AN EFFICIENT IMPLEMENTATION FOR MMSE BASED MIMO TIME DOMAIN EQUALIZER

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ABSTRACT

A time domain equalizer is a finite impulse response filter that shortens the channel impulse response to mitigate inter-symbol interference (ISI). Al-Dhahir and Cioffi proposed a design criteria for single input single output TEQ based on designing a MMSE decision feedback equalizer. They also extended this method to MIMO channels. In this paper, we propose a simple implementation of time domain equalizer for MIMO channels also based on the MMSE criteria. The solution is simplified compared to the above solution by eliminating the cross channel linear equalizers. This results in a set of M independent equalizers, each designed to meet a multi-objective channel shortening MMSE criterion. The new method is simple and provides good results for the MIMO problem. Finally we demonstrate the efficiency of the proposed approach compared to the Al-Dhahir's on measured channels, where the NEXT channels are simultaneously shortened with the direct channels. These are the *first* published results demonstrating MIMO-TEQ design on real life measured DSL channels.
keywords: Time domain equalization, Multicarrier, MIMO, ADSL, VDSL.

1. INTRODUCTION

Multicarrier modulations are used for many types of communication systems, such as DSL, wireless LAN, digital video and audio broadcast. In DSL applications the multicarrier modulation is usually called discrete multi-tone (DMT). DMT systems use a cyclic prefix (and in VDSL cyclic suffix as well) to transform the channel equalization problem into a set of one tap equalizers at each tone [1] in the single input single output case. In the multi-input multi-output case the channel is reduced to a single linear matrix channel at each frequency. In many cases the effective channel length is not shorter than the length of the cyclic prefix. When the assumption on the length of the prefix is violated the receiver is subject to very substantial interblock interference. Therefore the receiver is required to perform channel shortening to limit the effective length of the channel. This shortening is achieved by the addition of a relatively short equalizer in the time domain often called time domain equalizer (TEQ). TEQ is often required in DSL (ADSL/VDSL) systems that were the main motivation for the development of TEQ design techniques.

Several methods were proposed for the design of TEQs. Minimum mean-squared error (MMSE) based impulse response shortening was originally proposed by Falconer and Magee [2] for complexity reduction in maximum-likelihood receivers. It has been extended to time domain equalization in multicarrier systems [1], [3]. The basic idea is to design an MMSE optimal decision feedback equalizer for the channel under a length constrain on the feedback section. The linear part serves as the TEQ and the response of the feedback (which has predefined length shorter than the prefix length) is equalized in the frequency domain. An important parameter of the method is a delay in the feedback section that strongly affects the output MMSE. Other methods for single input single output TEQ include the maximum shortening signal-to-noise ratio (MSSNR) [4] and the minimum intersymbol interference (min-ISI) method [5]. An alternative approach is the use of frequency domain per tone equalizers [6] where the channel shortening is performed in the frequency domain. For a good overview of the various approaches for TEQ as well as other issues related to DSL the reader is referred to [7].

In recent years there is an increased interest in multiple input multiple output (MIMO) DSL system that enable partial or full binder coordination. For a good overview of the subject see [8]. In a MIMO system multiple channels need to be shortened simultaneously. In digital subscriber lines (DSL), joint channel shortening can be combined with multiuser detection and precoding to mitigate crosstalk. MIMO channel shortening using a generalized MIMO-MMSE-DFE structure has been studied by Al-Dhahir [9] and Wubben and Kammeyer [10]. These papers extended the single input single output TEQ to a full MIMO MMSE-TEQ. This has a large computational complexity due to the full matrix of feedforward filters in the DFE design. The complexity is approximately M^2 times the complexity of a single input single output TEQ.

In this paper, we propose a new implementation of MMSE-DFE based channel-shortening for MIMO ISI channels. Unlike the above methods, we find that M inputs and M outputs channel shortening can be decomposed into M parallel channel shortening systems each having M inputs and single output. Finally we use measured channels (including phase and amplitude of NEXT) to provide a *first experimental verification* of multichannel time domain equalization on DSL channels. This is an important indication for the viability of multichannel crosstalk cancellation in DSL and other multicarrier systems.

The structure of the paper is as follows. In Section 2, we present the general MIMO model and the mathematical formulation of MIMO channel shortening problem. We also describe the general MIMO TEQ design based on MMSE-DFE. The new simplified procedure is derived in Section 3. Simulation results based on measured DSL channels are presented in Section 4. We end up with some conclusions.

