

MSE-OFDM: A NEW OFDM TRANSMISSION TECHNIQUE WITH IMPROVED SYSTEM PERFORMANCE

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Abstract—A new multicarrier system, termed Multi-Symbol Encapsulated Orthogonal Frequency Division Multiplexing (MSE-OFDM), was proposed, in which one cyclic prefix (CP) is used for multiple OFDM symbols. The motivations for this new OFDM system are either to reduce the redundancy caused by the CP or to increase the system robustness to frequency offset, depending on the two different proposed implementations for the MSE-OFDM systems. The corresponding frequency offset and channel estimation algorithms are investigated. Possible ways to reduce the complexity of the joint maximum likelihood (ML) estimator, including the approximation of the joint ML estimator and FFT pruning, are discussed. The performance of the proposed estimators is also analyzed and verified through numerical simulations.

I. INTRODUCTION

One important advantage of Orthogonal Frequency Division Multiplexing system is its simple receiver structure, utilizing a frequency domain equalizer with only one complex multiplication per sub-carrier. This is achieved by introducing a time domain cyclic prefix, enabling the receiver to separate the steady-state response from the transient response of the communication channel. Redundancy is unavoidably introduced into conventional OFDM systems due to the insertion of CP. We studied a unique MSE-OFDM system proposed in [1] that uses a different type of cyclic prefix, i.e., instead of using one cyclic prefix for each OFDM symbol, a number of OFDM symbols are grouped together as a frame and protected by one single cyclic prefix. Different implementations of the MSE-OFDM scheme can be used to either improve the bandwidth efficiency for static channels, or improve the robustness to synchronization errors and reduce peak-to-average power ratio (PAPR) for mobile channels. As with conventional OFDM systems, MSE-OFDM relies on coherent quadrature amplitude modulation for higher spectral efficiency. Joint estimation of channel and frequency offset based maximum likelihood (ML) algorithm is therefore investigated. Simplification of the joint ML estimator is also studied.

II. MSE-OFDM SYSTEM

The block diagram of the proposed MSE-OFDM is depicted in Fig. 1, where N and M denote the size of IFFT modulator and the total number of OFDM symbols in one MSE-OFDM frame, respectively. The duration of the cyclic prefix, i.e., the largest expected channel duration, is P samples.

Transmitter: To realize the proposed MSE-OFDM frame structure, modifications have to be made to a conventional OFDM transceiver. M OFDM symbols have to be generated before the CP insertion at the transmitter side as in Fig. 1 (a). Each OFDM symbol is given by the N points complex modulation sequence through IDFT

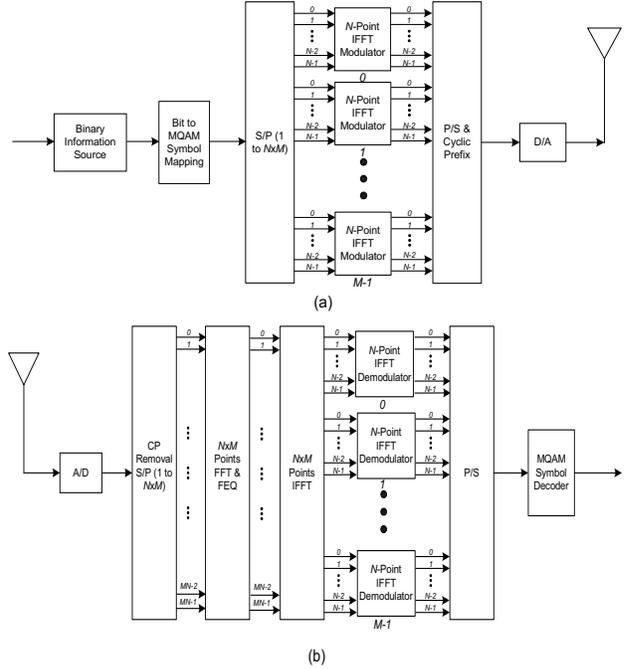


Fig. 1. Block diagram of the Multiple-Symbol Encapsulated OFDM system.

