JOINT SYNCHRONIZATION AND CHANNEL ESTIMATION IN UPLINK OFDMA SYSTEMS

Man-On Pun[†], C.-C. Jay Kuo

Department of Electrical Engineering University of Southern California Los Angeles, CA 90089-2564,USA

ABSTRACT

We consider the uplink of an OFDMA system and address the problem of estimating the carrier frequency offset, timing error and channel response of each active user. In doing so we follow a maximum likelihood (ML) approach and assume that a training sequence is available. Unfortunately, joint ML estimation of all the above parameters involves a multi-dimensional grid-search that is difficult to implement in practical systems. Therefore we resort to the alternating-projection algorithm and replace the multidimensional search with a sequence of one-dimensional searches. Compared to other existing methods, the proposed estimator has more flexibility since it can be used with any subcarrier assignment scheme. Simulations indicate that the accuracy of the frequency estimates asymptotically achieve the relevant Cramer-Rao bound (CRB).

1. INTRODUCTION

Orthogonal Frequency Division Multiple Access (OFDMA) has attracted much attention in the last few years and it is widely recognized as a promising technique for fourth generation (4G) broadband wireless networks. In an OFDMA system, several users simultaneously transmit their own data by modulating an exclusive set of subcarriers. Two critical issues in OFDMA uplink transmissions are frequency/timing synchronization and channel estimation. Similar to orthogonal frequency division multiplexing (OFDM), OFDMA is very sensitive to carrier frequency offsets (CFOs) due to Doppler shifts and/or oscillator instabilities. Inaccurate estimation of the CFO results in a loss of orthogonality between subcarriers, thereby leading to severe performance degradation. Timing errors may lead to interblock interference (IBI) between adjacent OFDMA blocks. The timing synchronization requirement may be relaxed by appending a sufficiently long guard interval (cyclic prefix) in front of each block.

In practical OFDM(A) applications, data transmission is organized in frames and training blocks carrying known symbols are located at the beginning of each frame for synchronization purposes. The training blocks usually have a very long cyclic prefix (CP) that comprises both the channel delay spread and the propagation delay incurred by the users' signals [1]. Vice versa, the CP of data blocks is made just greater than the length of the channel impulse response to minimize the system overhead. This means Michele Morelli

Department of Information Engineering University of Pisa 56100 Pisa, Italy

that accurate estimates of the timing offsets must be properly obtained during the training period so as to align all users in time and avoid IBI over the data section of the frame. Finally, estimating the channel response of each user is indispensable for coherent detection of the transmitted data. The synchronization and channel estimation task is particularly challenging in the uplink of an OFDMA system since each user has his/her own CFO, timing error and channel response.

Several frequency and timing recovery algorithms have been proposed in the technical literature for OFDM applications (see [2] therein). However, these schemes cannot be directly employed in the uplink of an OFDMA system since each user must be separated from the others at the base station (BS) before its synchronization parameters can be estimated. A possible separation method is to assign a group of adjacent subcarriers to each user and then pick them up through a filter bank [3]. However, grouping the subcarriers together prevents the possibility of exploiting more flexible generalized subcarrier assignment schemes (GCASs), where each user can select wathever subcarriers among those available at a given time instant. As there is no rigid association between subcarriers and users, GCAS allows dynamic resource allocation and provides more flexibility than conventional sub-band or interleaved subcarrier assignment schemes. A method for estimating the CFO and timing offset of a new user entering an OFDMA system was proposed in [4]. This scheme is suitable for GCAS but assumes that all existing users have already been synchronized, which could be a serious constraint in practical applications.