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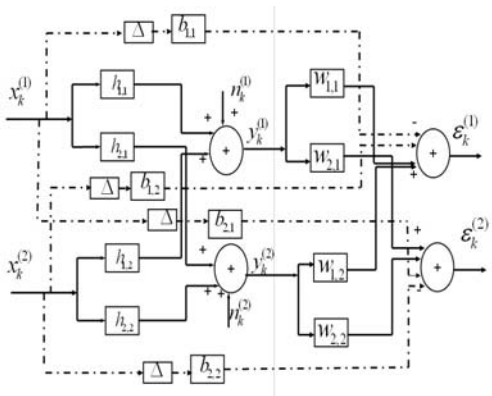

Figure 1: Two input and two output channels with conventional channel shortening

2. MIMO MODEL AND PROBLEM FORMULATION

In this section we present the basic MIMO model, formulate the MIMO TEQ design problem and describe the general MMSE-DFE based TEQ design.

Consider a communication system with M inputs and M outputs. The output of the i^{th} channel $(i = 1, \ldots, M)$ is given by

$$y_k^{(i)} = h_{i,i} * x_k^{(i)} + \sum_{l \neq i} h_{i,l} * x_k^{(l)} + n_k^{(i)} \tag{1}$$

where $y_k^{(i)}$ is the i^{th} channel output, $h_{i,l}$ is the channel inpulse response between l^{th} input and i^{th} output. $x_k^{(l)}$ is the input sequence at l^{th} transmitter. $n_k^{(i)}$ is the noise at i^{th} output.

We give a brief introduction to MIMO channel shortening. A general MIMO time domain equaliztion system is composed of an $M \times M$ matrix of feedforward filters. The special case of two input and two output system, is shown in Figure 1. The matrix of feedback filters is only used during training period, in order to enforce an MMSE optimal MIMO-DFE structure. In steady state these filters are removed and the frequency domain single tap matrix equalizers are used to compensate for this response as well as to remove the crosstalk.

Following [9], the TEQ $w_{i,j}$ and target impulse response (TIR) $b_{i,j}$ can be obtained by solving the following minimum mean square error (MMSE) with some constraint on b to avoid the trival solution:

$$\min_{\Delta, w_{i,j}, b_{i,j}} J\left(\Delta, w_{i,j}, b_{i,j}\right) \tag{2}$$

where

$$J\left(\Delta, w_{i,j}, b_{i,j}\right) = E\{\varepsilon_1^2(k) + \varepsilon_2^2(k)\} \tag{3}$$

$$\begin{aligned} \varepsilon_1(k) \quad &= \quad w_{1,1} * y_k^{(1)} + w_{1,2} * y_k^{(2)} \\ & \qquad - b_{1,1} * x_{k-\Delta}^{(1)} - b_{1,2} * x_{k-\Delta}^{(2)}, \end{aligned} \tag{4}$$

$$\begin{aligned} \varepsilon_2(k) \quad &= \quad w_{2,1} * y_k^{(1)} + w_{2,2} * y_k^{(2)} \\ & \qquad - b_{2,1} * x_{k-\Delta}^{(1)} - b_{2,2} * x_{k-\Delta}^{(2)}, \end{aligned} \tag{5}$$

$$y_k^{(1)} = h_{1,1} * x_k^{(1)} + h_{1,2} * x_k^{(2)} + n_k^{(1)} \tag{6}$$

$$y_k^{(2)} = h_{2,1} * x_k^{(1)} + h_{2,2} * x_k^{(2)} + n_k^{(2)} \tag{7}$$

Solving optimization problem (2) has a high computational complexity in MIMO case where M^2 filters are implemented. This complexity does not diminish during steady state transmission since the matrix of feedforward filters continues to operate.

3. SIMPLIFIED CHANNEL SHORTENING

In this section, we propose a new simplified method for MIMO channel shortening. The basic idea is to remove the cross filters $\langle w_{i,j} : i \neq j \rangle$. This is equivalent to imposing a diagonal structure on the feedforward matrix filters $\mathbf{w}_1, \ldots, \mathbf{w}_{N_w}$ of [9], i.e., we require that all $\mathbf{w}_k$ in the MIMO DFE will be diagonal. This constrain is not imposed naturally in the MMSE-MIMO-DFE design. The equivalent structure for the 2 × 2 system with diagonal constrain is depicted in Figure 2.