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k) e^{j2\pi \frac{nk}{N}} \quad n=0, 1, 2, \dots, N-1. \quad (1)$$

The MSE-OFDM signal in (1) consists of N complex exponentials, or sub-carriers, which have been modulated with the complex data X . The l -th frame MSE-OFDM signal with M symbols can be generated according to (1) as

$$s_l = \sum_{k=0}^{N-1} X_{l,M-1}(k) \psi_1(n, k) + \sum_{i=0}^{M-1} \sum_{k=0}^{N-1} X_{l,i}(k) \psi_2(n - iN - P, k) \quad (2)$$

where the two subscripts i and l indicate the i -th OFDM symbol of the l -th frame. $\psi_1(n, k)$ and $\psi_2(n, k)$ are two rectangular signal multiplexing window functions corresponding to the cyclic prefix and the M information carrying OFDM symbols defined as follows

$$\psi_1(n, k) = \begin{cases} \frac{1}{\sqrt{N}} e^{j2\pi \frac{k(N-P+n)}{N}}, & 0 \leq n \leq P-1 \\ 0, & \text{elsewhere} \end{cases} \quad (3)$$

and

$$\psi_2(n - iN - P, k) = \begin{cases} \frac{1}{\sqrt{N}} e^{j2\pi \frac{k(n-P-iN)}{N}}, & P \leq n \leq MN + P - 1 \\ 0, & \text{elsewhere} \end{cases} \quad (4)$$

Receiver: With the help of the CP, simple frequency domain equalization can be realized for the MSE-OFDM system. However, a new frequency domain equalizer has to be employed due to the unique frame structure of the MSE-OFDM signal. To illustrate the new equalizer principle, we rewrite \mathbf{s}_l in (2) as a vector

$$\mathbf{s}_l = \underbrace{[x_{l,M-1}(N-p+1), \dots, x_{l,M-1}(N-1)]}_{\text{prefix}} \underbrace{[x_{l,0}(0), \dots, x_{l,0}(N-1), x_{l,1}(0), \dots, x_{l,1}(N-1), \dots, x_{l,M-1}(0), \dots, x_{l,M-1}(N-1)]^T}_{M \text{ OFDM symbols}}. \quad (5)$$

The received signal \mathbf{r}_l corresponding to the above transmitted signal vector can be expressed as

$$\mathbf{r}_l = \begin{bmatrix} h_0 & 0 & \dots & \dots & \dots & 0 \\ h_1 & h_0 & & & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ h_{L-1} & \dots & h_1 & h_0 & \ddots & \vdots \\ 0 & \ddots & & \ddots & \ddots & 0 \\ \vdots & & h_{L-1} & \dots & h_1 & h_0 \\ \vdots & & \ddots & h_{L-1} & \dots & h_1 \\ \vdots & \ddots & & \ddots & \dots & \vdots \\ 0 & \dots & \dots & \dots & 0 & h_{L-1} \end{bmatrix} \cdot \mathbf{s}_l + \mathbf{w} \quad (6)$$

where the size of the channel matrix in (6) is $[MN+2P, MN+P]$. \mathbf{w} is an additive white Gaussian noise (AWGN) vector with the same size as \mathbf{s}_l . There exists a cyclic convolution between the CP removed signal $\tilde{\mathbf{s}}_l$ (the tilde symbol indicates the signal after the CP removal) and channel impulse response \mathbf{h} : the following DFT transform pair holds

$$\tilde{\mathbf{s}}_l \otimes \mathbf{h} + \mathbf{w}' \Leftrightarrow \text{DFT}(\tilde{\mathbf{s}}_l) \cdot \mathbf{H} + \tilde{\mathbf{w}} \quad (7)$$

where \otimes denotes the cyclic convolution while \mathbf{H} and $\tilde{\mathbf{w}}$ are the Fourier transform of \mathbf{h} and \mathbf{w}' . Note that the size of DFT here is MN points. Assuming that the channel transfer function \mathbf{H} is known from channel estimation, channel impairments can be compensated with (7) using a one-tap frequency domain equalizer. For the demodulation of each OFDM symbols in the same frame, the frequency domain equalized signal has to be converted back into the time domain for the IDFT demodulation. This process can be formulated as:

$$\tilde{\mathbf{r}}_l^{\text{FEQ}} = \text{IDFT} \left\{ \frac{\text{DFT}(\mathbf{r}_l)}{\mathbf{H}} \right\} + \mathbf{w}_l^{\text{FEQ}} \quad (8)$$

$\mathbf{w}_l^{\text{FEQ}}$ is the AWGN noise after the frequency domain equalization. The equalized signal, $\tilde{\mathbf{r}}_l^{\text{FEQ}}$, is then split into M OFDM symbols for demodulation with an N -point FFT.

