu = \sum_{m=1}^{M} \beta_m R_c = \beta R_c$, where R_c is the coding rate. And the "system" E_b/N_0 is

$$\left(\frac{E_b}{N_0}\right)_{\text{sys}} \triangleq \frac{\sum_{m=1}^{M} \beta_m \gamma_m}{2\sum_{m=1}^{M} \beta_m R_c} = \frac{\boldsymbol{\beta}^T \boldsymbol{\gamma}}{2\beta R_c}, \qquad (6)$$

where $\boldsymbol{\beta} \triangleq [\beta_1, \cdots, \beta_M]^T$ is the class load vector and $\boldsymbol{\gamma} \triangleq [\gamma_1, \cdots, \gamma_M]^T$ is the power profile vector. In practice, to make the problem tractable, we may resort to solving the following two simpler problems: (1) fix the received power levels $\boldsymbol{\gamma}$, and consider the optimization of the class loads $\boldsymbol{\beta}$; (2) fix the class loads $\boldsymbol{\beta}$ and optimize the power levels $\boldsymbol{\gamma}$. Note that for IDMA systems the class loads $\{\beta_m = K_m\}$ are discrete integers. To avoid a discrete optimization problem, here we treat the second problem, in which the optimization variables $\{\gamma_m\}$ are continuous. Then, the optimization problem can be stated as follows

$$\begin{aligned} \boldsymbol{\gamma}_{\text{opt}} &= \arg \min_{\boldsymbol{\gamma} \in \mathbb{R}^M} \left(\frac{\boldsymbol{\beta}^T \boldsymbol{\gamma}}{2\beta R_c} \right) \\ &\text{s.t.} \quad \max_m \ P_b(m) \leq \epsilon, \text{ and } \boldsymbol{\gamma} \geq \mathbf{0} \end{aligned} \tag{7}$$

where $P_b(m)$ is the corresponding BER of the mth class users with a rate-R_c channel code C and a certain detection strategy. Here we consider both the parallel turbo detection and the successive group stripping detection.

3.2.2. *The Optimization Procedure*

Note that for IDMA systems, extrinsic information is exchanged between the chip demodulator and single-user decoders. And the optimization problem considered here is similar to the one encountered in the irregular LDPC code design, where the nonlinear optimization procedure called differential evolution [4] is utilized to find the degree profile of the irregular LDPC codes [7]. Specifically, for a given $\boldsymbol{\beta}$, we first choose the system E_b/N_0 and find a vector $\boldsymbol{\gamma}$ using differential evolution that satisfies the BER constraint ϵ. If at least one $\boldsymbol{\gamma}$ exists, E_b/N_0 then is reduced and the procedure is repeated until no such a vector $\boldsymbol{\gamma}$ exists. The least value of E_b/N_0 for which a power profile $\boldsymbol{\gamma}$ satisfies the BER constraint is the desired minimum system SNR and the corresponding $\boldsymbol{\gamma}$ is the optimized power profile.

During differential evolution, we need to obtain the BER $P_b(m)$ for a given $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$. Since the received SNR γ_m might be different for different class, for parallel turbo detection, the input/output mutual information of the chip demodulator for different class users at each iteration might also be distinct, which can be denoted as $I_{\text{dem,in}}(m)$ and $I_{\text{dem,out}}(m)$, for $m = 1, \cdots, M$, respectively. With given $\{I_{\text{dem,in}}(m)\}$ and $\{\gamma_m\}$, we can obtain $\{I_{\text{dem,out}}(m)\}$ by Monte Carlo simulations, which is similar to the procedure for equal power case. Since all the users employ an identical single-user decoder, the mutual information of class m passing from the decoder to the demodulator for the next iteration is given as $I_{\text{dem,in}}(m) = v(I_{\text{dem,out}}(m))$, where the mutual information transfer function of the decoder $I_{\text{dec,out}} = v(I_{\text{dec,in}})$ can again be obtained by Monte Carlo simulations. After a given number of iterations, the BER performance $P_b(m)$ can be estimated by

$$P_b(m) = Q(J^{-1}(I_{\Lambda_{\text{dec}}}(m))/2), \tag{8}$$

with $Q(\cdot)$ being the Q-function and Λ_{dec} denoting the output LLR of the decoder for the information bit, and the mutual information $I_{\Lambda_{\text{dec}}}(m) = \varphi(I_{\text{dec,in}}(m))$, where the transfer function $\varphi(\cdot)$ depends upon the decoder, and similar to $v(\cdot)$, it can be obtained by Monte Carlo simulations.

To speed up the optimization procedure, the large-system approximation of the chip demodulator can be utilized. Extending from the Tse-Hanly equation [8], the output SNR of the chip demodulator for class m is the unique solution of the following equation

$$\gamma_{\text{dem}}(m) = \gamma_m \cdot \left(1 + \sum_{i=1}^{M} \beta_i \, \mathbb{E}_{\lambda_i} \left\{ \frac{\gamma_i \left(1 - \tanh^2\left(\frac{\lambda_i}{2}\right)\right)}{1 + \frac{\gamma_{\text{dem}}(m)\gamma_i}{\gamma_m}\left(1 - \tanh^2\left(\frac{\lambda_i}{2}\right)\right)} \right\} \right)^{-1}$$

where $\lambda_i \sim \mathcal{N}(\frac{\sigma_i^2}{2}, \sigma_i^2)$ with $\sigma_i = J^{-1}\left(I_{\text{dem,in}}(i)\right)$.

