EXPLOITING SPATIAL DIVERSITY IN COOPERATIVE COPPER UNITS

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ABSTRACT

This paper explains how spatial diversity can be exploited by Cooperative Copper Units (CCU) to increase the Shannon capacity in a crosstalk dominated environment. Contrary to the rank instability of frequency dispersive wireless channel, CCU always experience full rank MIMO channels. Nevertheless, the major gain inherent in CCU optimal joint processing does not stem from the channel full rank property but from the pair-to-pair alien crosstalk correlation. For that purpose, the probability distribution of the normalized correlation coefficient is modeled by a Beta probability distribution. Limited here to just two pairs, MIMO-CCU operates either in Full Multiplexing (FM) or Full Diversity (FD) modes, transmitting independent or identical data streams.

1. INTRODUCTION

Metallic access transmission systems can face various impairments inherent to high pair concentration. Interactions between lines, known as crosstalk, are the most disturbing interferences. Their mitigation remain an actual problematic for Digital Subscriber Lines (DSL) deployment. Dynamic Spectrum Management (DSM) [1] illustrates the benefits of transceiver coordination on this matter.

In this paper we introduce Cooperative Copper Units (CCU) and evaluates how it can make use of spatial diversity to increase the aggregate Shannon capacity in a crosstalk environment. We consider a system with N transceivers at both front-ends, modeled by the equation

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{r}. \qquad (1)$$

where $\mathbf{x}$ is the vector of transmit signals, $\mathbf{H}$ is the $N \times N$ complex narrowband channel matrix, $\mathbf{y}$ is the vector of receive signals, and $\mathbf{r}$ is a noise vector of i.i.d. complex Gaussian elements. Each transceiver transmits the same variance σ_x^2. From [2] and [3], assuming Gaussian transmit signaling, the mutual information of the vectors $\mathbf{x}$ and $\mathbf{y}$ yields

$$C = \log_2(det[\mathbf{I}_N + \sigma_x^2 \mathbf{H}^\dagger \mathbf{R}^{-1} \mathbf{H}]) \qquad (2)$$

where $\mathbf{R} = E[\mathbf{r}\mathbf{r}^\dagger]$, $\dagger$ is the hermitian operator, and $E[.]$ is the expectation operator. This system is displayed in Fig.1 for $N = 2$. Two categories of crosstalk noises are distinguished, both potential source of spacial diversity:

- ***the cooperative crosstalk***, or Far-End Crosstalk (FEXT) arises from inside of the CCU. It is characterized in Eq(2) by the off-diagonal entries of $\mathbf{H}$. Using FEXT loss statistical distribution model we come up with a close form of its average contribution to the capacity. In contrast to the MIMO wireless principle, we will establish that spatial crosspaths do not benefit to the MIMO capacity in the copper environment.
- ***the alien crosstalk*** radiates from the outside of the cooperative pairs and appears into $\mathbf{R}$. Copper capacity is increased when the crosstalk correlation experienced by the CCU paths of each impaired tone of the copper bandwidth is leveraged through joint processing at the receiver.

For motivation to look at their effects on the aggregate capacity, probability distribution model of both pair-to-pair correlation and FEXT loss are derived. For that purpose we use the free access crosstalk database collected by Kerpez and downloadable from [4]. Limited here to just two pairs, MIMO-CCU operates either in Full Multiplexing (FM) or Full Diversity (FD) modes, transmitting independent or identical data streams. Numerical estimates of the probability distribution of the processing gain exploiting noise correlation are proposed for both modes.

The rest of the paper is organized as follow. Section 2 characterizes the CCU channel and evaluates the contribution of crosspaths to the aggregate capacity. Section 3 appraises the spatial diversity due to alien crosstalk noise. For that purpose crosstalk pair-to-pair correlation probability distribution is modeled. Finally in section 4 we evaluate the processing gain that is a consequence of exploiting correlation in a two pair DSL receiver operating either in FM or FD.

2. CROSSPATH DIVERSITY

The coordination of several transceivers allows crosspath signal propagation. In the case of the CCU, cooperative crosstalk convey additional information if joint processing is considered at the receiver. This section evaluates this contribution.

