ON THE PERFORMANCE OF SPACE TIME TRANSMIT DIVERSITY IN THE DOWNLINK OF W-CDMA WITH AND WITHOUT EQUALIZATION

Belkacem Mouhouche[1,3], *Philippe Loubaton*[2], *Karim Abed-Meraim*[1], *Nicolas Ibrahim*[3]

1-Télécom Paris, TSI Department, 46 rue Barrault, Paris, France. {{mouhouch,abed}@tsi.enst.fr}

2-Univ. de Marne-La-Vallée, 5, Bld. Descartes, 77454 Marne-La-Vallée, France. {loubaton@univ-mlv.fr}

3- WAVECOM, 3 Esplanade du Foncet, 92442 Issy-les-Moulineaux, France. {nicolas.ibrahim@wavecom.fr}

ABSTRACT

In this paper, we discuss the performance of Space Time Transmit Diversity (STTD) in the downlink of DS-CDMA over frequency-selective fading channels. We consider two kinds of receivers: the RAKE receiver and the chip-level MMSE equalizer-based receiver. These two receivers comparison turns out to be a very difficult task because their output Signal to Interference plus Noise Ratios (SINRs) depend in a complex way on the spreading and scrambling codes. To obtain tractable expressions, we study the SINRs in the asymptotic regime, i.e. we suppose that the spreading factor and the number of users both tend to infinity while their ratio remains constant. We further suppose that the code matrix is a random matrix obtained by multiplying a random scrambling code by a Walsh-Hadamard matrix. Under these conditions, the SINRs of the two receivers tend to deterministic values. We compare the asymptotic SINRs and draw some conclusions about the effect of the channel transfer function on the performance. Simulation results show that the asymptotic results allow to predict the performances of real life systems like the UMTS-FDD.

1. INTRODUCTION

Third generation (3G) mobile communications systems such cdma2000 and W-CDMA are intended to provide higher data rates than current second generation systems. Diversity is one way to combat channel fading. Multiple antennas at the receiver can be used to provide diversity. The dilemma is that, in the downlink, multiple antennas at the receiver induces an increase in the size of the mobile unit, while significant effort is being done to make wireless mobile devices smaller and cheaper. Alamouti [1] has shown that the diversity provided by using two transmit antennas and one receive antenna is the same as that provided by one transmit antenna and two receive antennas. However, this result is valid for flat fading channels only. The Alamouti scheme allows to provide diversity without the need to include multiple antennas at the receiver side.

Space Time Transmit Diversity has been adopted in the W-CDMA norm [2]. In W-CDMA, the propagation channels are known to be frequency selective. It is then of great importance to study the performance of STTD in frequency selective fading channels when associated with the conventional receiver of CDMA systems (the RAKE receiver).

A promising alternative to the RAKE reception is chip-rate Minimum Mean Squared Error (MMSE) equalization prior to descrambling and desreading [4]. The orthogonality between the spreading codes is destroyed due to the multipath propagation channel. MMSE equalization allows to partially restore the orthogonality. Thus, after descrambling and despreading the symbol estimate is better than that obtained by the RAKE receiver. It is thus very useful to study the performance of STTD in frequency selective fading channels when associated with a MMSE equalizer-based receiver. In this paper, we consider the use of STTD in the downlink of W-CDMA. We discuss the applicability of the Alamouti scheme in the case of multipath (frequency-selective) channels when using a RAKE receiver or a MMSE equalizer-based receiver. We follow the classical approach used for the first time in [5], and assume that the spreading factor N and the number of users K tend to $+\infty$ at the same rate. The spreading codes are supposed to coincide with Walsh Hadamard codes scrambled by an Independent Identically Distributed (i.i.d) sequence. In this context, the SINRs of the two receiver tend to deterministic limits independent of the scrambling and the spreading codes. We derive the asymptotic SINRs, compare the two receivers and discuss the gain that we obtain by using STTD for both of them.

Notations: Throughout the paper, we denote by $\mathbf{A}^H$ and $\mathbf{A}^T$ the conjugate and the transpose of $\mathbf{A}$ respectively. $\overline{\mathbf{A}}$ denotes $(\mathbf{A}^H)^T$. $\mathbf{A} \otimes \mathbf{B}$ denotes the Kronecker product of $\mathbf{A}$ and $\mathbf{B}$.

