OPTIMAL ONE-TAP EQUALIZATION FOR DMT TRANSCEIVERS WITH INSUFFICIENT GUARD INTERVAL

Tanja Karp

Texas Tech University
ECE Dept., Box 43102
Lubbock TX, 79409-3102
tanja.karp@ttu.edu

Carsten Bauer and Norbert J. Fliege

Mannheim University
Institute of Computer Engineering
B6 23–29, D-68131 Mannheim, Germany
{carsten.bauer, fliege}@ti.uni-mannheim.de

ABSTRACT

In this paper we present an improved algorithm to calculate the one-tap equalizer per subchannel in a DMT receiver in the case of a guard interval of insufficient length. The new equalizer reduces the symbol error rate at no additional computational and hardware cost. It is designed for a scenario where standard modems have to be used that do not allow time-domain or per-tone equalization or where the extra cost of these modules is undesired since in the majority of setups the guard interval is of sufficient length.

1. INTRODUCTION

Discrete Multi-Tone (DMT) modulation, a multicarrier transmission technique where the input data stream is divided among different narrow frequency bands and then simultaneously transmitted, has been introduced as the modulation scheme for fast data-rate transmission over twisted copper-wire in order to bridge the fiber optic gap between a private home and the telecommunication service provider's central office. Having applications such as fast internet access from private homes in mind, DMT was designed to work well with channels that have a relatively short impulse response, corresponding to twisted pair copper wires of several hundred meters of length.

DMT prevents interference between consecutive DMT symbols due to dispersion of the channel by introducing a guard interval in form of a cyclic prefix at the beginning of each DMT symbol. No interference occurs as long as the guard period is at least as long as the channel memory. In this case a one-tap equalizer per frequency band is sufficient to compensate for the attenuation and phase shift in the band. If the guard interval is too short, intersymbol interference (ISI) and intercarrier interference (ICI) occur and result in a loss of orthogonality of the tones.

More recently, efforts have been made to increase the achievable distance for DMT modems to ranges of $10-20$ km, e.g. for remote control applications. Since the guard interval does not cover the entire channel impulse response for these distances, various equalizer schemes such as time-domain equalization [1] or per-tone frequency-domain equalization [2] have been presented in the literature. While time-domain equalization aims at reducing the channel impulse response through introduction of an FIR filter at the receiver input, per-tone equalization presents a more elaborate equalizer design in the frequency domain. Both schemes require additional computational cost for the implementation of these algorithms.

In this paper we look at cases where transmission over a long distance or a severely corrupted channel has to be performed with standard modems such as the ones used for ADSL [3] that do not offer the improved equalization schemes. We show that symbol error rates in presence of ISI and ICI can be significantly reduced, if the one-tap equalizer per subcarrier is not calculated as the inverse of the channel frequency response at the subcarrier frequency but takes interference into consideration. The scheme allows DSL internet providers to offer their services to remote areas and reduces the number of copper-wire lines that have to be replaced when introducing DSL.

Note that although we concentrate on wireline DMT transceivers, our results can also be applied to OFDM wireless transmission in a severe multipath environment.

2. THE DMT TRANSCEIVER

The block diagram of a DMT transceiver with M subcarriers is shown in Figure 1. Its input/output relationship can be calculated as [4, 5, 6]:

$$\hat{\mathbf{u}}(k) = \mathbf{E}\,\frac{\mathbf{W}_M}{\sqrt{M}}\left(\begin{bmatrix}\tilde{\mathbf{C}}_0 & \tilde{\mathbf{C}}_1\end{bmatrix}\begin{bmatrix}\frac{\mathbf{W}_M^H}{\sqrt{M}} & \mathbf{0}\\ \mathbf{0} & \frac{\mathbf{W}_M^H}{\sqrt{M}}\end{bmatrix}\begin{bmatrix}\mathbf{u}(k-1)\\ \mathbf{u}(k)\end{bmatrix} + \mathbf{r}(k)\right) \tag{1}$$

where $\mathbf{E} = \mathrm{diag}(e_0, \ldots, e_{M-1})$ denotes the one-tap equalizer per subcarrier and $\mathbf{W}_M/\sqrt{M}$ and $\mathbf{W}_M^H/\sqrt{M}$ describe