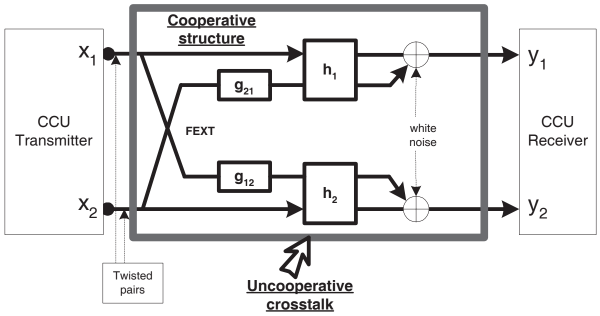


Fig. 1 . MIMO-CCU for $N = 2$

2.1. Definition of the channel

Given a CCU with 2 pairs, the MIMO channel is constituted of two direct paths and two crosspaths. They are displayed on the diagonal entries, and off-diagonal entries of **H**, respectively. Because of the principle of reciprocity [5], the FEXT coupling from pair 1 to pair 2, noted g_{12} is the same as the crosstalk coupling g_{21}. Therefore without loss of generality, we assume a single coupling g. On the other hand, twisted pairs with equal length and located in the same binder are likely to display similar transfer functions. Therefore we assume $h_1 = h_2 = h$. Consequently the MIMO channel, observed at tone k, is given by

$$\mathbf{H}_k = h_k \begin{pmatrix} 1 & g_k \\ g_k & 1 \end{pmatrix} \qquad (3)$$

Note that **H** is deterministic on its main diagonal, and stochastic everywhere else.

2.2. FEXT statistical distribution

The coupling statistical distribution of the pair-to-pair FEXT is modeled in [6] by $g_k(\chi, \psi) = \chi k \Delta e^{\jmath \psi_k}$ where Δ is the frequency width of a tone, ψ_k is a random phase with unknown distribution. χ is a random parameter that varies from pair-to-pair. The database [4] supplies with pair-to-pair FEXT loss measurements collected in a 25 pair binder constituted of 24 American Wire Gauge (AWG) twisted pairs. Measurements are performed at several distances and from 100kHz to 2Mhz with 401 points. Data processing reveal that χ follows a lognormal distribution. Therefore the expectation of χ^2 is the second raw moment of the lognormal distribution [7] defined as $e^{2(\mu_\chi + \sigma_\chi^2)}$ with μ_χ the mean and σ_χ the standard deviation of χ. Then the average FEXT loss is given by

$$E[|g_k(\psi)|^2] = (k\Delta)^2 e^{2(\mu_\chi + \sigma_\chi^2)} \qquad (4)$$

The lognormal parameter of χ, $\{\mu_\chi \,;\, \sigma_\chi\}$, were found to be equal to $\{-115; 4\}$dB. Although variations were observed from tone to tone, we assume a single set of parameters to stay simple.

2.3. FEXT capacity

We substitute Eq(3) into Eq(2), expand the determinant and generalize the result to Discrete MultiTone modulation (DMT) with $k \in [1:256]$ and $\Delta = 4.3125\text{kHz}$. Then the capacity can be broken into two pieces

$$C = 2C_{SISO}^{awgn} + C_{FEXT} \qquad (5)$$

where C_{SISO}^{awgn} is the white noise capacity of a single line, given by

$$C_{SISO}^{awgn} = \sum_k \log_2(1 + Snr_k^{awgn})$$

and the second term is the FEXT capacity of the CCU. Assuming $|g|^4 \ll 1$ it is approximated by

$$C_{FEXT} \simeq \sum_k \log_2(1 + \frac{2Snr_k^{awgn}|g_k|^2}{(1+Snr_k^{awgn})^2} \times \quad \ldots$$
$$\ldots \qquad (1 + Snr_k^{awgn}(1 - 2\cos^2(\psi_k)))) \qquad (6)$$

with

$$Snr_k^{awgn} = \frac{\sigma_{x_k}^2 |h_k|^2}{\sigma_{w_k}^2} \qquad (7)$$

the Signal-to-Noise Ratio (SNR) under white noise. We consider here only white noise with equal variance $\sigma_{w_k}^2$ on both pairs. Clearly the total capacity, for a given Snr_k^{awgn}, varies only according to C_{FEXT}. We assume $\psi_k = \pi/2$ for all k as this maximizes the FEXT path capacity. Therefore substituting Eq[4] into Eq[6], the average FEXT capacity becomes

$$\bar{C}_{FEXT}^{upper} \simeq \frac{2}{\ln 2} \sum_k (k\Delta)^2 e^{2(\mu_\chi + \sigma_\chi^2)} \qquad (8)$$

