ON THE PERFORMANCE OF CERTAIN FIXED-COMPLEXITY MULTIUSER DETECTORS IN FEXT-LIMITED VECTORED DSL SYSTEMS

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ABSTRACT

Multiple-input multiple-output (MIMO) communication has recently attracted considerable interest for transmission over multiple twisted copper pairs in a common binder. The motivation is to overcome the limitations imposed by crosstalk, and thus boost capacity. Far-end crosstalk (FEXT) is the dominant impairment in frequency-division duplex (FDD) digital subscriber line (DSL) systems. While the per-tone MIMO channel matrix is diagonally dominated in this context, noise is colored due to alien FEXT, which is particularly pronounced for short DSL loops - such as those encountered in fiber to the curb (FTTC) or fiber to the basement (FTTB) architectures. This calls for effective multiuser detection strategies at the receiver. We test three candidate detection algorithms (probabilistic data association, sphere decoding, and a recently proposed hybrid thereof) using measured channel data for 300 meter cable, and a fixed upper bound on vector decoding complexity. The results indicate that, when half of the loops in the binder are coordinated, contrary to the case of Rayleigh fading wireless channels, the sphere decoder provides the best performance even when a pragmatic complexity restriction is employed.

1. INTRODUCTION

Multiple-input multiple-output (MIMO) twisted-pair communication was first proposed in the early 90's [8], for two-pair coordinated transmission. More recently, so-called *vectored* transmission has attracted attention [3] due to its potential of roughly doubling the per-line capacity. When frequency-division duplex (FDD) is employed for upstream - downstream isolation, far-end crosstalk (FEXT) is the main performance-limiting factor in twisted-pair transmission, and MIMO techniques help in mitigating FEXT interference.

In parallel, there has been growing interest in fiber to the curb (FTTC) or fiber to the basement(FTTB) architectures as promising low-overhead solutions for broadband network access to businesses and residential premises. These architectures rely on transmission over twisted copper pairs for the last few hundred meters to the customer premises. Self-FEXT can be (at least approximately) pre-equalized in a MIMO subsystem, making the effective per-tone direct channel matrix approximately diagonal. However, alien FEXT from the remaining non-coordinated loops is spatially colored. This necessitates whitening, which destroys the diagonality of the direct channel matrix, and thus calls for multiuser detection.

Sphere Decoding (SD) [15], Probabilistic Data Association (PDA) [9, 11], and Semi-Definite Relaxation (SDR) [10] are three MIMO detectors that can provide near-optimal performance at relatively low complexity in certain scenarios. Among them, SD appears to be prevalent in the recent literature. Numerous variants and improvements of SD have recently been developed, e.g., [2, 1, 16, 17], incorporating more sophisticated schemes for increasing the associated search radius and organizing the computations in a more efficient manner. A drawback of the SD family of detectors is that, for close-to-ML performance, complexity remains high in the low Signal-to-Noise Ratio (SNR) regime, or when the number of symbols to be jointly detected is large [5, 6].

The PDA is a simpler detection method, which however generally provides worse performance than SD. SD, PDA, SDR, and several other algorithms have recently been compared in the context of CDMA multiuser detection [4].

A hybrid PDA-SD algorithm has been recently proposed in [7]. The key idea of [7] is to reduce the dimension of the problem solved via SD by first running a single stage of the PDA to fix symbols which can be decoded with high reliability. After cancelling the effect of those symbols, a reduced-dimensionality problem is passed to SD for decoding. Simulations under a multiple antenna Rayleigh fading scenario show that this two-step algorithm attains a better performance-complexity trade-off than either of its constituent components.

In this paper we test the PDA, SD, and hybrid PDA-SD using measured DSL channel data for 300 meter cable, and a fixed upper bound on vector decoding complexity. Using a fixed upper bound on complexity is different from the comparisons in [7], and is dictated by modem implementation considerations. DSL modems, particularly those to be employed over short loops in the context of FTTC/FTTB architectures, operate at much higher rates than wireless modems.