2. SYSTEM MODEL

We consider a single base station transmitting the sum of K users chip signals given by:

$$d(n) = s(n) \sum_{k=1}^{K} c_k(n \; mod \; N) b_k(\lfloor \frac{n}{N} \rfloor) \qquad (1)$$

where $s(n)$ is the base-station dependent QPSK (long) scrambling code, N is the spreading factor, K is the number of users, $b_k(\lfloor \frac{n}{N} \rfloor)$ and $c_k(n \; mod \; N)$ are the QPSK symbol sequence and the (N-periodic) normalized spreading code of user k, respectively. (mod stands for the modulo and $\lfloor . \rfloor$ for the integer part).

Throughout the paper, we will assume that the scrambling sequence is i.i.d, and that users symbols are independent zero mean QPSK signals. The index of the user of interest is 1.

The transmitted chip vector in one symbol period $\mathbf{d}(m) = \left[d(mN), d(mN+1), ..., d(mN+N-1)\right]^T$ is given by:

$$\mathbf{d}(m) = \mathbf{S}(m)\mathbf{C}\mathbf{b}(m) \qquad (2)$$

where $\mathbf{S}(m)$ is the $N \times N$ diagonal matrix whose diagonal elements are $s(mN), s(mN+1), ..., s(mN+N-1)$ and $\mathbf{C}$ is a

$N \times K$ matrix whose columns are the spreading codes assigned to different users and $\mathbf{b}(m) = [b_1(m), ..., b_K(m)]^T$.
The sum chip signal (1) is transmitted through two multipath frequency-selective fading channels whose impulse responses are given by

$$h_i(t) = \sum_{q=0}^{P-1} \lambda^i(q) p(t - \tau_q^i) \qquad i = (1, 2) \tag{3}$$

where $p(t)$ is the total shaping filter (including the transmitter and the receiver matched filters), $\lambda^i(q)$ and τ_q^i are the complex gain and the delay associated with path q of the channel between transmit antenna $i = (1, 2)$ and the receiver, and P is the total number of resolvable paths. For the sake of simplicity we suppose that the number of resolvable paths is the same for both channels.
A symbol-level Alamouti STBC is applied at the base station.This is equivalent to transmitting the chip vectors defined by equation 2 according to Table. 1.

time Antenna	$m-2$	$m-1$	m	$m+1$
1	$\mathbf{d}(m-2)$	$\mathbf{d}(m-1)$	$\mathbf{d}(m)$	$\mathbf{d}(m+1)$
2	$\mathbf{d}^*(m-1)$	$-\mathbf{d}^*(m-2)$	$\mathbf{d}^*(m+1)$	$-\mathbf{d}^*(m)$

Table 1. The Alamouti STBC for W-CDMA

If we call the chips transmitted from antenna 1 $d_1(n)$ and the chips transmitted from antenna 2 $d_2^*(n)$ then the chip-rate sampled received signal is given by:

$$x(n) = \sum_{l=1}^{L-1} h_{1,l} d_1(n-l) + \sum_{l=1}^{L-1} h_{2,l} d_2^*(n-l) + v(n) \tag{4}$$

where $h_{i,l} \triangleq h_i(t)|_{t=lT_c}$, L is the overall channel length (in chip periods) and $v(n)$ is a centered white gaussian noise process with variance σ^2.

3. ASYMPTOTIC PERFORMANCE OF STTD

To study the asymptotic performance of the two considered receivers, we suppose that the spreading factor and the number of users tend to infinity while their ratio remains constant (see for example [5, 3]). In this scenario, it can be shown that the ISI term has no effect on the asymptotic SINR (see for example [3]). Chip-rate model 4 can be replaced by the following symbol-rate model:

$$\begin{bmatrix} \mathbf{x}(m) \\ \overline{\mathbf{x}}(m+1) \end{bmatrix} = \begin{bmatrix} \mathbf{H}_1 & \mathbf{H}_2 \\ -\overline{\mathbf{H}}_2 & \overline{\mathbf{H}}_1 \end{bmatrix} \begin{bmatrix} \mathbf{d}(m) \\ \overline{\mathbf{d}}(m+1) \end{bmatrix} + \begin{bmatrix} \mathbf{v}(m) \\ \overline{\mathbf{v}}(m+1) \end{bmatrix}$$