We assume here $Snr_k^{awgn} \gg 1$ (high regime SNR) and also $\ln(x+1) \simeq x$ for x very small. The contribution of $\bar{C}_{FEXT}^{upper}$ is numerically evaluated in the following realistic DSL environment: both the transmit signals and the white noise have flat Power Spectral Densities (PSDs) with $\sigma_x^2 = -40dBm/Hz$ and $\sigma_w^2 = -140dBm/Hz$, respectively. The loop attenuation $|h|^2$ is modeled from a 24 AWG twisted pair. From simulations, at 1km, the ratio of capacities $\frac{\bar{C}_{FEXT}^{upper}}{(2C_{SISO}^{awgn})}$ is approximatively equal to 10^{-5}. Then the FEXT paths have a negligible influence on the total capacity. This is a significant difference with regard to the MIMO wireless key principle, which uses specifically multi-paths to enhance capacity.

3. ALIEN CROSSTALK

In addition to white noise we consider alien crosstalk. Joint processing at the receiver benefits from pair-to-pair correlation characterized by the off-diagonal entries of **R**. Exploitation of pair-to-pair noise correlation in the copper environment was first proposed in [8] however with a too simplistic correlation distribution model.

3.1. Definition of the correlation

The alien crosstalk signal u_i measured on the pair i is the sum of the crosstalk noises that arise from N_D uncoordinated interfering pairs. $\sigma^2_{u_i}$ is the corresponding variance. The correlation between the signals u_i and u_j is computed by $\rho_{i,j} = E[u_i u_j^*] = \sigma_{u_i}\sigma_{u_j}\ell_{u,j}e^{\jmath\theta_{i,j}}$ where $\ell_{i,j}$ is the normalized correlation that takes values in the interval $[0:1]$, and $\theta_{i,j}$ is the phase with $0 \leq \theta_{i,j} \leq 2\pi$. Therefore in our case, the noise auto-covariance matrix at tone k shows the following form

$$\mathbf{R}_k = \begin{pmatrix} \sigma^2_{w_k} + \sigma^2_{u_{1,k}} & \sigma_{u_{1,k}}\sigma_{u_{2,k}}\ell_k e^{\jmath\theta_k} \\ \sigma_{u_{1,k}}\sigma_{u_{2,k}}\ell_k e^{-\jmath\theta_k} & \sigma^2_{w_k} + \sigma^2_{u_{2,k}} \end{pmatrix} \quad (9)$$

Here we simplify $\{\ell_{12}, \theta_{12}\}_k$ into $\{\ell, \theta\}_k$. Note that the case $N_D = 1$ leads to $\ell = 1$.

3.2. Modeling of the correlation

The distribution models of $\{\ell; \theta\}_k$ are derived from [4] for increasing N_D. Near-end crosstalk complex couplings are measured for each pair combination formed of pairs within a common 25 pair binder, over 401 frequencies from 5kHz up to 2MHz. To ensure the consistency of the statistics, the pairs of the CCU have to be reasonably collocated. Unfortunately, the cable geometry was not provided and we extrapolated a craft cable organization by using the following procedure: since the attenuation of a crosstalk signal is likely to be proportional to its distance between its origin and the pair victim, the average coupling magnitude (over the 401 frequencies) gives a *strong* sense of the nearness between pairs. Based on this assertion, 25 distinct sets of two pairs (each constitutes a CCU) are formed. For each set, the 8 strongest interferers are detected and sorted. The $C_8^{N_d}$ possible pair-to-pair correlation are computed. Thus, $\{700; 1400; 1750; 1400; 700\}$ observations were computed for $N_D = \{2; 3; 4; 5; 6\}$, respectively. For each N_D, the histogram of ℓ was fitted with accuracy by a beta distribution. For that purpose we used the *Matlab Statistical Toolbox*. The Beta probability density function [7] is given by

$$\mathcal{B}(x) = \xi\, x^{a-1}(1-x)^{b-1} \quad (10)$$

where ξ is the Beta function and $x \in [0:1]$.