Once this constrain has been enforced, we can see that the MSE function is given by

$$J\left(\Delta, w_{i,j}, b_{i,j}\right) = E\{\varepsilon_1^2(k) + \varepsilon_2^2(k)\} = J_1 + J_2 \tag{8}$$

where

$$\varepsilon_1(k) = w_1 * y_k^{(1)} - b_{1,1} * x_{k-\Delta}^{(1)} - b_{1,2} * x_{k-\Delta}^{(2)} \tag{9}$$

$$\varepsilon_2(k) = w_2 * y_k^{(2)} - b_{2,1} * x_{k-\Delta}^{(1)} - b_{2,2} * x_{k-\Delta}^{(2)} \tag{10}$$

The MIMO TEQ and TIR are obtained by minimizing MMSE

$$\begin{aligned} &\min_{\Delta} \min_{w_1, w_2, b_{1,1}, b_{1,2}, b_{2,1}, b_{2,2}} J\left(\Delta, \mathbf{w}_i, b_{i,j}\right) \\ &= \min_{\Delta} \min_{w_1, w_2, b_{1,1}, b_{1,2}, b_{2,1}, b_{2,2}} J_1 + J_2 \end{aligned} \tag{11}$$

For fixed Δ, $\varepsilon_1(k)$ is only dependent on $w_1, b_{1,1}, b_{1,2}$ and $\varepsilon_2(k)$ only on $w_2, b_{2,1}, b_{2,2}$. Therefore, for any given Δ minimizing J is equaivalent to mininizing both J_1 and J_2 independently.

Based on this analysis, we see that the M inputs and M outputs time-domain equalizer (TEQ) can be decomposed into M parallel TEQs with M inputs and single output (This is also valid for the general M inputs and N outputs case). This architecture is shown in Figure 3. Let $w_i, b_{i,l}$ be the $N_{w_i} + 1$ and $N_{b_l} + 1$ length filters

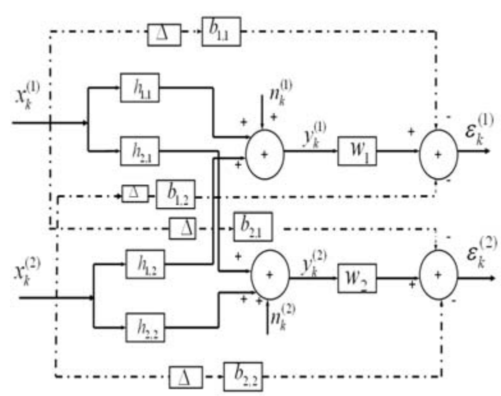

Figure 2: Two inputs and two outputs channels with simplified channel shortening

respectively, then from the i^{th} channel output given in (1), we have

$$\varepsilon_i(k) = w_i * y_k^{(i)} - \sum_{l=1}^{M} b_{i,l} * x_{k-\Delta}^{(l)} \tag{12}$$

or more explicitly

$$\varepsilon_i(k) = \sum_{p=0}^{N_{w_i}} w_i(p) y_{k-p}^{(i)} - \sum_{l=1}^{M} \sum_{p=0}^{N_{b_l}} b_{i,l}(p) x_{k-\Delta-p}^{(l)} \tag{13}$$

In vector form we can rewrite (13):

$$\begin{aligned} \varepsilon_i \;\; = \;\; & \mathbf{w}_{(i)}^H \mathbf{y}_k^{(i)} - \textstyle\sum_{l=1}^{M} \mathbf{b}_{i,l}{}^H \mathbf{x}_{k-\Delta}^{(l)} \\ & = \mathbf{w}_{(i)}^H \mathbf{y}_k^{(i)} - \mathbf{b}_{(i)}^H \mathbf{x}_{k-\Delta} \end{aligned} \tag{14}$$

where

$$\mathbf{w}_{(i)} = \left(w_i(0), w_i(1), \ldots, w_i(N_{w_i})\right)^T, \tag{15}$$

$$\mathbf{y}_k^{(i)} = \left(y_k^{(i)}, y_{k-1}^{(i)}, \ldots, y_{(k-N_{w_i})}^{(i)}\right)^T, \tag{16}$$

$$\mathbf{b}_{i,l} = \left(b_{i,l}(0), b_{i,l}(1), \ldots, b_{i,l}(N_{b_l})\right)^T, \tag{17}$$