where $\mathbf{x}(m)$ and $\mathbf{v}(m)$ are defined as $\mathbf{d}(m)$,

$$\mathbf{H}_i = \begin{bmatrix} h_{i,0} & 0 & h_{i,L-1} & \cdots & h_{i,1} \\ \vdots & h_{i,0} & & \ddots & \vdots \\ h_{i,L-1} & & & & h_{i,L-1} \\ & \ddots & \ddots & & \\ & & & & \\ 0 & & h_{i,L-1} & & h_{i,0} \end{bmatrix}$$

It is more convenient to use the following equivalent model:

$$\mathbf{y} = \mathcal{H}\mathcal{C}\mathcal{B} + \mathcal{V} \tag{5}$$

where

$$\mathbf{y} = [x(mN+1)\, x^*((m+1)N+1) ... x(mN+N)\, x^*((m+1)N+N)]^T$$

$\mathcal{H}$ is a block Toeplitz matrix of the same structure as $\mathbf{H}_i$ whose 2×2 blocks are equal to $\begin{bmatrix} h_{1,l} & h_{2,l} \\ -(h_{2,l})^* & (h_{1,l})^* \end{bmatrix}$

$$\mathcal{C} = (\mathbf{S}(m)\mathbf{C}) \otimes \mathbf{A}_{1,1} + (\overline{\mathbf{S}}(m+1)\mathbf{C}) \otimes \mathbf{A}_{\mathbf{2},\mathbf{2}}$$

$\mathbf{A}_{i,j}$ stands for a 2 by 2 matrix whose entry (i, j) is equal to 1 and all other entries are equal to zero,

$$\mathcal{B} = [b_1(m)\, b_1(m+1)\, b_2(m)\, b_2(m+1) ... b_K(m)\, b_K(m+1)]^T$$

and $\mathcal{V}$ has the same structure as $\mathbf{y}$. $\mathcal{C}$ can be interpreted as the overall code matrix. Note we have omitted the time index as it is irrelevant.

3.1. The receivers

The RAKE receiver is a matched filter matched to the signature of the user of interest. Suppose that we want to retrieve $b_1(m)$, that is the symbol transmitted by user 1 at time instant m from antenna 1. Let $\mathcal{C} = [\mathbf{w}_1 \ \ \mathbf{U}]$, where $\mathbf{w}_1$ is the overall code of the user of interest and $\mathbf{U}$ represents the matrix of interferers codes.
The soft estimate of $b_1(m)$ is given by:

$$\tilde{b}_1(m) = \mathbf{w}_1^H \mathcal{H}^H \mathbf{y} \tag{6}$$

The SINR, that we index by the spreading factor, corresponding to this receiver is given by :

$$\beta_{RAKE}^{(N)} = \frac{|\mathbf{w}_1^H \mathcal{H}^H \mathcal{H} \mathbf{w}_1|^2}{\mathbf{w}_1^H \mathcal{H}^H (\mathcal{H} \mathbf{U}_1 \mathbf{U}_1^H \mathcal{H}^H + \sigma^2 \mathbf{I}) \mathcal{H} \mathbf{w}_1} \tag{7}$$

The MMSE equalizer-based receiver consists of a MMSE channel-equalizer followed by a despreader. The MMSE equalizer is given by:

$$\mathbf{G} = \mathcal{H}^H (\mathcal{H}^H \mathcal{H} + \frac{N\sigma^2}{K} \mathbf{I})^{-1} \tag{8}$$

The soft estimate of $b_1(m)$ is given by:

$$\tilde{b}_1(m) = \mathbf{w}_1^H \mathbf{G} \mathbf{y} \tag{9}$$

Note that this is exactly the Wiener receiver that would be implemented if the chip sequence were considered i.i.d with variance $\frac{K}{N}$. The corresponding SINR is:

$$\beta_{MMSE}^{(N)} = \frac{|\mathbf{w}_1^H \mathbf{G} \mathcal{H} \mathbf{w}_1|^2}{\mathbf{w}_1^H \mathbf{G} (\mathcal{H} \mathbf{U}_1 \mathbf{U}_1^H \mathcal{H}^H + \sigma^2 \mathbf{I}) \mathbf{G}^H \mathbf{w}_1} \tag{10}$$