The fitting is performed on the shaping parameters a_k and b_k with $\{a_k; b_k\} > 0$. We found that the frequency does not influence their values. Therefore, a unique set $\{a; b\}$ is defined per N_D. Estimates are displayed in Table 1. Subsequently, Table 2 extracts from the survival function[1] of $\mathcal{B}(\ell)$ the probability for ℓ to exceed $\{0.5; 0.9; 0.95\}$. As one could expect, the probability to have a "strong" correlation diminishes dramatically as N_D increases. This is the consequence of noise whitening due to the central limit theorem.

[1]Describes the probability that a variate X exceeds a number x: $S(x) = P(X > x)$

N_D	2	3	4	5	6
a	2	1.8	1.8	1.8	1.8
b	1.2	2.1	3	4	4.8

Table 1. Estimated of $\{a; b\}$ according to N_D

N_D	2	3	4	5	6
$\ell \geq 0.50$	68.2	43.5	26.7	16.2	9.8
$\ell \geq 0.90$	12	2	0.3	0	0
$\ell \geq 0.95$	5.3	0.5	0	0	0

Table 2. Probability (%) that ℓ exceeds $\{0.5; 0.9; 0.95\}$

Finally results show that likelihood of θ is uniform in the interval $[0:2\pi]$.

4. CAPACITY GAIN BY EXPLOITING PAIR-TO-PAIR CORRELATION FOR $N = 2$

Outcomes in sections 2 and 3 demonstrate the benefit of joint optimal processing in presence of pair-to-pair correlation. To highlight the gain, we derive the capacity of the CCU for two signalling scheme strategies: first when performing Full Multiplexing (FM) and second for Full Diversity (FD). The achievements are displays in terms of ratio between the SNR of the CCU considered (FM or FD), and the SNR of a scheme that does not perform joint processing at the receiver (known as Bonding, with each pair engineered as if they were in isolation). We implicitly assume that the transmitter knows **R**. The developments are done for a single tone.

4.1. Full multiplexing signaling scheme

We consider the case where both pairs convey independent data with $\mathbf{H} = h\mathbf{I}_2$. By using Eq(2) and Eq(9), the processing gain of the FM-CCU over the Bonding ($G_{FM/B}$) scheme is given by

$$G_{FM/B} = \frac{1 + \xi_1 + \xi_2 + \xi_1\xi_2}{1 + \xi_1 + \xi_2 + \xi_1\xi_2\,(1 - \ell^2)} \quad (11)$$

where $\xi_1 = \sigma^2_{u_1}/\sigma^2_w$ and $\xi_2 = \sigma^2_{u_2}/\sigma^2_w$ are the crosstalk to white noise ratio on pair one and pair two, respectively. Assuming the same $\xi = 40dB$ on both pair the related survival function, generated from Monte Carlo simulations, is displayed on Fig2. Random values of ℓ are generated from the beta distribution as described in subsection 3.2. Curves are displayed for 2 to 6 interferes. For $\mathrm{N_D} = 2$, there is a better than 40% chance that $G_{FM/B} > 3\mathrm{dB}$. When $\mathrm{N_D} = 4$, the gain drops to 1dB for the same probability.

4.2. Full diversity signaling scheme

We perform Full Diversity (FD) as transmit signalling strategy. Both pairs convey the same information up to a phase