In this paper we consider the uplink of an OFDMA system employing GCAS and extend our previous work on CFO estimation [5] to jointly estimate the frequency offsets, the timing errors and the channel responses of all active users. In doing so we resort to ML reasoning and assume that all users send one pilot block at the beginning of each frame. The exact ML solution to the above problem involves a multidimensional grid-search that is prohibitively complex for practical implementation. Therefore, we replace the multi-dimensional search with a sequence of one-dimensional (1D) searches using the alternating-projection algorithm. The resulting scheme has affordable complexity and is effective for practical OFDMA transmissions

2. SIGNAL MODELS

We consider the uplink of an OFDMA network in which K active users simultaneously communicate with the BS as depicted in Fig. 1. We denote N the total number of subcarriers and call $s_k(n)$ the *n*th block of frequency-domain symbols sent by the *k*th user,

[†]Author for all correspondence, mpun@usc.edu,telephone/fax (213)-748-7663.

where $k \in \{1, 2, \dots, K\}$. The corresponding time-domain vector is given by $\boldsymbol{x}_k(n) = \boldsymbol{F}^H \boldsymbol{s}_k(n)$, where \boldsymbol{F} is the N-point discrete Fourier transform (DFT) matrix. A CP of length N_g is appended in front of $\boldsymbol{x}_k(n)$ to eliminate the interblock interference (IBI). The resulting vector $\boldsymbol{u}_k(n)$ (with length $Q = N + N_g$) is then transmitted over the channel. The discrete-time composite channel impulse response of the kth user (encompassing the shaping filters and the transmission medium) is denoted by $\{h_k(l)\}$, and the corresponding channel response vector with channel order L_k is given by $\boldsymbol{h}_k \stackrel{\text{def}}{=} [h_k(0), \dots, h_k(L_k - 1)]^T$. Since L_k is usually unknown, we replace \boldsymbol{h}_k by the following $L_h \times 1$ vector

$$\boldsymbol{h}_{k}^{\prime} \stackrel{\text{def}}{=} \begin{bmatrix} \boldsymbol{h}_{k}^{T} & \boldsymbol{0}_{(L_{h}-L_{k})\times 1}^{T} \end{bmatrix}, \qquad (1)$$

where $L_h \ge \max_k \{L_k\}$ is a design parameter that depends on the duration of the transmit/receive filters and the *maximum expected* channel delay spread.



Fig. 1. The discrete-time OFDMA baseband model.

The waveform arriving at the BS is given by the superposition of signals from all active users. In the presence of both CFOs and timing errors, the discrete-time output of the BS receive filter is given by

$$r(m) = \sum_{k=1}^{K} \left\{ e^{j\omega_k m} \sum_{l=0}^{L_h - 1} h_k(l) u_k(m - l - \mu_k) \right\} + v(m),$$
(2)

where $\omega_k = \frac{2\pi\Delta f_k}{N}$ with Δf_k being the *k*th CFO normalized to the subcarrier spacing, μ_k is the integer-valued timing error of the *k*th user and v(m) is zero-mean white Gaussian noise with variance σ_v^2 . As done in [4], the fractional part of the timing error is incorporated into the channel response.

As shown in Fig. 1, samples r(m) corresponding to the *n*th block of received data are serial-to-parallel (S/P) converted to form r(n) at the receiver. Next, the CP is removed and the remaining samples are collected into the *N*-dimensional vector y(n). We consider a quasi-synchronous system where user's timing is locked to a signal received from the BS through a downlink synchronization channel [4]. In this way, the timing errors in the uplink are only due to the (two-way) line-of-sight propagation delay and are limited to $\mu_{\text{max}} = 2R/c$, where *R* is the cell radius and *c* the speed of light. In the following, we assume that each user transmits pilot symbols over its preassigned subcarriers during the *n*th block (training block). Also, we let $N_g \ge L_h + \mu_{\text{max}}$ so that vector

y(n) is not affected by IBI. This assumption is not restrictive since training blocks are preceded by CPs of long duration in practical applications. For simplicity, we omit the temporal index n in the sequel. Then, from Eq. (2) it follows that

$$\boldsymbol{y} = \sum_{k=1}^{K} e^{j\bar{\omega}_k} \boldsymbol{\Gamma}(\omega_k) \boldsymbol{A}_k \boldsymbol{\xi}_k + \boldsymbol{v}, \qquad (3)$$

or, equivalently,

$$\boldsymbol{y} = \sum_{k=1}^{K} e^{j\bar{\omega}_k} \boldsymbol{\Gamma}(\omega_k) \boldsymbol{D}_k(\mu_k) \boldsymbol{h}'_k + \boldsymbol{v}, \qquad (4)$$