Vectoring generally yields a significant capacity benefit only when at least half of the active loops in the binder are vectored [3, 13]. We therefore consider a scenario wherein half of the loops are vectored, while the other half act as interferers. The results indicate that SD offers the best performance in this setting, despite the severe complexity restriction. The hybrid PDA-SD algorithm is a relatively close second, while the PDA has about an order of magnitude worse performance on more than half of the tones. The results are different from those obtained in the case of Rayleigh flat fading wireless MIMO channels, for a number of reasons which

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are thoroughly discussed in the sequel.

The rest of this paper is structured as follows. In Section 2 we lay out the per-tone MIMO system model, and its reduction to real form. The hybrid PDA-SD algorithm is briefly reviewed in Section 3. Our main results are presented in section 4, and conclusions are drawn in Section 5.

2. SYSTEM MODEL

We consider a vectored transmission subsystem comprising L coordinated loops. Under a FDD band plan, FEXT is the dominant impairment. Let L_{AF} denote the number of alien FEXT interferers from the remaining loops in the binder. For multicarrier modulation, we may focus on the following discrete - time baseband - equivalent model on a per-tone basis:

$$\widetilde{\mathbf{y}} = \widetilde{\mathbf{Q}}_f \widetilde{\mathbf{s}} + \widetilde{\mathbf{v}}_f, \tag{1}$$

where $\widetilde{\mathbf{y}} = [\widetilde{y}_1, \widetilde{y}_2, \dots, \widetilde{y}_L]^T, \widetilde{\mathbf{s}} = [\widetilde{s}_1, \widetilde{s}_2, \dots, \widetilde{s}_L]^T$ are the receive and the transmit vector, respectively, $\widetilde{\mathbf{Q}}_f$ is a complex $L \times L$ direct channel matrix with entries given by the insertion loss and self-FEXT crosstalk channel frequency responses at the f-th subcarrier, and $\widetilde{\mathbf{v}}_f$ models the frequency-dependent alien FEXT interference plus noise. The covariance of $\widetilde{\mathbf{v}}_f$ is given by $p_{AF}\widetilde{\mathbf{A}}_f\widetilde{\mathbf{A}}_f^H + \sigma_N^2\mathbf{I}$, where $\widetilde{\mathbf{A}}_f$ is the $L \times L_{AF}$ complex coupling matrix from the L_{AF} alien FEXT interferers to the L vectored loops at frequency f, p_{AF} is the transmit power of alien interferers relative to the coordinated loops, and σ_N^2 is the power of the complex circular white Gaussian noise component. The overall interference is modelled as approximately Gaussian, with the given covariance matrix. The symbols in $\widetilde{\mathbf{s}}$ are assumed to be drawn from a QAM constellation.

In order to transform the above model to a real-valued one, define (dropping the explicit dependence on f for brevity):

$$\mathbf{s} := [\Re(\widetilde{\mathbf{s}}^T) \quad \Im(\widetilde{\mathbf{s}}^T)]^T, \tag{2}$$

$$\mathbf{y} := [\Re\{\widetilde{\mathbf{y}}^T\} \quad \Im\{\widetilde{\mathbf{y}}^T\}]^T, \tag{3}$$

$$\mathbf{Q} := \begin{bmatrix} \Re\{\widetilde{\mathbf{Q}}\} & -\Im\{\widetilde{\mathbf{Q}}\}, \\ \Im\{\widetilde{\mathbf{Q}}\} & \Re\{\widetilde{\mathbf{Q}}\}, \end{bmatrix} \tag{4}$$

$$\mathbf{v} := [\Re\{\widetilde{\mathbf{v}}^T\} \quad \Im\{\widetilde{\mathbf{v}}^T\}]^T, \tag{5}$$

where $\Re$, $\Im$ denote real and imaginary part, respectively. Using the above vectors and matrices we obtain the real-valued vector equation

$$\mathbf{y} = \mathbf{Q}\mathbf{s} + \mathbf{v}. \tag{6}$$

The noise covariance matrix in the the above real system is $\mathbf{R}_{vv} = E\{\mathbf{v}\mathbf{v}^T\} = \frac{p_{AF}}{2}\mathbf{A}\mathbf{A}^T + \frac{\sigma_N^2}{2}\mathbf{I}$, where $\mathbf{A}$ is derived from $\widetilde{\mathbf{A}}$ analogously to the derivation of $\mathbf{Q}$ from $\widetilde{\mathbf{Q}}$.