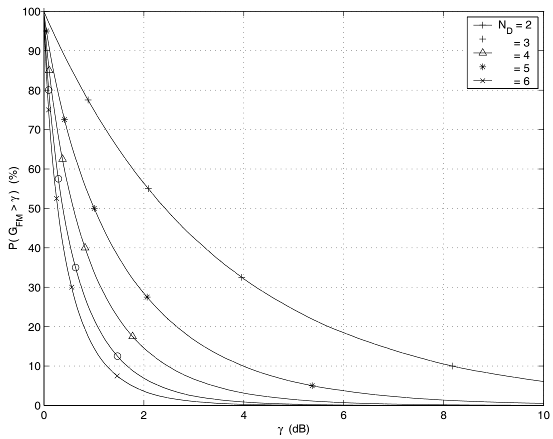


Fig. 2 . Probability of $G_{FM/B}$ to be beyond γ

π. To prevent the system to degenerate, the transmit signal is precoded, such as $\mathbf{H} = h[1\ 1]^T$ if $\cos(\theta) \leq 0$, and $h[1\ \text{-}1]^T$ otherwise. Similarly to Eq[11] we display the gain of the FD-CCU over the bonding scheme. This yields

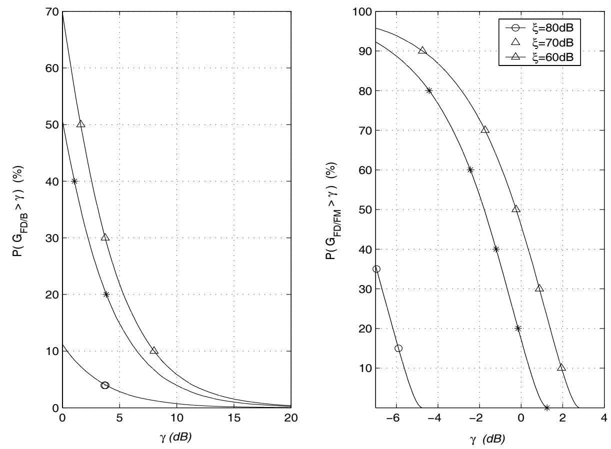
$$G_{FD/B} = G_{FM/B} \underbrace{\left(\frac{2 + \xi_1 + \xi_2 + 2\sqrt{\xi_1 \xi_2}\ell|\cos(\theta)|}{2 + \xi_1 + \xi_2 + Snr^{awgn}} \right)}_{G_{FD/FM}} \quad (12)$$

which interestingly is function of $G_{FM/B}$. $G_{FD/FM}$ exhibits the gain of the FD-CCU over the FM-CCU. It comes easily that $G_{FD/B} \geq G_{FM/B}$ only for $2\ell|\cos(\theta)| \geq \frac{Snr^{awgn}}{\sqrt{\xi_1\xi_2}}$. Consequently the FD-CCU benefits from strong crosstalk environment compared with Snr^{awgn}, as is it showed in Fig.3 (***a***). In the case of $N_D = 2$ and $Snr^{awgn} = 70$dB, the probability that $G_{FD/B}$ exceeds 3dB when $\xi = \{80; 70; 60\}$dB is $\{30; 20; 4\}\%$, respectively. Again the same ξ on both pairs is supposed. Random observations of θ are uniformly generated from within $[0 : 2\pi]$. On the other hand, the subplot (***b***) shows that the FD-CCU has 20% chance to outperforms the FM-CCU when $\xi = 70$dB. In the best case, $G_{FD/FM}$ tends asymptotically towards 2, i.e. 3dB.

Generalization to DMT allows to exploit both space and frequency diversity. Therefore the overall gain becomes significant. Future works will address a Space-Frequency modulation that combines the selection of the best spatial signalling strategy with the multicarrier modulation.

5. CONCLUSION

This paper introduces the Copper Cooperative Units (CCU) that adapt the concept of MIMO organization to the copper medium. It is shown that MIMO-CCU do not benefit from multipath spatial diversity but rather from pair-to-pair uncoordinated crosstalk correlation. For that purpose the correlation magnitude is modeled by a Beta distribution. Limited here to just two pairs, MIMO-CCU operates either in Full



(a) (b)

Fig. 3 . Probability of $G_{FD/B}$ (case **a**) and $G_{FD/FM}$ (case **b**) to be beyond γ

Multiplexing (FM) or Full Diversity (FD) modes, transmitting independent or identical data streams. In some environments, simulations show a probability of 40% that the gain of the FM-CCU exceeds by 3dB the SNR of two lines engineered as if they were uncoordinated. On the other hand, FD-CCU has a probability of 20% to display better performances than the FM-CCU in a environment with strong crosstalk.

6. REFERENCES

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