where $\bar{\omega}_k = \omega_k \left(nQ + N_g \right)$ and

$$\boldsymbol{\Gamma}(\omega_k) = \operatorname{diag}\left(1, e^{j\omega_k}, \cdots, e^{j(N-1)\omega_k}\right),$$

$$[\boldsymbol{A}_k]_{n,q} = [\boldsymbol{x}_k]_{|n-q|_{\infty}}, \quad 1 \le p \le N, 1 \le q \le N_q(6)$$

$$\boldsymbol{\xi}_{k} \stackrel{\text{def}}{=} \begin{bmatrix} \mathbf{0}_{\mu_{k}\times1}^{T} & \mathbf{b}_{k}^{T} & \mathbf{0}_{(N_{g}-\mu_{k}-L_{h})\times1}^{T} \end{bmatrix}^{T} (\mathcal{I})$$
$$\begin{bmatrix} \boldsymbol{D}_{k}(\mu_{k}) \end{bmatrix}_{p,q} = \begin{bmatrix} \boldsymbol{x}_{k} \end{bmatrix}_{|p-q-\mu_{k}|_{N}}, \quad 1 \le p \le N, 1 \le q \le L (\$)$$

where $[\boldsymbol{x}_k]_l$ denotes the *l*th entry of \boldsymbol{x}_k for $0 \le l \le N-1$, the modulo-*N* operation $|n|_N$ shifts *n* to interval [0, N-1], $[\boldsymbol{x}_k]_l$, with $k = 1, 2, \dots, K$ and $l = 0, 1, \dots N-1$ are pilot symbols and assumed to be known to the base station.

Our scheme exploits the signal model of Eq. (3), to jointly estimates $\boldsymbol{\omega} = [\omega_1, \dots, \omega_K]^T$ and $\boldsymbol{\xi} = [\boldsymbol{\xi}_1^T, \dots, \boldsymbol{\xi}_K^T]^T$. After obtaining the CFO estimates, the model of Eq. (4) is used to estimate $\boldsymbol{\mu} = [\mu_1, \dots, \mu_K]^T$ and $\boldsymbol{h}' = [\boldsymbol{h}_1'^T, \dots, \boldsymbol{h}_K'^T]^T$. In this way, the estimation of CFOs is decoupled from the estimation of timing errors.

3. JOINT ESTIMATION OF SYNCHRONIZATION PARAMETERS

3.1. CFO Estimation

A. The Alternating-Projection Estimator

It can be shown that joint ML estimation of ω and ξ based on the observation of y requires a numerical search over a multidimensional space. This prevents the possibility of using the ML frequency estimator in practical systems. A simpler solution was proposed in [5] to estimate the CFO of each user sequentially using the alternating-projection algorithm [6]. In this way, the multidimensional problem is reduced to a succession of 1D searches. The resulting scheme has affordable complexity and it is referred to as the Alternating-Projection Frequency estimator (APFE). As shown in [5], the complexity of the CFO estimator can be reduced further if the matrix inversion involved in APFE is approximated by the von Neumann series. This leads to a sub-optimal but simple estimation algorithm called the Approximate APFE (AAPFE). We refer to [5] for more details and explanations about APFE and AAPFE.

B. CRB Analysis

The computation of the CRB for the estimation of Δf_k is lengthy and is not pursued here for space limitations. Carrying out the calculations, it turns out that

$$\operatorname{CRB}\left(\Delta f_{k}\right) = \frac{2\pi^{2}\sigma_{v}^{2}}{N^{2}} \left[\left(\Re \left\{ \boldsymbol{\Psi}^{H} \boldsymbol{\Pi}_{\tilde{\boldsymbol{A}}}^{\perp} \boldsymbol{\Psi} \right\} \right)^{-1} \right]_{k,k}, \quad (9)$$

where $\Re \{z\}$ denotes the real part of z, while $\Pi_{\tilde{A}}^{\perp} \stackrel{\text{def}}{=} I - \tilde{A} (\tilde{A}^{H} \tilde{A})^{-1} \tilde{A}^{H}$ where n_{k} is a disturbance vector and $S(\mu_{k})$ is an $N_{g} \times L_{h}$ matrix with $\tilde{A} = [\Gamma_{1}A_{1} \cdots \Gamma_{K}A_{K}]$. Also, $\Psi = [\psi_{1} \ \psi_{2} \cdots \psi_{K}]$ with entries with $\psi_{k} = M\Gamma_{k}A_{k}\xi_{k}$ and $M = \text{diag}\{0, 1, \dots, N-1\}$. The derivation is skipped here for space limitation but available in [7].