Computing the Cholesky decomposition $\mathbf{R}_{vv} = \mathbf{L}^T\mathbf{L}$ and multiplying (6) from the left by $\mathbf{L}^{-T}$ we obtain the equivalent white noise system

$$\mathbf{r} = \mathbf{L}^{-T}\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n}, \tag{7}$$

where $\mathbf{H} = \mathbf{L}^{-T}\mathbf{Q}$ and $\mathbf{n} = \mathbf{L}^{-T}\mathbf{v}$ is a noise vector with covariance matrix $\mathbf{I}$.

3. THE HYBRID ALGORITHM

The hybrid algorithm consists of the following steps. As in [11], we pre-multiply (7) with $\mathbf{H}^T$, which yields

$$\mathbf{z} = \mathbf{H}^T\mathbf{r} = \mathbf{G}\mathbf{s} + \mathbf{v}, \tag{8}$$

where $\mathbf{G} := \mathbf{H}^T\mathbf{H}$, and $\mathbf{v} = \mathbf{H}^T\mathbf{n}$ is a noise vector with covariance matrix $\mathbf{G}$.

We then apply one stage of the PDA detector (steps 1-5 in [9]) to the system in (8), and thus obtain a vector $\mathbf{p}$ which contains the associated probabilities for the elements of $\mathbf{s}$. Let D denote the subset of bits that satisfy

$$\mathbf{p}(i) \in [0, \tau] \cup [1 - \tau], \tag{9}$$

with τ to be suitably chosen. $\overline{D}$ will henceforth denote the complement of D. We then make hard decisions for the bits in D; that is, set $\widehat{s}_i = sign(\mathbf{p}(i) - 0.5), \forall i \in D$, and collect these decisions in a vector $\widehat{\mathbf{s}}_D$. Now expand (6) as

$$\mathbf{r} = [\mathbf{H}_D \;\; \mathbf{H}_{\overline{D}}] \begin{bmatrix} \mathbf{s}_D \\ \mathbf{s}_{\overline{D}} \end{bmatrix} + \mathbf{n},$$

with obvious notation. Assuming perfect decisions for the bits in D (that is, $\widehat{\mathbf{s}}_D = \mathbf{s}_D$), the residual subsystem after cancellation is

$$\mathbf{y}_{\overline{D}} := \mathbf{r} - \mathbf{H}_D\mathbf{s}_D = \mathbf{H}_{\overline{D}}\mathbf{s}_{\overline{D}} + \mathbf{n}.$$

After compacting,

$$\mathbf{y}_c := \mathbf{H}_{\overline{D}}^T\mathbf{y}_{\overline{D}} = \mathbf{H}_{\overline{D}}^T\mathbf{H}_{\overline{D}}\mathbf{s}_{\overline{D}} + \mathbf{H}_{\overline{D}}^T\mathbf{n} = \mathbf{G}_{\overline{DD}}\mathbf{s}_{\overline{D}} + \mathbf{v}_{\overline{D}},$$

the noise vector $\mathbf{v}_{\overline{D}}$ is colored Gaussian with zero mean and covariance matrix $\mathbf{G}_{\overline{DD}}$. Introduce the Cholesky factorization

$$\mathbf{G}_{\overline{DD}} = \mathbf{L}_{\overline{DD}}^T\mathbf{L}_{\overline{DD}}, \tag{10}$$

and pre-multiply the system with $\mathbf{L}_{\overline{DD}}^{-T}$ to obtain

$$\mathbf{x} := \mathbf{L}_{\overline{DD}}^{-T}\mathbf{y_c} = \mathbf{L}_{\overline{DD}}\mathbf{s}_{\overline{D}} + \mathbf{w}, \tag{11}$$

where the noise vector $\mathbf{w}$ is Gaussian with covariance matrix $\mathbf{I}$. We now apply SD to (11). Let K be the number of elements in $\overline{D}$. As suggested in [5], the initial radius for SD is set to $C = aK\sigma^2$, with a such that

$$\int_0^{aK/2} \frac{x^{(K/2-1)}}{\Gamma(K/2)} e^{-x} dx = 0.99. \tag{12}$$

The threshold parameter τ should be small enough to ensure that the PDA stage makes reliable decisions. On the other hand, τ should not be too small, for otherwise the inclusion of the PDA stage will yield little if any dimensionality reduction benefit. For the simulations of the vectored DSL system presented below we choose τ to be 10^{-8}. With this choice, over half of the symbols are detected by the single PDA stage of the hybrid algorithm, while the rest are passed on to the SD stage for decoding.