Different Implementations and System Performance: Based on an conventional OFDM system, MSE-OFDM can be implemented in two different ways, i.e., either keeping the symbol size of the MSE-OFDM (i.e., number of the subcarriers) unchanged to increase the bandwidth efficiency, or keeping the bandwidth efficiency unchanged (ratio between CP and useful data transmission time) for system robustness to synchronization errors and a lower peak-to-average power ratio. For the latter case, the MSE-OFDM symbol duration is

reduced to $1/M$ of the conventional system. This is equivalent to an OFDM system with smaller number of subcarriers. For this FFT size-reduced MSE-OFDM, robustness to synchronization errors is improved considerably due to the smaller number of subcarriers. The PAPR is also expected to be reduced.

III. FREQUENCY OFFSET AND CHANNEL ESTIMATION FOR MSE-OFDM SYSTEM

As discussed in Section I, channel estimation has to be realized before equalization and demodulation of the OFDM signal. The accuracy of the channel estimation is also crucial to the performance of the overall system in terms of bit/symbol error rate. The frequency offset of the OFDM system has also to be estimated and corrected to avoid intercarrier interference due to the loss of orthogonality among the subcarriers. In this section, the joint maximum likelihood (ML) estimation of frequency offset and channel impulse response is investigated. The structure of the MSE-OFDM preamble is also exploited to reduce the complexity of the estimators.

Joint Estimation of Frequency Offset and Channel Impulse Response: Consider an MSE-OFDM preamble vector \mathbf{a} of length N , i.e., the duration of one OFDM symbol. The preamble is also extended by a CP with length of P . If the intersymbol interference is completely mitigated by the CP, the received preamble vector \mathbf{y} after CP removal can be expressed as

$$\mathbf{y} = \mathbf{\Gamma}(\Delta k) \mathbf{A} \mathbf{h} + \mathbf{w} \quad (9)$$

where Δk is the relative frequency offset and $\mathbf{\Gamma}(\Delta k)$ is a diagonal matrix

$$\mathbf{\Gamma}(\Delta k) = \text{diag}\{1, e^{j2\pi\Delta k/N}, e^{j4\pi\Delta k/N}, \dots, e^{j2\pi(N-1)\Delta k/N}\} \quad (10)$$

and \mathbf{A} is $N \times P$ matrix with entries

$$[\mathbf{A}]_{i,j} = a_{i-j}, \quad 0 \leq i \leq N-1, \quad 0 \leq j \leq P-1. \quad (11)$$

$\mathbf{w} = [w(0), w(1), \dots, w(N-1)]^T$ is a zero-mean Gaussian vector with covariance matrix $\mathbf{C}_w = E\{\mathbf{w}\mathbf{w}^H\} = \sigma_n^2 \mathbf{I}_N$, where \mathbf{I}_N is the $N \times N$ identity matrix. For a given channel \mathbf{h} and frequency offset Δk , the vector of the received signal \mathbf{y} is Gaussian with mean $\mathbf{\Gamma}(\Delta k) \mathbf{A} \mathbf{h}$ and covariance matrix $\sigma_n^2 \mathbf{I}_N$. Thus, the likelihood function for the parameters $(\mathbf{h}, \Delta k)$ takes the form [2][3]

$$\Lambda(\mathbf{y}|\mathbf{h}, \Delta k) = \frac{1}{(\pi\sigma_n^2)^N} \exp\left\{-\frac{1}{\sigma_n^2} \left\{ [\mathbf{y} - \mathbf{\Gamma}(\Delta k) \mathbf{A} \mathbf{h}]^H [\mathbf{y} - \mathbf{\Gamma}(\Delta k) \mathbf{A} \mathbf{h}] \right\}\right\}. \quad (12)$$

Maximum likelihood channel estimation can be achieved choosing \mathbf{h} and Δk such that the above maximum likelihood function is maximized. This is equivalent to minimizing

$$\Lambda_L(\mathbf{y}|\mathbf{h}, \Delta k) = \text{Tr}\left\{ [\mathbf{y} - \mathbf{\Gamma}(\Delta k) \mathbf{A} \mathbf{h}]^H [\mathbf{y} - \mathbf{\Gamma}(\Delta k) \mathbf{A} \mathbf{h}] \right\}. \quad (13)$$