3.2. Joint Estimation of Timing Offsets and Channel Responses

A. ML Estimation

We begin by rewriting Eq. (4) into the following form

$$\boldsymbol{y} = \boldsymbol{G}\left(\boldsymbol{\omega}, \boldsymbol{\mu}\right)\boldsymbol{h}' + \boldsymbol{v}, \tag{10}$$

where

$$\boldsymbol{G}(\boldsymbol{\omega},\boldsymbol{\mu}) = \begin{bmatrix} e^{j\bar{\omega}_1}\boldsymbol{\Gamma}(\omega_1)\boldsymbol{D}_1(\mu_1) & e^{j\bar{\omega}_2}\boldsymbol{\Gamma}(\omega_2)\boldsymbol{D}_2(\mu_2) \\ \cdots & e^{j\bar{\omega}_K}\boldsymbol{\Gamma}(\omega_K)\boldsymbol{D}_K(\mu_K) \end{bmatrix}$$
(11)

If the frequency offsets ω were perfectly known, from Eq. (10) we see that the ML estimates of μ and h' could be obtained by examining the minimum of

$$\Pi\left(\tilde{\boldsymbol{\mu}}, \tilde{\boldsymbol{h}}'\right) = \left\|\boldsymbol{y} - \boldsymbol{G}\left(\boldsymbol{\omega}, \tilde{\boldsymbol{\mu}}\right) \tilde{\boldsymbol{h}}'\right\|^2$$
(12)

with respect to $\tilde{\mu}$ and \tilde{h}' .

In practice, however, ω is unknown and only its estimate $\hat{\omega}$ is available. Therefore, we propose to replace $G(\omega, \tilde{\mu})$ in Eq. (12) by $G\left(\hat{\omega}, \tilde{\mu}
ight)$. To proceed further, we keep $\tilde{\mu}$ fixed and minimize $\Pi\left(\tilde{\mu}, \tilde{h}'\right)$ with respect to \tilde{h}' . This produces

$$\hat{\boldsymbol{h}}'\left(\hat{\boldsymbol{\omega}},\tilde{\boldsymbol{\mu}}\right) = \left[\boldsymbol{G}^{H}\left(\hat{\boldsymbol{\omega}},\tilde{\boldsymbol{\mu}}\right)\boldsymbol{G}\left(\hat{\boldsymbol{\omega}},\tilde{\boldsymbol{\mu}}\right)\right]^{-1}\boldsymbol{G}^{H}\left(\hat{\boldsymbol{\omega}},\tilde{\boldsymbol{\mu}}\right)\boldsymbol{y} \quad (13)$$

Then, substituting Eq. (13) into Eq. (12) and minimizing with respect to $\tilde{\mu}$ yields

$$\hat{\boldsymbol{\mu}} = \operatorname*{arg\,max}_{\tilde{\boldsymbol{\mu}}} \left\{ \| \boldsymbol{P}_{\boldsymbol{G}}\left(\hat{\boldsymbol{\omega}}, \tilde{\boldsymbol{\mu}}\right) \boldsymbol{y} \|^{2} \right\}, \tag{14}$$

where

$$P_{G}(\hat{\omega},\tilde{\mu}) = G(\hat{\omega},\tilde{\mu}) \left[G^{H}(\hat{\omega},\tilde{\mu}) G(\hat{\omega},\tilde{\mu}) \right]^{-1} G^{H}(\hat{\omega},\tilde{\mu}).$$
(15)