3.2. Asymptotic analysis

The expressions of the MMSE and the RAKE SINRs depend in a complex way on the codes. To overcome the difficulty of interpreting them, we study their limit in the asymptotic regime, i.e. we suppose that $N \to \infty$, $K \to \infty$ while $\frac{K}{N} \to \alpha$ where $1 > \alpha > 0$. Under these conditions $\beta_{MMSE}^{(N)}$ and $\beta_{RAKE}^{(N)}$ can be shown to converge to deterministic limits β_{MMSE} and β_{RAKE}

$$\beta_{RAKE} = \frac{\sum_k(|h_{1,k}|^2+|h_{2,k}|^2)}{\alpha\big(\sum_{k\neq 0}(|R_k(h_1)|^2+|R_k(h_2)|^2)+\sum_k|R_k(f_1)|^2\big)+\sigma^2(\sum_k(|R_k(h_1)|^2+|R_k(h_2)|^2))}$$

$$\beta_{MMSE} = \frac{\sum_k(|f_{2,k}|^2)}{\alpha(\sum_{k\neq 0}|R_k(f_2)|^2+\sum_k|R_k(f_3)|^2)+\sigma^2(\sum_k(|R_k(g_1)|^2+|R_k(g_2)|^2))}$$

respectively. These limits depend only on the channel, the noise variance and the load factor (and not on the spreading codes or the specific realization of the scrambling code anymore). Note that, asymptotically, model 5 is equivalent to the following chip-rate 2×2 MIMO system:

$$\left[\begin{array}{c} x(n) \\ \overline{x}(n+N) \end{array}\right] = H(z)\left[\begin{array}{c} d(n) \\ \overline{d}(n+N) \end{array}\right] + \left[\begin{array}{c} v(n) \\ \overline{v}(n+N) \end{array}\right] \quad (11)$$

for $2kN < n \leq (2k+1)N$,
where $H(z) = \left[\begin{array}{cc} h_1(z) & h_2(z) \\ -\overline{h}_2(z) & \overline{h}_1(z) \end{array}\right]$
The MMSE equalizer designed to recover $d(n)$ from $x(n)$ is thus given by:

$$\left[\begin{array}{cc} g_1(z) & g_2(z) \end{array}\right] = [\overline{h}_1(z^{-1}) - h_2(z^{-1})](H(z)H^H(z^{-1}) + \frac{\sigma^2}{\alpha})^{-1} \quad (12)$$

where we have replaced $\frac{K}{N}$ by α. We introduce the following notation: for the function $p(e^{2i\pi f})$ let the series $R_k(p)$ be such that

$$|p(e^{2i\pi f})|^2 = \sum_k R_k(p)e^{-2i\pi kf} \quad (13)$$

We are now in a position to give the two main results of this paper. The limit SINR of the RAKE and MMSE-equalizer are given in theorems 1 and 2. The proofs are omitted due to the lack of space.

Theorem 1 *Under the assumption that the scrambling sequence is i.i.d with variance* 1*,*

$$\lim_{N\to\infty,\frac{K}{N}\to\alpha} \beta_{RAKE}^{(N)} \to \beta_{RAKE}$$

given on top of the page, where:

$$f_1(e^{2i\pi f}) = \overline{h_1}(e^{-2i\pi f})h_2(e^{2i\pi f}) - h_2(e^{-2i\pi f})\overline{h_1}(e^{2i\pi f}) \quad (14)$$

and the convergence stands for the convergence in probability.

Theorem 2 *Under the assumption that the scrambling sequence is i.i.d with variance* 1*,*

$$\lim_{N\to\infty,\frac{K}{N}\to\alpha} \beta_{MMSE}^{(N)} \to \beta_{MMSE}$$

given on top of the page, where:

$$f_2(e^{2i\pi f}) = g_1(e^{2i\pi f})h_1(e^{2i\pi f}) - g_2(e^{2i\pi f})\overline{h_2}(e^{2i\pi f})$$

$$f_3(e^{2i\pi f}) = g_1(e^{2i\pi f})h_2(e^{2i\pi f}) + g_2(e^{2i\pi f})\overline{h_1}(e^{2i\pi f})$$

and the convergence stands for the convergence in probability.