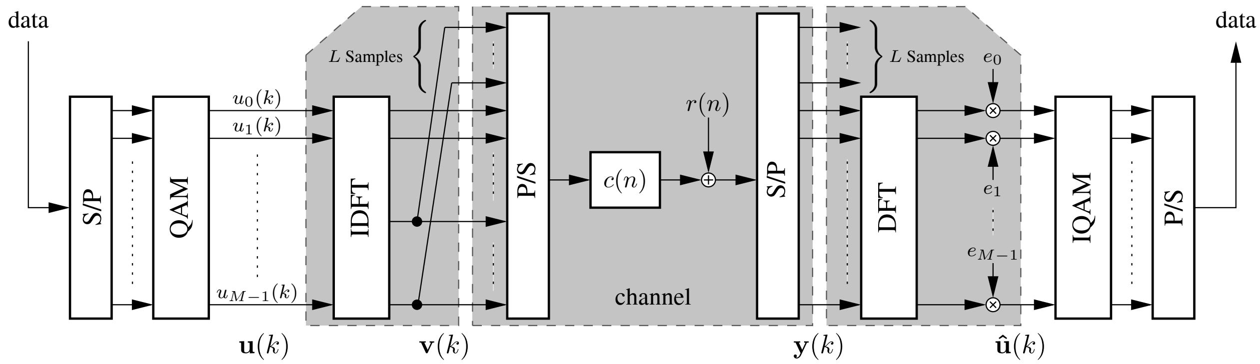

Fig. 1. DMT Transceiver

the orthonormal DFT and IDFT matrix, respectively. The matrix $\tilde{\mathbf{C}} = [\tilde{\mathbf{C}}_0 \quad \tilde{\mathbf{C}}_1]$ is a size $M \times 2M$ transmission matrix combining the introduction of the guard interval in form of a cyclic prefix of size L, the parallel-to-serial conversion at the transmitter, the convolution with the channel impulse response, the serial-to-parallel conversion at the receiver and the removal of the guard interval. Its entries are given by

$$\tilde{\mathbf{C}}_0 = \begin{bmatrix} 0 & \cdots & 0 & c_{L_c-1} & \cdots & c_{L+1} \\ \vdots & & & \ddots & \ddots & \vdots \\ \vdots & & & & \ddots & c_{L_c-1} \\ \vdots & & & & & 0 \\ \vdots & & & & & \vdots \\ 0 & \cdots & \cdots & \cdots & \cdots & 0 \end{bmatrix}$$

$$\tilde{\mathbf{C}}_1 = \begin{bmatrix} c_0 & & & & c_L & \cdots & c_1 \\ \vdots & \ddots & & & \vdots & & \vdots \\ \vdots & & \ddots & & c_{L_c-1} & & \vdots \\ \vdots & & & \ddots & & \ddots & \vdots \\ c_{L_c-1} & & & & \ddots & & c_{L_c-1} \\ & \ddots & & & & \ddots & \\ & & c_{L_c-1} & \cdots & \cdots & \cdots & c_0 \end{bmatrix}$$

where L_c denotes the length of the discrete channel impulse response. Vector $\mathbf{r}(k)$ in (1) describes the serial-to-parallel converted additive channel noise after removal of the guard interval at the receiver. The channel matrix $\tilde{\mathbf{C}}$ can be split into a desired part $\tilde{\mathbf{C}}_{\text{circ}}$, that describes the form it takes for a sufficiently long guard interval, and an error matrix $\tilde{\mathbf{C}}_{\text{err}}$ due to interference occurring if the guard interval is too short:

$$\tilde{\mathbf{C}} = \underbrace{\left[\mathbf{0}_{M\times M} \quad (\tilde{\mathbf{C}}_1 + \tilde{\mathbf{C}}_0\,\mathbf{P})\right]}_{\tilde{\mathbf{C}}_{\text{circ}}} + \underbrace{\left[\tilde{\mathbf{C}}_0 \quad -\tilde{\mathbf{C}}_0\,\mathbf{P}\right]}_{\tilde{\mathbf{C}}_{\text{err}}} \qquad (2)$$

where $\mathbf{P}$ shifts the entries of $\tilde{\mathbf{C}}_0$ by L columns to the left:

$$\mathbf{P} = \begin{bmatrix} \mathbf{0}_{L\times(M-L)} & \mathbf{I}_L \\ \mathbf{I}_{M-L} & \mathbf{0}_{(M-L)\times L} \end{bmatrix} \qquad (3)$$

such that $\tilde{\mathbf{C}}_{\text{circ}}$ is a circular matrix. Note that in case of a sufficiently long guard interval, $\tilde{\mathbf{C}}_0$ is a zero matrix. Using (2) we can rewrite (1) and separate the desired, equalized signal from ISI components created by the previous transmit symbol, ICI components that leak from one frequency band into the others, and additive channel noise:

$$\begin{aligned} \hat{\mathbf{u}}(k) = {} & \mathbf{E}\,\mathbf{C}_{\text{freq}}\,\mathbf{u}(k) && \text{desired signal} \\ & + \mathbf{E}\,\frac{\mathbf{W}_M}{\sqrt{M}}\tilde{\mathbf{C}}_0\,\frac{\mathbf{W}_M^H}{\sqrt{M}}\,\mathbf{u}(k-1) && \text{ISI} \\ & - \mathbf{E}\,\frac{\mathbf{W}_M}{\sqrt{M}}\tilde{\mathbf{C}}_0\,\mathbf{P}\,\frac{\mathbf{W}_M^H}{\sqrt{M}}\,\mathbf{u}(k) && \text{ICI} \\ & + \mathbf{E}\,\frac{\mathbf{W}_M}{\sqrt{M}}\,\mathbf{r}(k) && \text{channel noise} \end{aligned} \qquad (4)$$

$\mathbf{C}_{\text{freq}}$ in the upper equation is a diagonal matrix, containing the frequency response of the channel at the subcarrier frequencies:

$$\begin{aligned} \mathbf{C}_{\text{freq}} &= \frac{\mathbf{W}_M}{\sqrt{M}}\tilde{\mathbf{C}}_{\text{circ}} \begin{bmatrix} \mathbf{0} \\ \frac{\mathbf{W}_M^H}{\sqrt{M}} \end{bmatrix} \\ &= \text{diag}([C(e^{j0}),\ C(e^{j2\pi 1/M}), \ldots,\ C(e^{j2\pi(M-1)/M})]) \end{aligned}$$

The one-tap equalizer e_ℓ in subband ℓ is generally calculated as

$$e_\ell = 1/\hat{C}_\ell \qquad (5)$$

where $\hat{C}_\ell$ denotes the estimated channel frequency response at the subcarrier with index ℓ and is obtained during initialization. Assuming a correct estimate of the channel frequency response and a guard interval of sufficient length such that no ISI and ICI occur, this yields in an output signal of

$$\hat{\mathbf{u}}(k) = \mathbf{u}(k) + \mathbf{E}\,\frac{\mathbf{W}_M}{\sqrt{M}}\,\mathbf{r}(k) \qquad (6)$$

If, however, the guard interval is of insufficient length, interference corrupts the performance of the transceiver. It not only introduces a spread of the received values around the true QAM constellation points, which could be modeled as an additional additive noise source, but also introduces a rotation and scaling of the received constellation points resulting in a significant increase of of symbol errors if the constellation detector is not aware of it.