The iterative procedure discussed in [5] can be applied to solve the multidimensional minimization problem in Eq. (14). This results in a scheme that we call the alternating-projection timing estimator (APTE). Note that APTE requires an initial estimate of μ , say $\hat{\mu}^{(0)}$, which can be obtained by the following steps. First, vector $\hat{\boldsymbol{\xi}}$ is partitioned into K adjacent blocks $\hat{\boldsymbol{\xi}}_k$ $(k = 1, 2, \dots, K)$, each of size N_g . Then, bearing in mind the structure of $\boldsymbol{\xi}_k$ shown in Eq. (7), we take $\hat{\mu}^{(0)}$ as the index of the first significant element of $\hat{\xi}_k$. Once $\hat{\mu}$ is computed using the alternating-projection algorithm, it is employed in Eq. (13) to estimate the users' channel responses as

$$\hat{\boldsymbol{h}}' = \left[\boldsymbol{G}^{H}\left(\hat{\boldsymbol{\omega}}, \hat{\boldsymbol{\mu}}\right) \boldsymbol{G}\left(\hat{\boldsymbol{\omega}}, \hat{\boldsymbol{\mu}}\right)\right]^{-1} \boldsymbol{G}^{H}\left(\hat{\boldsymbol{\omega}}, \hat{\boldsymbol{\mu}}\right) \boldsymbol{y}.$$
 (16)

B. Suboptimal Method

We now discuss a simple yet suboptimal method to estimate μ_k and h'_k based on $\hat{\boldsymbol{\xi}}_k$. To this purpose, we return to Eq. (7) and observe that ξ_k can be written as

$$\hat{\boldsymbol{\xi}}_k = \boldsymbol{S}(\mu_k)\boldsymbol{h}'_k + \boldsymbol{n}_k, \qquad (17)$$

$$\left[\boldsymbol{S}(\mu_k)\right]_{i,j} = \begin{cases} 1, & \text{if } i-j = \mu_k, \\ 0, & \text{otherwise.} \end{cases}$$
(18)

From Eq. (17) we see that the least-squares (LS) estimates of μ_k and h'_k can be found by minimizing the cost function

$$J\left(\tilde{\mu}_{k}, \tilde{\boldsymbol{h}}_{k}^{\prime}\right) = \left\|\hat{\boldsymbol{\xi}}_{k} - \boldsymbol{S}(\tilde{\mu}_{k})\tilde{\boldsymbol{h}}_{k}^{\prime}\right\|^{2}, \qquad (19)$$

with respect to $\tilde{\mu}_k$ and \tilde{h}'_k . Minimizing with respect to \tilde{h}'_k and exploiting the identity $S^{H}(\tilde{\mu}_{k})S(\tilde{\mu}_{k}) = I_{L_{h}}$ yields

$$\hat{\boldsymbol{h}}_{k}^{\prime}(\tilde{\mu}_{k}) = \boldsymbol{S}^{H}(\tilde{\mu}_{k})\hat{\boldsymbol{\xi}}_{k}.$$
(20)

Next, substituting Eq. (20) into Eq. (19) and minimizing with respect to $\tilde{\mu}_k$ produces

$$\hat{\mu}_{k} = \operatorname*{arg\,max}_{\tilde{\mu}_{k}} \left\{ \left\| \boldsymbol{S}^{H}(\tilde{\mu}_{k}) \hat{\boldsymbol{\xi}}_{k} \right\|^{2} \right\}$$
(21)

Finally, the estimate of h'_k is obtained by inserting $\hat{\mu}_k$ into Eq. (20), i.e.,

$$\hat{\boldsymbol{h}}_{k}^{\prime} = \boldsymbol{S}^{H}(\hat{\boldsymbol{\mu}}_{k})\hat{\boldsymbol{\xi}}_{k}.$$
(22)

Compared with APTE, the estimator in Eq. (21) is easier to implement as it avoids any matrix inversion. For this reason, it is called the low-complexity timing estimator (LCTE).