3.3. Discussion of the two theorems

The expression of the RAKE receiver SINR contains the desired signal term in the numerator and three undesired terms in the denominator. The third term stems from the effect of noise and will not be discussed. The first undesired term

$$\alpha\big(\sum_{k\neq 0}(|R_k(h_1)|^2+|R_k(h_2)|^2)$$

is the classical Multi User Interference (MUI) which is due to the non-perfect nature of each channel separately. The second undesired term $(\alpha\sum_k|R_k(f_1)|^2)$ is more interesting and can be interpreted as the Cross-Channel Interference (CCI) due to the simultaneous use of two multipath channels (see equation 14). Note that if the channels were single path (flat-fading), then we would have (by virtue of equation 13) $R_k(h_1)=0$ and $R_k(h_2)=0$ for $k\neq 0$. This means that the first term in the denominator would vanish. The second term would also vanish because:

$$f_1(e^{2i\pi f}) = (h_{1,0})^* h_{0,2} - h_{2,0}(h_{1,0})^* = 0$$

and only the noise term would remain in the denominator. On the other hand, when there is no transmit diversity (i.e. $h_2(z)=0$), part of the first term $(\alpha\sum_{k\neq 0}|R_k(h_1)|^2)$ would still be present, while the second term would vanish.
The remark that the CCI vanishes for single path channels was behind the original Alamouti STBC proposed for single-user flat-fading channels. For multipath channels, however, the CCI can be very high, and the STBC may deteriorate the performances when used with a RAKE receiver. The MUI and CCI terms are both weighted by the load factor α. This explains the fact that the SINR is higher for lightly loaded systems and vice versa.
Concerning β_{MMSE}, we first mention how $f_2(e^{2i\pi f})$ and $f_3(e^{2i\pi f})$ behave. The MMSE-equalizer tries to recover $\mathbf{d}(m)$ from $\mathbf{x}(m)$ and $\mathbf{x}(m+1)$ (see equation 11). It strives to make $f_2(e^{2i\pi f})$ close to a single path channel (which is the case in the absence of noise). This is done by concentrating the energy of $f_2(e^{2i\pi f})$ in the central term $R_0(f_2)$. On the other hand, $f_3(e^{2i\pi f})$ is made as close to zero as possible. Now, looking at the expression of β_{MMSE}, we see that the first term in the denominator decreases with respect to the first term in the denominator of β_{RAKE}. The second term, the CCI, also decreases and the noise is this time filtered by the two equalizers. The numerator, on the other hand, remains comparable to the RAKE case. By decreasing the first and second terms in the denominator while keeping the third term and the numerator comparable, the SINR is increased.

4. SIMULATION RESULTS

In this section, we verify that our asymptotic analysis allows to predict the performance of W-CDMA. We have implemented the physical layer of the downlink of the UMTS-FDD, and we have compared the measured Bit Error Rate (BER) obtained for $N=256$ and $K=128$ with its asymptotic evaluation given by $Q(\sqrt{\beta_{MMSE}})$

