# IMPULSE NOISE PROTECTION FOR MULTICARRIER COMMUNICATION SYTEMS

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# ABSTRACT

Impulse noise in multicarrier communication systems behaves effectively as a modulating signal that controls the first moment of the background Gaussian noise. The composite noise, which is the aggregate of the Gaussian noise and impulse noise, has a probability density function that is conditionally Gaussian with non-zero average, hence referred to as Biased-Gaussian. The BER-equivalent power of the composite noise source is defined as the power of a pure Gaussian noise source that yields the same bit-error rate (BER). The BER-equivalent noise for a Biased-Gaussian noise is simply the amplified version of the underlying Gaussian noise source. The amplification factor is derived from the characteristics of the impulse interference. Any bit-loading algorithm designed for Gaussian noise sources is also applicable to Biased-Gaussian noise sources provided that the BER-equivalent SNR is used in place of the measured SNR.

## **1. INTRODUCTION**

In a multicarrier communication system, such as Asymmetric Digital Subscriber Line (ADSL) based on Discrete Multi-Tone (DMT), data is transmitted over a number of mutually independent sub-channels or tones. Each sub-channel carries only a certain portion of data through Quadrature Amplitude Modulation (QAM) of the sub-carrier. The number of information bits loaded on each tone and the size of corresponding QAM constellation may potentially vary from one tone to another and depend generally on the relative power of signal and noise at the receiver. When the characteristics of signal and noise are known for all tones, a bit-loading algorithm can determine the optimal distribution of data bits and signal power amongst sub-channels.

The goal of bit loading is to find the right balance between three key quantities: data rate, transmission power and the probability of error. The algorithm usually optimizes for one of these quantities while the other two are fixed at their target values. This optimization is often constrained on conditions such as limited transmission power, power spectrum mask and integer number of bits per tone. The research on bit-loading has been fairly extensive with many proposed algorithms with various optimization goals, constraints, and complexities [1-7]. These algorithms, however, mostly share the same model of additive white Gaussian (AWG) for the noise source. Although this model matches the characteristics of many noise sources, it is not quite applicable to some important sources of interference and error. This paper addresses specifically one such source: impulse noise.

Major sources of impulse noise in ADSL systems are electric AC power switches and motors. For instance, an electric light dimmer switch with wires running in the proximity of the telephone wiring in the customer premises is potentially a strong impulse interferer. ADSL systems are equipped with Reed-Solomon error correction coding to increase their resilience to impulse noise sources. However, the use of Reed-Solomon coding is optional. Some applications are sensitive to the interleaving latency often associated with Reed-Solomon coding and many ADSL lines are provisioned without that type of coding.

The AWG noise model generally under-estimates the effect of impulse noise. Consequently, bit-loading based on AWG model may yield a significantly higher error rate in the presence of impulse interferers. Another model that better matches the impulse noise characteristics is Biased-Gaussian model, which is simply an AWG noise with non-zero average. This paper presents an analysis of the error rate for Biased-Gaussian noise sources and its implications on bit-loading. It is shown how to apply this model to impulse noise and how to measure the parameters of the model.

### 2. BIASED-GAUSSIAN NOISE MODEL

A DMT transmitter modulates each sub-carrier with a data point in a QAM constellation. Because of the noise and other sources of interference, the receiver detects the data point with some distance from the transmitted constellation point. In order to avoid error in decoding data, the demodulated point should not pass the so-called decision boundary – the midway border between neighboring constellation points. Given the probability distribution of the noise, one can calculate the minimum distance between constellation points such that the probability of error is less than some target value. When the power of noise is low, the minimum distance can be chosen to be small. This results in a denser constellation that carries more data bits.

Fig. 1-a shows the probability density function of a one dimensional noise source. A decoding error occurs when the value of noise is larger than half the minimum distance. The probability of error is proportional to the area of the highlighted region. For a Gaussian noise source, this area is determined solely by the ratio of the minimum distance of constellation points, d, to the standard deviation of noise,  $\sigma$ . Therefore, in order to maintain a target probability of error, this ratio has to be set to a constant value as

$$\frac{d}{\sigma} = C \tag{1}$$

where C is a factor that depends on the target error rate and also coding gain and noise margin. For instance, for an error rate of  $10^{-7}$  with no coding gain and noise margin, the value of C is close to 20.5 dB. Although this factor is also a function of constellation size, the relationship is fairly weak and is ignored in this analysis.