4. RESULTS

We test the three algorithms using measured short-loop DSL channel data collected by France Telecom R&D as part of the EU-FP6 U-BROAD project # 506790. We use insertion loss (IL) and FEXT measured data for S88 cable comprising 14 quads, i.e., 28 loops. The length of the cable is 300 meters. All 378 (28 picks 2) FEXT channels were measured. For each channel, a log-frequency sweeping scheme was used to measure the I/Q components of the frequency response from 10 KHz - 30 MHz, yielding 801 complex samples per channel. Cubic spline complex interpolation was used to convert these samples to a linear frequency scale.

The source power was 15 dBm. A power splitter was used to direct half of the source's power to the cable, while the other half was directed to the reference (R) input of a network analyzer. The network analyzer was a HP 4395A, and the resolution was set to 100 Hz. The return signal from the cable was directed to the A input of the analyzer, and IL/NEXT/FEXT were measured as the ratio A/R. North Hills 413BF (100 kHz – 100 MHz) baluns were used to connect the measured pairs to the measurement device, and a calibration procedure was employed to offset the effects of the baluns and the coaxial cable connections.

In our simulations, we compare the bit error rate (BER) of the three aforementioned algorithms over the 21.5 to 30 MHz frequency range. Transmission over the 20-30 MHz band is viable albeit challenging at this cable length. Insertion loss drops between -40 and -45 dB in this range of frequencies, while FEXT coupling is between -77 and -82 dB in the mean, with over 10 dB standard deviation and significant variation across frequency as well [12]. Note that, besides magnitudes, the relative phases of the elements of $\widetilde{\mathbf{Q}}_f$ and $\widetilde{\mathbf{A}}_f$ are also important performance-wise.

Loops 1-14 are coordinated and the remaining 14 loops are taken as alien FEXT interferers. It is assumed that FDD is employed and the 21.5 - 30 MHz band is allocated to transmission in one direction, thus NEXT is not considered. 4-QAM modulation is employed, with unit transmitted symbol power. The transmission power of alien FEXT is set to $p_{AF} = 64$ and $\sigma_N^2 = 10^{-7}$ - thus a severe alien interference environment is simulated. For every tone, $3 \cdot 10^6$ Monte-Carlo iterations are employed to assess BER. Note that, since each symbol vector carries 2×14 bits, the averaging order is over 10^7.

The implementation of PDA does not incorporate the bit-flip stage [9]. The internal threshold parameter of PDA is set to $\epsilon = 3 \cdot 10^{-4}$ (note that this is different from our hard decoding threshold τ). The initial radius of SD is set as in Section 3; if SD fails to find a point inside the sphere, the radius is increased by 1, up to 5 times (6 searches at most). When SD is forced to stop abruptly due to reaching the complexity limit, it returns the quantized zero-forcing solution. In the same vain, the SD part of the hybrid PDA-SD detector returns the quantized zero-forcing solution for the undecided part of the overall symbol vector when forced to terminate prematurely.

4.1. Discussion

In the figures presented below, we show the performance of each algorithm subject to a specific upper bound on computational complexity. For the results shown in Fig. 1 the complexity restriction is 2 times the computational cost of a single PDA stage. In Fig. 2 this complexity bound is set to 10 times the computational cost of a PDA stage.

We observe that the PDA has worse performance than the other two algorithms for most of the frequencies examined. The SD exhibits the smallest probability of error uniformly across all frequencies, followed by the proposed hybrid algorithm whose BER performance is close to that of the SD in most cases.

An interesting observation is that the performance of all three algorithms does not improve appreciably when the complexity restriction is relaxed from 2 to 10 PDA stages - with the exception of the hybrid PDA-SD algorithm around 27 MHz. The PDA does not improve with an increased stage budget, while there is a substantial gap in BER performance relative to SD - roughly an order of magnitude between 24 and 28 MHz.