In order to visualize the effect of ISI and ICI, we simulate a DMT transceiver with $M = 128$ subbands and a guard interval of length $L = 8$ using the channel impulse response shown in Figure 2 that significantly exceeds the guard interval length.

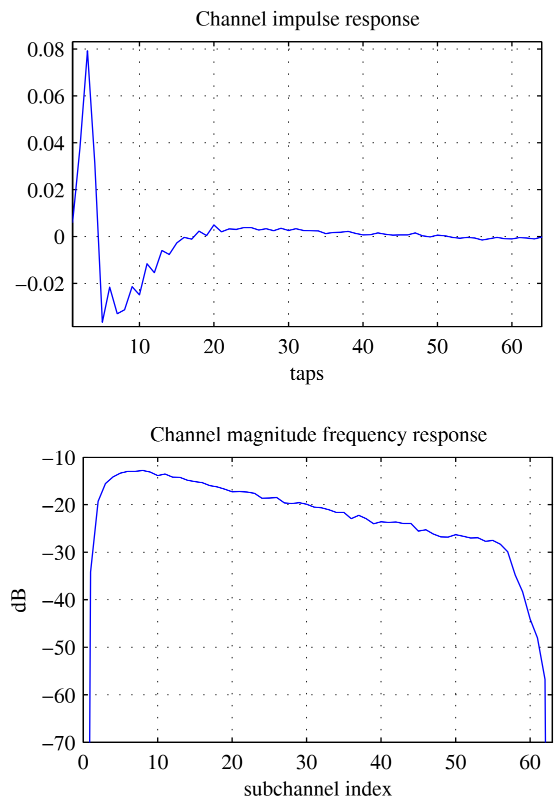

Fig. 2. Channel impulse response (upper figure) and magnitude frequency response (lower figure).

No additive channel noise is present during this simulation. We transmit a constant power, i.i.d. 16-QAM symbol in each subband. Figure 3 shows receive QAM symbols at the equalizer output for subchannels 26 and 41. The red crosses denote the QAM constellation points the receiver assumes, the green crossed show the center of mass of the equalized constellation points, and the dashed lines show the detection boundaries when remapping a received value to its constellation point. The one-tap equalizer per subband was calculated according to (5), i.e. inverting the channel frequency response at the subcarrier. One can clearly see that the equalized values are attenuated in subchannel 26 and rotated in subchannel 41.

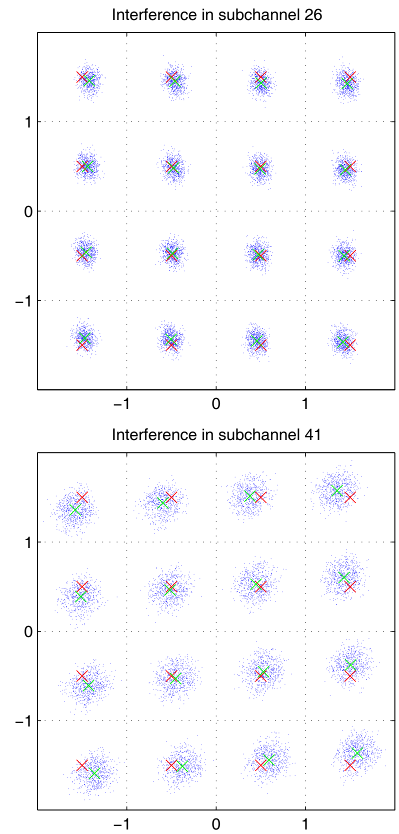

Fig. 3. Received QAM symbols at the equalizer output in subchannels 26 and 41. The red cross denotes the transmit QAM symbols. The green crossed show the center of mass of the equalized constellation points.