Similarly, when the Gaussian noise source is biased with a non-zero average of  $\mu$ , the probability of error is dominated by the highlighted area shown in Fig. 1-b. In this case, the effective minimum distance, which is twice the distance between decision boundary and the center of distribution function, is smaller than the minimum distance of constellation and consequently the probability of error is higher than the case of unbiased noise. The effective minimum distance,  $d_{eff}$ , can be easily derived as

$$d_{eff} = d - 2\mu \tag{2}$$

To maintain a target error rate, the effective minimum distance has to satisfy equation (1) as follows

$$\frac{d_{eff}}{\sigma} = \frac{d - 2\mu}{\sigma} = C \tag{3}$$

It is insightful to define a BER-equivalent noise source for a biased Gaussian noise source as an unbiased Gaussian noise with the same error rate as the original biased noise. In order to obtain the same error rate the standard deviation of the equivalent noise source,  $\sigma_{eq}$ , has to satisfy the following

$$\frac{d}{\sigma_{eq}} = C \tag{4}$$

From equations (3) and (4), the standard deviation of the equivalent noise source is calculated from the parameters of the biased Gaussian noise source as



Fig. 1- Probability density functions of (a) an unbiased Gaussian, and (b) biased Gaussian noise sources. The highlighted area indicates the probability of receiving a data point out of the decision boundary, hence the probability of error.

$$\sigma_{eq} = \left(1 + \frac{2}{C}\frac{\mu}{\sigma}\right)\sigma \tag{5}$$

This equation indicates that, for the sake of error rate analysis, a non-zero bias of a Gaussian source effectively amplifies the power of noise. Similarly, for the sake of bitloading, a biased Gaussian noise source can be treated as an unbiased noise with the same standard deviation provided that the bit-loading algorithm uses the following additional term for noise margin:

$$M = \left(1 + \frac{2}{C}\frac{\mu}{\sigma}\right)^2 \tag{6}$$

#### **3. COMPENSATION FOR IMPULSE NOISE**

Impulse noise is an additive source that is only active for very short intervals in time. Because of its small duty cycle, the average power of impulse noise is much lower than its instantaneous power during active intervals. This results in a large peak-to-average ratio (PAR) - the salient feature of this type of noise. A bit-loading algorithm designed for Gaussian noise sources underestimates the effect of impulses and yields more frequent errors than expected. Impulse noise is a more problematic source of error when it activates frequently with impulse power larger than background noise. In this case, the error events are dominated by the impulse noise and the error rate will be in the order of impulse activation rate. For instance, in ADSL systems, which are prone to impulse interference from AC power switches, the activation rate of the impulse noise is in the order of AC frequency, resulting in an error rate much higher than the target value.

Fig. 2 shows a typical error scatter plot when both Gaussian and impulse noise sources are present in the communication channel. This plot shows a QAM cell corresponding to the constellation point at the center. Each marked point in this plot represents a detected data point at the receiver. The distance of these points from the center shows the detection error. The cluster of points at the center has a Gaussian distribution and represents the detection error due to the background noise when there is no impulse interference. The outlying points, which form a ring in the scatter plot, consists of detected points received during the active intervals of the impulse noise.

Fig. 3 shows a simple noise model that combines the effects of the background Gaussian noise and the impulse noise. The value of impulse noise during active periods is expressed as  $re^{j\theta}$ , where *r* is the impulse magnitude and  $\theta$  is the impulse phase. A simple model for impulse noise assumes a uniform distribution for impulse phase and a Gaussian distribution with an average of  $\mu_1$  and a standard deviation of  $\sigma_1$  for impulse magnitude.

Although it is possible to derive the parameters of this model from a set of measurements using, for instance, a Maximum-Likelihood parameter estimation of a Gaussian mixture, the following reasonable assumptions simplify the analysis significantly:

A1 - The impulse noise activation frequency is much higher than the target error rate.