Some remarks are in order:

- While the signal to interference plus noise (SINR) is not necessarily a good performance predictor in the context of multiuser detection (e.g., the conditioning of the mixing matrix matters a lot), it is true that SINR is much higher in the given DSL application than in typical MIMO wireless settings. The BER of PDA flares up, while the complexity of SD drops quickly at high SINR.
- The pre-whitened equivalent mixing for such broadband FEXT-limited MIMO DSL channels is far from being i.i.d. Rayleigh - distributed. In particular, ill-conditioning is more likely to be encountered.

We have considered 14 vectored loops plus 14 interferers, because boosting the per-loop capacity requires vectoring at least half of the active loops in the binder. Vectoring the full binder is often not possible, due to regulatory "unbundling" considerations. If, however, the full binder can be vectored, then the ordering of algorithms may be reversed, because:

- The complexity of SD grows very quickly with the number of coordinated loops. With 28 loops, SD will often be forced to terminate abruptly, due to the complexity limit. At the same time:
- In the absence of alien crosstalk, there will be no need to whiten the noise, and thus the mixing matrix will remain diagonally-dominated. This type of mixing is well-suited for the PDA.

5. CONCLUSIONS

We have tested three multiuser detection algorithms (PDA, SD, and hybrid PDA-SD) in the context of FEXT-limited vectored DSL MIMO communication over short loops, using actual measured channel data. For the realistic scenario wherein half of the loops in the binder are vectored, and a strict complexity limit is placed on vector decoding complexity for all three algorithms, we have found that SD provides the best solution. This conclusion has been placed in context relative to earlier performance assessments for typical MIMO wireless channels.

In our experiments, we have used the Fincke-Pohst SD in [15]. Using more advanced variants of SD, e.g., the Schnorr-Euchnerr [14] SD [2, 1], or recent developments in [17], should tilt the balance more in favor of SD in the half-binder vectoring scenario.

Acknowledgment: We would like to thank Dr. Meryem Ouzzif and Dr. Rabah Tarafi of France Telecom R&D, who conducted the channel measurements on behalf of France Telecom R&D under the auspices of the U-BROAD project.

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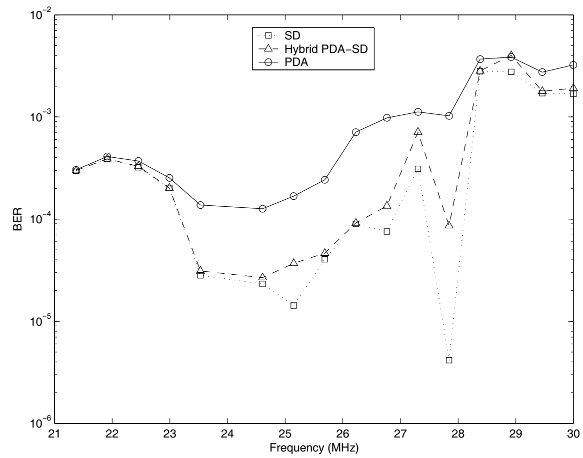

Fig. 1. BER performance versus frequency, 14 coordinated loops, 14 FEXT interferers, 4-QAM, complexity restriction at 2 times the cost of one PDA stage, $3 \cdot 10^6$ Monte Carlo runs.

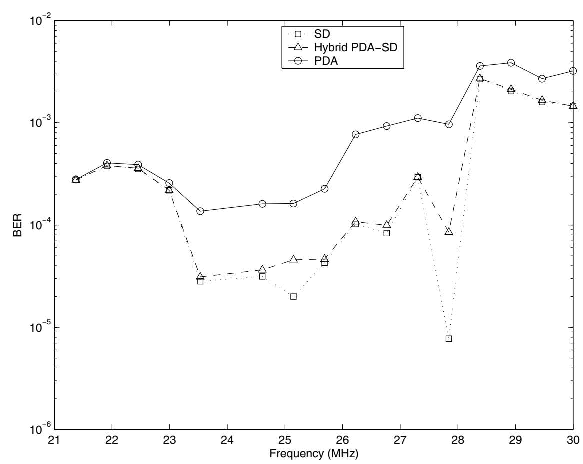

Fig. 2. BER performance versus frequency, 14 coordinated loops, 14 FEXT interferers, 4-QAM, complexity restriction at 10 times the cost of one PDA stage, $3 \cdot 10^6$ Monte Carlo runs.