Since $\Lambda_L(\mathbf{y}|\mathbf{h}, \Delta k)$ is a convex function over \mathbf{h} and Δk , the estimation of \mathbf{h} can be obtained by choosing \mathbf{h} that satisfies the following condition

$$\frac{\partial \Lambda_L(\mathbf{y}|\mathbf{h}, \Delta k)}{\partial \mathbf{h}} = 0. \quad (14)$$

This implies that the ML channel estimate is given by

$$\hat{\mathbf{h}} = (\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H \mathbf{\Gamma}^H(\Delta k) \mathbf{y}. \quad (15)$$

If we substitute $\hat{\mathbf{h}}$ back into $\Lambda_L(\mathbf{y}|\mathbf{h}, \Delta k)$, it is found that maximizing the likelihood function, $\Lambda_L(\mathbf{y}|\mathbf{h}, \Delta k)$, is equivalent to maximizing

$$\Psi(\Delta k) = \mathbf{y}^H \mathbf{\Gamma}(\Delta k) \mathbf{B} \mathbf{\Gamma}^H(\Delta k) \mathbf{y} \quad (16)$$

where $\mathbf{B} = \mathbf{A}(\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H$. The frequency offset estimator can be formulated as

$$\Delta \hat{k} = \arg \max_{\Delta k} \{\Psi(\Delta k)\}. \quad (17)$$

Simplification of the Joint Estimation Algorithms: Eq. (16) indicates that the estimates of Δk and \mathbf{h} can be separated, i.e., Δk can be estimated before the estimation of \mathbf{h} . This observation coincides with the results in [2]. Once $\Delta \hat{k}$ is obtained, channel estimation can be achieved using

$$\hat{\mathbf{h}} = (\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H \mathbf{\Gamma}^H(\Delta \hat{k}) \mathbf{y} \quad (18)$$

The estimation of Δk , i.e., maximization of the $\Psi(\Delta k)$ in (16) can be obtained by adopting a proper search strategy. In this paper, we adopt the gradient search algorithm (GSA) described below. We first rewrite (18) as

$$\Psi(\Delta k) = \gamma(\Delta k) (\mathbf{y}_D^H \mathbf{B} \mathbf{y}_D) \gamma^H(\Delta k), \quad (19)$$

where

$$\mathbf{y}_D = \text{diag}\{y(0), y(1), \dots, y(N-1)\} \quad (20)$$

is the observation \mathbf{y} in diagonal matrix form, and

$$\gamma(\Delta k) = [1, e^{j2\pi\Delta k/N}, e^{j4\pi\Delta k/N}, \dots, e^{j2\pi(N-1)\Delta k/N}] \quad (21)$$

is the pattern of multiplicative distortions caused by the frequency offset Δk . The structure in (19) is appealing as it suggests that the term in the middle, $\mathbf{y}_D^H \mathbf{B} \mathbf{y}_D$, needed to be calculated only once. Since $\Psi(\Delta k)$ is a quadratic function in $\gamma(\Delta k)$, we can calculate its derivative, i.e. the slope, at Δk and search for the maximum value using the gradient algorithm. This is equivalent to searching for the Δk at which

$$\frac{d}{d\Delta k} \Psi(\Delta k) = 0. \quad (22)$$

Since $\Psi(\Delta k)$ can be rewritten as

$$\Psi(\Delta k) = \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} b_{n,m} y^*(n) y(m) \exp\left\{j \frac{2\pi\Delta k}{N} (n-m)\right\}, \quad (23)$$

where $b_{n,m}$ is the (n, m) -th element of the matrix \mathbf{B} , it follows that

$$\begin{aligned} G(\Delta k) &= \frac{d}{d\Delta k} \Psi(\Delta k) = \Delta k \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} j \frac{2\pi(n-m)}{N} b_{n,m} y^*(n) y(m) \exp\left\{j \frac{2\pi\Delta k}{N} (n-m)\right\} \\ &= \Delta k \cdot \gamma(\Delta k) (\mathbf{y}_D^H \mathbf{B} \mathbf{y}_D) \gamma^H(\Delta k), \end{aligned} \quad (24)$$