$$\mathbf{x}_{k-\Delta}^{(l)} = \left(x_{k-\Delta}^{(l)}, x_{k-\Delta-1}^{(l)}, \ldots, x_{k-\Delta-N_{b_l}}^{(l)}\right)^T, \tag{18}$$

and

$$\mathbf{b}_{(i)} = \left(\mathbf{b}_{i,1}^T, \ldots, \mathbf{b}_{i,M}^T\right)^T, \tag{19}$$

$$\mathbf{x}_{k-\Delta} = \left(\mathbf{x}_{k-\Delta}^{(1)T}, \ldots, \mathbf{x}_{k-\Delta}^{(M)T}\right)^T, \tag{20}$$

Based on these notations, we have

$$\begin{aligned} J_i &= E\{(\mathbf{w}_{(i)}^H \mathbf{y}_k^{(i)} - \mathbf{b}_{(i)}^H \mathbf{x}_{k-\Delta})^2\} \\ &= \mathbf{w}_{(i)}^H R_{y^{(i)}} \mathbf{w}_{(i)} - \mathbf{w}_{(i)}^H R_{y^{(i)}x} \mathbf{b}_{(i)} - \\ & \quad \mathbf{b}_{(i)}^H R_{xy^{(i)}} \mathbf{w}_{(i)} + \mathbf{b}_{(i)}^H R_x \mathbf{b}_{(i)} \end{aligned} \tag{21}$$

where $R_{y^{(i)}}$ is the i^{th} channel output covariance matrix, R_x is the input covariance matrix, $R_{xy^{(i)}}$ is the covariance between the input and i^{th} channel output data, and $R_{y^{(i)}x}$ is covariance between i^{th} channel output and the input data and equals the conjugate of $R_{xy^{(i)}}$.

To obtain the MMSE solution in this case, we first differentiate with respect to $\mathbf{w}_{(i)}^H$ obtaining

$$\frac{\partial J_i}{\partial \mathbf{w}_{(i)}^H} = 2R_{y^{(i)}} \mathbf{w}_{(i)} - 2R_{y^{(i)}x} \mathbf{b}_{(i)} = 0 \tag{22}$$

Hence the optimal $\mathbf{w}_{(i)}$ is given by

$$\mathbf{w}_{i,opt} = R_{y^{(i)}}^{-1} R_{y^{(i)}x} \mathbf{b}_{(i)} \tag{23}$$

Substituting this back to (21) yields

$$\begin{aligned} J_i &= \mathbf{b}_{(i)}^H (R_x - R_{y^{(i)}x}^H R_{y^{(i)}}^{-1} R_{y^{(i)}x}) \mathbf{b}_{(i)} \\ &= \mathbf{b}_{(i)}^H R_{x|y^{(i)}}^{\perp} \mathbf{b}_{(i)} \end{aligned} \tag{24}$$

To avoid trivial null solution, we impose a unit norm constraint $\mathbf{b}_{(i)}^H \mathbf{b}_{(i)} = 1$. Similiar to the analysis in [3], we know that $\mathbf{b}_{i,opt}$ is the eigenvector of the matrix $R_{x|y^{(i)}}^{\perp}$ corresponding to the minimum eigenvalue. The optimum equalizer is determined from (23)

$$w_{i,opt} = R_{y^{(i)}}^{-1} R_{y^{(i)}x} \mathbf{b}_{i,opt} \tag{25}$$

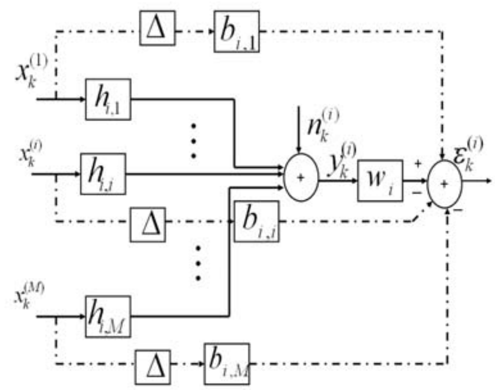

Figure 3: M inputs and one output channel shortening

4. COMPUTER SIMULATIONS

This section presents simulations of the proposed algorithm. We consider a 3 input and 3 output systems. The insertion loss and NEXT crosstalk channels are formed from the measured data obtained by direct measurement by France Telecom over 600 m. In the simulation the input power was 20 dB. The frequency band is 1.1 MHz, width of each frequency bin was 4.3125 KHz. The signal was oversampled by a factor of 2, so FFT size was 512. CP length for each transmitted signal was 32, so $N_b = 32$. The TEQ length was set to $N_w = 16$.