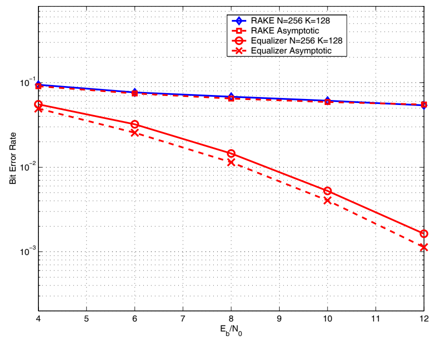

Fig. 1. Comparison of empirical and theoretical BER

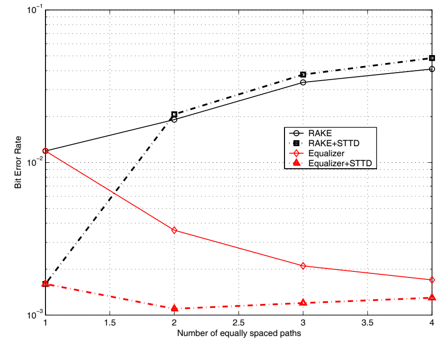

Fig. 3. BER with and without transmit diversity Vs the number of channel paths

and $Q(\sqrt{\beta_{RAKE}})$. The results are presented in Figure 1. The propagation channel is the Vehicular A channel. It is noteworthy that the receiver we implemented is based on the correct model (4), thus showing that the approximation (5) is justified in this context. Figure 1 shows that our asymptotic evaluations allow to predict rather accurately the BER performance for $N = 256$. We next study the gain obtained by using the Alamouti scheme in CDMA with multipath channels. For this, we represent in the following the asymptotic BER for a half-loaded CDMA system obtained by using a RAKE receiver and a MMSE equalizer-based receiver. We compare the performances in the case where we use transmit diversity with the case where there is no transmit diversity [1]. The propagation channel is assumed to have three equal power paths spaced by twice the chip period. The results are shown in Figure 2. We

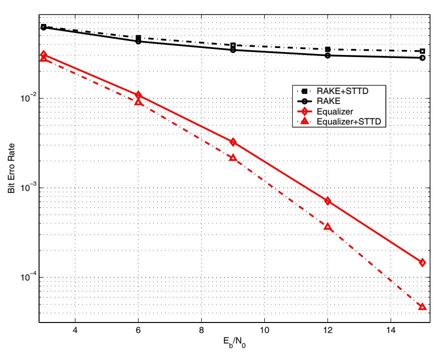

Fig. 2. The BER of the two receivers with and without transmit diversity for a three equal path channel,$\alpha = 0.5$

note that in this setting, the transmit diversity deteriorates the performances of the RAKE receiver because the CCI is greater than the diversity provided. In the case of the equalizer-based receiver, not only does it outperform the RAKE receiver in both cases, but it gives a better performance in the case of STTD because the CCI is partially cancelled out.
To have a clearer idea about the effect of multipath channels on the performance of STTD, we plot the BER obtained by using the two receivers (with and without diversity) as a function of the number of the channel paths. All the paths are assumed to have the same power and to be spaced by a chip period, $E_b/N_0 = 10dB$. The results are shown in Figure 3. The MMSE equalizer is known to outperform the RAKE receiver (without diversity). We note that the use of STTD deteriorates the BER performance when using a RAKE receiver, while it improves the BER performance when using a MMSE equalizer. This is a very important remark since it is another argument toward the use of equalizer-based receivers for third generation systems.

[1]For the comparison to be fair, the total transmitted power should be the same in both cases.

5. CONCLUSION.

In this paper, we have addressed the performance of Space Time Transmit Diversity in the downlink of W-CDMA over frequency-selective fading channels. We have derived asymptotic expressions of SINR provided by two kinds of receiver: the RAKE receiver and the chip-level MMSE equalizer-based receiver. Simultion results show that our asymptotic expressions allow to predict the performance of UMTS-FDD for $N = 256$. We have noticed that for some channels, the RAKE receiver deteriorates the BER performance when using STTD, while the equalizer based receiver still gives some improvement. This is another reason to use equalizer based receiver for 3G systems other than the fact that the MMSE equalizer outperforms the RAKE receiver when used without diversity.

6. REFERENCES

[1] S. Alamouti, "A simple transmit diversity technique for wireless communications ", *IEEE Journal on Selected Areas in communications* , vol. 16. No. 8, pp 1451-1458,Oct 1998.

[2] 3GPP TSG RAN, "Physical channels and mapping of transport channels onto physical channels (FDD) , Release 5" Tech. Spec. 3G TS 25.211, Version 5.5.0 *3G Partnership Project,*, Sep 2003.

[3] J.M. Chaufray, W. Hachem, Ph. Loubaton, "Asymptotic analysis of optimum and sub-optimum CDMA downlink MMSE receivers", *Accepted for Publication in IEEE Trans. on Information Theory*.

[4] T.P. Krauss, M. D. Zoltowski and G. Leus, "Simple MMSE equalizer for CDMA downlink to restore chip sequence: Comparison to Zero-Forcing and RAKE," *ICASSP, Istanbul, Turkey*, 5-9 june 2000.

[5] D.N.C Tse and S. Hanly, "Linear Multi-User Receiver: Effective Interference, Effective Bandwidth and User Capacity", *IEEE Trans. on Information Theory*, vol. 45, no. 2, pp. 641–657, February 1999.