with \mathbf{B} being a matrix whose (n, m) -th element is

$$b_{n,m} = j \frac{2\pi(n-m)}{N} b_{n,m}. \quad (25)$$

At this point, we can use the GSA to look for the optimal Δk . Let $\Delta k^{(i)}$ and $G(\Delta k^{(i)})$ be the frequency offset estimate and the corresponding gradient in iteration i of the GSA. Then the frequency estimate used in the next iteration is

$$\Delta k^{(i+1)} = \Delta k^{(i)} + \delta \cdot G(\Delta k^{(i)}), \quad (26)$$

where δ is the step-size used in the search. $\Delta k^{(i+1)}$ is then used in (24) to compute the new gradient and this process continues until convergence occurs. Note that the matrix \mathbf{B} can always be pre-calculated and the matrix $\mathbf{y}_D^H \mathbf{B} \mathbf{y}_D$ in (24) needs to be computed only once. These properties help to reduce the computational complexity of the GSA. Indeed, we found that the GSA is much less time consuming than the one based on quantized search and interpolation

With the estimated $\Delta \hat{k}$, the frequency offset in the received signal can be compensated before the channel impulse response estimation. By doing so, the impact of the frequency offset to the channel estimation reduces to the residual frequency offset $\Delta k - \Delta \hat{k}$. The compensation process is equivalent to a shift to the received signal by $-\Delta \hat{k}$ in the frequency domain. Let \mathbf{y}' be the vector of the received signal after the removal of the estimated frequency offset.

$$\mathbf{y}' = \mathbf{\Gamma}(-\Delta \hat{k}) \mathbf{y} \quad (27)$$

where

$$\mathbf{\Gamma}(-\Delta \hat{k}) = \text{diag}\{1, e^{-j2\pi\Delta \hat{k}/N}, e^{-j4\pi\Delta \hat{k}/N}, \dots, e^{-j2\pi(N-1)\Delta \hat{k}/N}\}. \quad (28)$$

When the frequency offset estimation error is small, the diagonal matrix with the residual frequency offset becomes

$$\mathbf{\Gamma}(\Delta k - \Delta \hat{k}) \approx \text{diag}\{1, 1, 1, \dots, 1\} = \mathbf{I}_N. \quad (29)$$

Now the channel impulse response can be estimated using the simplified estimator as

$$\hat{\mathbf{h}} \approx (\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H \mathbf{y}'. \quad (30)$$

To reduce ML estimation complexity, $(\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H$ can be pre-calculated and stored. In this case, only $N \times P$ complex multiplications are needed (one complex multiplication corresponds to four real multiplications and two real additions). Once $\hat{\mathbf{h}}$ is obtained, $\hat{\mathbf{H}}$ can be determined using pruning FFT to reduce the computational complexity [4][5]. This process requires

$$N_{mp} = 2MN \lfloor \log_2 P \rfloor - 2MN - 4P + 4 + \frac{2MNP}{2^{\lfloor \log_2 P \rfloor}} \quad (31\text{-a})$$

real multiplications and

$$N_{add} = 3MN \lfloor \log_2 P \rfloor - 2P - 3MN + 2 + \frac{3MNP}{2^{\lfloor \log_2 P \rfloor}} \quad (31\text{-b})$$

real additions, where the function $\lfloor \cdot \rfloor$ returns the integer part

of the argument. To further reduce the computation complexity, the length of $\hat{\mathbf{h}}$ can be truncated using a threshold approach. This is because P represents the maximum expected channel duration. The true channel duration is often much smaller than P . Note here if $\Delta k - \hat{\Delta k} = 0$, the above estimator is identical to the conventional ML channel estimator. The mean square error of the channel estimation can be evaluated by

$$\begin{aligned} \text{MSE} &= E \left[\text{Tr} \left\{ \left[\hat{\mathbf{h}} - \mathbf{h} \right]^H \left[\hat{\mathbf{h}} - \mathbf{h} \right] \right\} \right] \\ &= E \left[\text{Tr} \left\{ \left(\mathbf{A}^H \mathbf{A} \right)^{-1} \mathbf{A}^H \mathbf{w} \mathbf{w}^H \mathbf{A} \left(\mathbf{A}^H \mathbf{A} \right)^{-1} \right\} \right] \\ &= \sigma_n^2 \text{Tr} \left\{ \left(\mathbf{A}^H \mathbf{A} \right)^{-1} \right\}. \end{aligned} \quad (32)$$