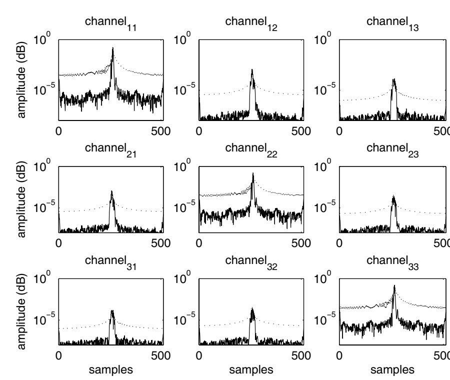

Figure 4: 3 × 3 channels. Dashed line - Original channels and crosstalk channels. Solid line - shortened channel based on our method.

The additive white Gaussian noise had power spectral density of -140 dBm/Hz. We also assumed uncorrelated signals at the various transmitters and that the noise was uncorrelated to the signals. Fig. 4 and Fig. 5 respectively show the original and shortenned channels of the 3 × 3 channel systems based on our method and Al-Dhahir's full TEQ method. Fig. 6 shows the original insertion loss and NEXT in the frequency domain. Table 1 and Table 2 respectively compare the norm of the TIR channels inside the N_b windows over the norm of the residual channel outside the win-

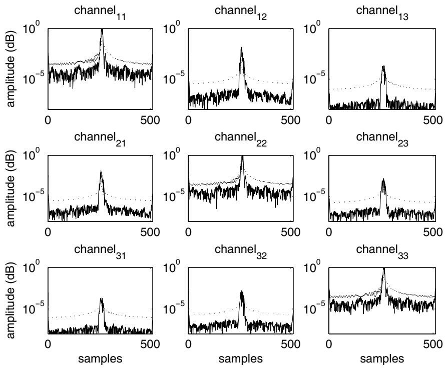

Figure 5: 3 × 3 channels. Dashed line - Original channels and crosstalk channels. Solid line - shortened channel based on Al-Dhahir's full TEQ method.

channel	1	2	3
1	55.80	55.86	50.37
2	55.84	55.68	49.18
3	50.46	49.20	55.37

Table 1: The norm of the TIR channels inside the N_b windows divided by the norm of the TIR outside the windows given by $20\log_{10}\frac{\|\mathbf{w}^H\mathbf{h}_{ij}\|_{in}}{\|\mathbf{w}^H\mathbf{h}_{ij}\|_{out}}$ for our propose method

dow for each of the 9 direct or crosstalk channels between our method and Al-Dhahir's method. From these results, we see that the performance loss of our method is not apparent as compared to Al-Dhahir's method, but with very low computational complexity.

5. CONCLUSIONS

In this paper we discussed the problem of MIMO channel shortening and proposed an efficient algorithm based on MMSE. The simplified approach is based on a set of M MISO time domain equalizers, each jointly shortening M cross channels. This yields substantial computational saving compared to the full MIMO-DFE. Computer simulation results verified the effeciency of the proposed method.

channel	1	2	3
1	56.80	56.19	50.71
2	56.19	56.71	49.26
3	50.70	49.25	56.52

Table 2: The norm of the TIR channels inside the N_b windows divided by the norm of the TIR outside the windows given by $20\log_{10}\frac{\|\mathbf{w}^H\mathbf{h}_{ij}\|_{in}}{\|\mathbf{w}^H\mathbf{h}_{ij}\|_{out}}$ for Al-Dhahir's full TEQ method

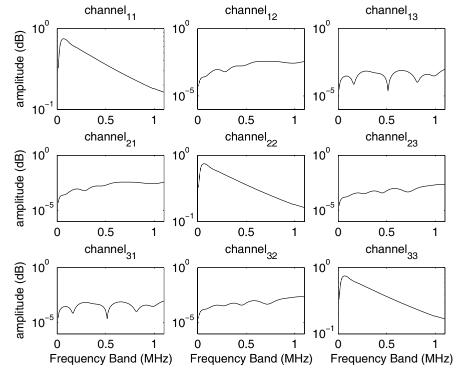

Figure 6: 3 × 3 channels in the Frequency domain for 600m.

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