A2 - The average value of impulse noise amplitude is much larger than the standard deviation of the background Gaussian noise.

A3 - The variation of the impulse amplitude is negligible comparing to the background noise.

A4 - The impulse noise has a very small duty cycle.

With these assumptions, the outliers in the scatter plot dominate the error events. Therefore, for the sake of errorrate analysis and bit-loading, one can only consider these data points. Given the value of impulse noise, these points have a biased-Gaussian distribution with a mean value equal to the impulse amplitude and a standard deviation identical to that of background Gaussian noise. These two parameters can be measured as the peak and average (standard deviation or root-mean-square) of the error values, respectively.

The compensation margin associated with this biased-Gaussian noise source can be derived as discussed in section 2. In this case, the ratio of mean to standard



Fig. 2- The error scatter plot when the communication channel suffers from both Gaussian and impulse noise sources. The cluster of points at the center represents the effect of Gaussian noise when impulse noise is not active. The outer ring shows the impact of impulse noise.



Fig. 3- A noise model that combines the effects of background Gaussian noise and the impulse noise.

deviation in equation (6) is simply the peak-to-average ratio of the measured error samples. Therefore, the compensation margin for impulse noise is expressed as

$$M_{I} = \left(1 + \frac{2}{C} PAR_{e}\right)^{2} \tag{7}$$

where  $PAR_e$  denotes the peak-to-average ratio of the of error samples in each dimension.

### 4. RESULTS

We simulated a 12,000-foot ADSL line with -140 dBm/Hz of white Gaussian noise. We chose a 120 Hz impulse noise that generated an average PAR of 5 over the

downstream band. The bit-loading algorithm targeted a bit-error rate of BER=10<sup>-7</sup>. Table 1 shows that a bit-loading using impulse compensation margin results in a bit-error rate close to the target value.

	Bit rate	BER
Gaussian noise only	4512 Kbps	6.3 x 10 <sup>-8</sup>
Gaussian + Impulse noise No Impulse Compensation	4128 Kbps	1.4 x 10 <sup>-3</sup>
Gaussian + Impulse noise With Impulse Compensation	3392 Kbps	2.7 x 10 <sup>-7</sup>

TABLE I. DATA RATE AND BIT ERROR RATE IN PRESENCE OF IMPULSE NOISE

We also conducted several experiments using a real ADSL modem on various loop lengths with a -140 dBm/Hz of AWG background noise and a dimmer light switch as a realistic source of impulse interference. The impulse frequency for this source is twice the AC frequency. Given strong impulse interference, every single ADSL superframe contains a CRC error and the effective data packet rate drops to zero. To compensate for impulse noise, its presence is first detected automatically when the frequency of outliers in scatter plot is measured to be high. Once the line is identified as impulsive, the compensation margin of equation (7) is applied to each tone with large error PAR.

Figure 4 compares the data rate at physical layer and the effective rate achieved in a file transfer protocol (FTP) both with and without impulse compensation. The connection provisioned for 6 dB of target noise margin with a maximum bit rate of slightly higher than 8 Mbps. Impulse compensation reduces the bit rate at physical layer but it also adds enough noise margin to reduce the CRC rate such that the packet transfer rate at upper network layers is significantly improved.

# **5. CONCLUSIONS**

This paper presented a method to compensate for a class of non-Gaussian impairments in multicarrier bit-loading. It also presented the application of this method to impulse noise, a practically important source of disturbance in multicarrier DSL systems. Experiments and simulations have shown that with simple noise models, this method provides a robust bit-loading which reduces the CRC error significantly and supports a high data packet rate even in the presence of strong impulse interferers. Although the problem is studied in the context of DMT in an ADSL system, the results are also applicable to any multicarrier system that allows flexible distribution of bits and power among sub-channels.



Fig. 4- Data rate versus loop length in presence of impulse noise. Solid and dashed lines indicate the bit rate at physical layer and the average FTP throughput, respectively. Stars and circles mark the rates with and without impulse compensation, respectively.

#### 6. REFERENCES

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