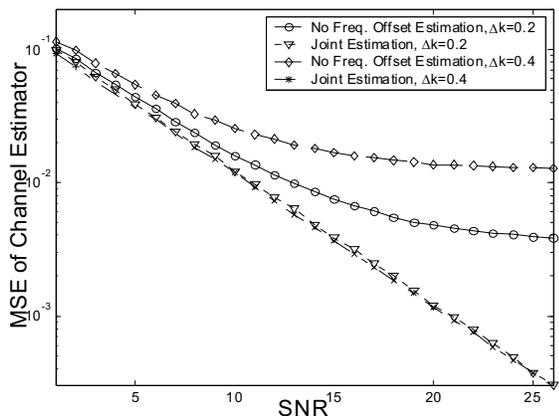


Fig.2. Mean square error (MSE) of the ML channel estimators at different simulation conditions.

IV. SIMULATIONS AND DISCUSSIONS

Computer simulations have been conducted to verify and extend the analytical results of the previous sections. The system parameters of the MSE-OFDM system are: $N=128$, $M=4$, $P=16$. The chosen modulation scheme is 64QAM. The preamble used here is from [6] with eight primary repetitive slots.

The MSE of the ML channel estimators for MSE-OFDM system are plotted in Fig. 2 for different frequency offsets. The channel estimation errors from the standalone channel ML estimator and the joint ML frequency offset and channel estimator were simulated. With the estimated frequency offset, the channel estimation accuracy was substantially improved. This is because the ICI from the loss of orthogonality among the subcarriers is suppressed when the frequency offset is compensated during the joint estimation of the frequency and channel impulse response. The MSE was reduced by more than 10 times at SNR of 20dB when the joint ML estimator was compared with the standalone channel estimator with a frequency offset of 0.4. Similar results were also obtained for the ML estimators with a frequency offset of 0.2. Another important observation is that there is an irreducible MSE floor at high signal to noise ratios for the ML channel estimator without joint frequency offset estimation.

The symbol error rates (SER) of the MSE-OFDM systems were also simulated, as seen in Fig. 3. A static multipath channel and an additive white Gaussian noise (AWGN) chan-

nel were used to evaluate the SER performance. The multipath channel was defined as $[0.9285 \ 0 \ 0.3714 \ 0 \ 0 \ 0]$. Curve (a) and (c) are the SER for the conventional OFDM system with AWGN and multipath channel, respectively. Similarly, the SER for MSE-OFDM system with AWGN channel and multipath channel were plotted (curves (b) and (d)). The normalized frequency offset for (e) and (f) is 0.02. The difference is that the joint estimator in this paper was used in (e). Comparing curves (d), (e) and (f), it can be found that the performance of the MSE-OFDM system with the proposed joint frequency offset and channel estimation (curve e) is very close to that of MSE-OFDM system with ideal frequency and channel estimation (curve d). The small gap between (d) and (e) is due to the system implementation loss and residual errors of the frequency and channel estimations.

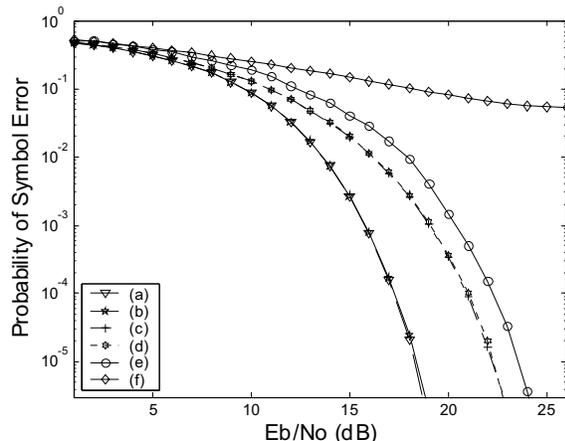


Fig.3. Probability of symbol error for the OFDM and MSE-OFDM systems.

V. CONCLUSIONS

Multi-Symbol Encapsulated Orthogonal Frequency Division Multiplexing (MSE-OFDM) and the corresponding joint frequency offset and channel estimator were proposed in this paper. Possible ways to reduce the estimator complexity, approximation of the joint ML estimator, and FFT pruning were discussed. The performance of the proposed MSE-OFDM system and the joint estimators were also analyzed and verified through numerical simulations. It is found that the performances of the MSE-OFDM system with the simplified joint frequency and channel estimator are very close to the MSE-OFDM system with ideal frequency and channel estimations.

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