4. SIMULATION RESULTS

We consider an OFDMA system employing N = 128 subcarriers and operating in the 5 GHz frequency band. The signal bandwidth is 20 MHz, corresponding to a sampling period $T_s = 50ns$. The channel response of each user is generated according to the HIPERLAN/2 channel model with eight paths $(L_k = 8)$. The channel coefficients are modeled as independent and complex-valued Gaussian random variables with zero-mean and an exponential basis an random variables with Zero-fican and an exponential power delay profile $E \{|h_k(l)|^2\} = \lambda_k e^{-l}, l = 0, 1, \dots, 7$. The constant λ_1 is chosen such that the signal power of user#1 is nor-malized to unity, i.e., $E \{\sum_n |H_1(n)|^2\} = 1$, where $H_1(n)$ is the channel frequency response of user#1 over the *n*th subcarrier and reads $H_1(n) = \sum_{l=0}^{7} h_1(l)e^{-j2\pi nl/N}$ for $n = 0, 1, \dots, 127$. Correspondingly, we define the average SNR as $1/\sigma_v^2$, where σ_v^2 is the variance of the Gaussian noise.

We assume an overall instability of the transmit/receive oscillators of 10 ppm, corresponding to $|\Delta f_k| \leq 0.32$. The cell radius is 150 m, so that the maximum propagation delay is $1\mu s$. Bearing in mind that $T_s = 50ns$, we see that the the maximum of μ_k equal to 20. The training blocks have a CP of length $N_q = 28$ so as to accommodate both the channel response duration and the maximum propagation delay. To reduce the system overhead, a shorter CP of length $N'_q = 8$ is employed during the data section of the frame. We assume that two users with equal power are active in the system and 50 subcarriers are randomly assigned to each of them. Without loss of generality, only the results for user #1 are illustrated below.

The performance of the timing estimators is evaluated in terms of the average IBI power due to imperfect estimates of μ_k . For a fixed error $\Delta \mu_k = \hat{\mu}_k - \mu_k$ and a channel response $\{h_k(l)\}$, the IBI power is given by [8]

$$\boldsymbol{I}(\Delta \epsilon_{k,l}, \boldsymbol{h}_k) = \sum_{l=0}^{L_k - 1} |h_k(l)|^2 \left[2 \frac{\Delta \epsilon_{k,l}}{N} - \left(\frac{\Delta \epsilon_{k,l}}{N}\right)^2 \right], \quad (23)$$

where

$$\Delta \epsilon_{k,l} = \begin{cases} \Delta \mu_k - l & \text{if } \Delta \mu_k > l \\ -N'_g + l - \Delta \mu_k & \text{if } \Delta \mu_k < l - N'_g \\ 0 & \text{otherwise.} \end{cases}$$
(24)

Fig. 2 illustrates the mean square error (MSE) of the frequency estimates as a function of the SNR. The CRB and the performance of the frequency synchronizer proposed by Morelli and Mengali (MMFE) in [2] are also shown for comparison. The parameter M with AAPFE indicates the order of approximation in the von Neumann series. We see that APFE gives the best results and approaches the CRB. The MMFE has the worst performance since it was originally derived for single-user systems and it has no protection against MAI. As for AAPFE, its accuracy improves with M and approaches that of APFE as M grows large.



Fig. 2. MSE of the CFO estimates vs. SNR.

Fig. 3 shows the average IBI power vs. SNR for both APTE and LCTE. The frequency estimates are ideal, i.e., $\hat{\omega} = \omega$, and the number of iterations with APTE is $N_c = 2$. It turns out that APTE performs much better than LCTE. The price for this is an increase of the computational load.

The overall system performance is computed in terms of uncoded bit-error-rate (BER). Fig. 4 shows the BER of a coherent QPSK system employing the proposed frequency and timing estimators. Channel estimates are computed according to Eq. (16). The zero-forcing equalization technique is employed in obtaining Fig. 4. The curve labeled "Ideal" is obtained with perfect knowledge of the channel response and synchronization parameters ($\hat{\omega} = \omega$, $\hat{\mu} = \mu$ and $\hat{h}_k = h_k$). At a bit error rate of 10^{-2} , we see that the use of APFE in conjunction with APTE entails a loss of approximately 1dB with respect to the ideal system.

5. REFERENCES

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Fig. 3. Average IBI power vs. SNR due to timing errors



Fig. 4. BER with uncode QPSK

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