3. OPTIMAL ONE-TAP EQUALIZER

The complex scaling factor observed in the QAM constellations in Figure 3 can only be caused by an interference component that is correlated to the transmit signal. Since previous QAM transmit symbols as well as transmit symbols from other subchannels are uncorrelated to the current transmit symbol in a particular subchannel for the i.i.d. QAM sequence, they do not influence the center of mass of a constellation point in the equalized signal. When having a closer look at (4) we observe that ICI also has an intra-channel or self-induced interference component due to the entries of the diagonal of

$$-\mathbf{E}\,\frac{\mathbf{W}_M}{\sqrt{M}}\,\tilde{\mathbf{C}}_0\,\mathbf{P}\,\frac{\mathbf{W}_M^H}{\sqrt{M}} \tag{7}$$

This in fact means that ICI does not only leak into the other subchannels, but also into the creating subchannel itself. Extracting the part of $\hat{u}_\ell(k)$ that depends on $u_\ell(k)$ in (4) results in

$$\hat{u}_\ell(k) = e_\ell \left(\hat{C}_\ell - \left[\frac{\mathbf{W}_M}{\sqrt{M}} \tilde{\mathbf{C}}_0 \, \mathbf{P} \, \frac{\mathbf{W}_M^H}{\sqrt{M}} \right]_{\ell,\ell} \right) u_\ell(k) \qquad (8)$$

We therefore suggest to calculate the one-tap equalizer coefficient as

$$e_\ell = \frac{1}{\hat{C}_\ell - \left[\frac{\mathbf{W}_M}{\sqrt{M}} \tilde{\mathbf{C}}_0 \, \mathbf{P} \, \frac{\mathbf{W}_M^H}{\sqrt{M}} \right]_{\ell,\ell}} \qquad (9)$$

The additional term in (9) takes the complex scaling factor observed in Figure 3 into consideration and compensates for it. Figure 4 shows the equalizer output signal for subchannel 41 using the same QAM symbols as in Figure 3 when calculating the one-tap equalizer according to (9). The rotation that was originally observed has been compensated and the centers of mass of the received QAM symbols denoted by the green crosses in Figure 4 coincide with the original QAM constellation points, denoted by the red crosses.

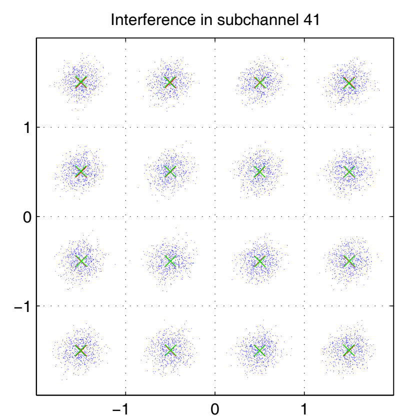

Fig. 4. Received QAM symbols at the equalizer output in subchannels 41 when applying (9) to calculate the equalizer. The red cross denotes the transmit QAM symbols, the green cross the center of mass for of the equalized symbols.

In practice, there are two ways to calculate the equalizer coefficients. Either, one estimates the channel impulse response in a first step by using a training sequence where ICI and ISI cancel, see e.g. [3, 7] and then calculates the coefficients according to (9). Alternatively, one can apply a pseudo-random training sequence $\mathbf{u}_{\text{train}}(k)$ where ISI and ICI do not cancel any more and estimate the equalizer coefficient e_ℓ in subchannel ℓ as

$$e_\ell = E\left\{ \frac{u_{\text{train},\ell}(k)}{\hat{u}_{\text{train},\ell}(k)} \right\}. \qquad (10)$$

4. CONCLUSION

In this paper we have presented a new way to calculate the one-tap equalizer coefficients per subcarrier in a DMT transceiver with insufficient guard interval. The equalizer design compensates a scaling and rotation of the equalized symbols that can be observed when calculating the equalizer as the inverse of the channel impulse response. The improved DMT system reduces the symbol error rate for channels with long impulse responses at no additional cost. Results can be directly applied to OFDM systems in a severe multipath environment.

5. REFERENCES

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