In the case that the successive group stripping detection and the capacity approaching codes (e.g., LDPC codes, turbo codes and turbo-Hadamard codes) are utilized, the above optimization procedure can be further simplified. Assuming that all users in classes $m+1, \cdots, M$ have been perfectly canceled, the successive decodability condition is that the output SNR of the chip demodulator for the mth class $\gamma_{\text{dem}}(m)$ should be no less than g, for $m = M, \cdots, 1$. With large-system analysis, $\gamma_{\text{dem}}(m)$ with group stripping decoding can be easily extended from the Tse-Hanly equation, which is given by

$$\gamma_{\text{dem}}(m) = \gamma_m \cdot \left(1 + \sum_{i=1}^{m} \beta_m \frac{\gamma_i}{1 + \gamma_{\text{dem}}(m)\frac{\gamma_i}{\gamma_m}}\right)^{-1}.$$

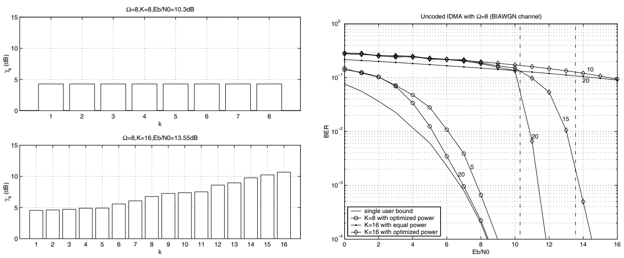
Again, the optimal power profile can be obtained by differential evolution.

3.3. Numerical Results

Uncoded IDMA with parallel processing:

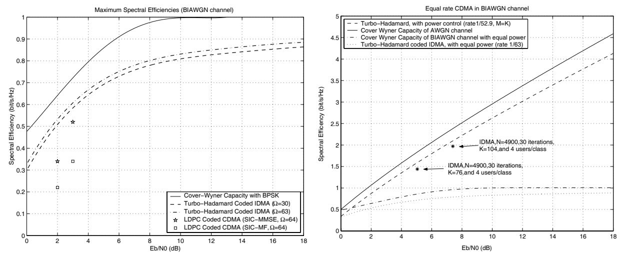
To illustrate the power optimization process, consier an uncoded IDMA system with $\Omega = 8$ and parallel iterative detection. Setting the class number $M = K$, and a target BER $\epsilon = 10^{-4}$ for 20 iterations, we obtain the optimized power profile $\boldsymbol{\gamma}$ using differential evolution for $K = 8$ and 16, respectively. As shown in Fig.2(a), for $K = 8$, the equal power configuration turns out to be the optimal one, whereas for $K = 16$, a nonuniform power profile is obtained. The BER performance of the worst user is given in Fig. 2(b) with a block size of 4000. For $K = 16$, a significant improvement is observed with the optimized power.

Spectral efficiency of Turbo-Hadamard coded IDMA



(a) Power profile (b) BER performance

Fig. 2. Power profile optimization and BER performance.



(a) Equal power case (b) Unequal power case

Fig. 3. Spectral efficiency of turbo-Hadamard coded IDMA.

Assuming equal-power and parallel iterative processing, Fig. 3(a) depicts the spectral efficiency of a turbo-Hadamard coded IDMA system in binary input AWGN (BIAWGN) channels. It is seen that the low-rate coded IDMA system provides a near-capacity performance in the single-cell case (around 1.4dB away from the capacity for $\nu = 0.5$ with $\Omega = 63$) at the low SNR region.

Now consider the unequal power case. Here we use the rate-$1/52.9$ turbo-Hadamard code which is only 0.4dB (measured at BER $= 10^{-5}$) away from the ultimate Shannon limit with a interleaver size of 65534. The SNR threshold of the code $g = -15.43$dB. With stripping decoding, with 4 users/class, we obtain the optimized power profiles for $K = 76$ and 104 with $E_b/N_0 = 4.8$dB and 7.4dB, respectively. We simulated the above two IDMA systems with soft-stripping decoding which are marked in Fig. 3(b). For $K = 76$, the simulated $E_b/N_0 = 5.0$dB with $N = 4900$ and 30 iteration for a BER less than 10^{-5}, which is only around 1.5dB away from the capacity. For $K = 104$, the simulated $E_b/N_0 = 7.5$dB with $N = 4900$ and 30 iteration for a BER less than 10^{-5}, which is around 1.6dB away from the capacity. For $M = K$, the spectral efficiency with the large-system of the turbo-Hadamard coded IDMA system is also shown in Fig. 3(b) with the optimized power. It is seen that with increasing group number, less power is required to reach the same spectral efficiency. In practical, less class number can increase the decoding speed since parallel decoding can be applied within the same class. Hence there is a tradeoff between the class number and the system E_b/N_0.

4. CONCLUSIONS

The spectral efficiencies of the coded IDMA system were analyzed. We have considered optimal power allocation among users in IDMA to maximize the spectral efficiency with finite-alphabet constellation (BPSK). Similar to the degree profile design of the irregular LDPC codes, the differential evolution is adopted to solve the power profile optimization problem. With optimized power profiles, the optimal spectral efficiency can be approached even with finite input constellations. In summary, with chip-level interleaving and iterative detection, IDMA is a promising and flexible air-interface technique for heavily-loaded systems.

5. REFERENCES

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