MAXIMUM LIKELIHOOD INTER-CARRIER INTERFERENCE SUPPRESSION FOR WIRELESS OFDM WITH NULL SUBCARRIERS

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ABSTRACT

Orthogonal Frequency Division Multiplexing (OFDM) transmission is robust to frequency-selective channels but sensitive to time-selective channels. Time variations of channels destroy the orthogonality between subcarriers, resulting in a considerable performance loss due to inter-carrier interference (ICI) between subcarriers. In this paper, we propose a Viterbi-type algorithm to effectively suppress the ICI, by exploiting the property of ICI terms and null subcarriers embedded in OFDM symbols for the reduction of interferences from/to adjacent bands. Simulations show that the proposed equalizer works well with affordable complexity and outperforms linear equalizers.

I. INTRODUCTION

Severe time-varying multipath channels often arise in high-rate digital transmissions. Conventional single-carrier transmissions with linear equalization significantly suffer from inter-symbol interference (ISI) resulting from the multipath. Orthogonal Frequency Division Multiplexing (OFDM) is a promising high-rate transmission technique, which mitigates ISI by inserting cyclic prefix (CP) at the transmitter. If the channel delay spread is shorter than the duration of CP, ISI is completely removed. Moreover, if the channel remains constant within one OFDM symbol duration, OFDM renders a convolution channel into parallel flat channels, which enables simple one-tap frequency-domain equalization. OFDM has been adopted in wireless LAN standards, e.g. , IEEE802.11a and HIPERLAN/2, and in digital audio/video broadcasting. However, OFDM is sensitive to channel variations, which arise from the relative motion between the transmitter and the receiver as well as from the presence of the carrier frequency offset (CFO). They destroy the orthogonality between subcarriers and generate inter-carrier interference (ICI). While the ICI induced by a CFO may be compensated for at the receiver, the ICI by the Doppler spread is not easily suppressed, since signals from different paths have different Doppler frequencies.

Assuming that the channel varies temporally in a linear fashion, a frequency-domain equalizer was presented in [1] but cannot be applied to rapidly time-varying channels where the assumption does not hold true. Block-wise equalizers based on zero forcing (ZF) and minimum mean squared error (MMSE) criteria were considered in [2]. To enhance the performance, a successive interference cancellation was incorporated into the equalization. A low-complexity decision feedback equalizer (DFE) was developed in [3] to reduce the computational complexity of block-wise equalizers. Both equalizers exhibit better performance than linear equalizers without decision feedback but suffer from error propagations at low and moderate SNR.

Maximum likelihood sequence estimator (MLSE) [4, Sec. 10.1] is the optimal equalization scheme for minimizing bit error rate (BER), while DFE is sub-optimal. The Viterbi algorithm (VA) in the time domain for MLSE is available [5]. VA is more efficient than the exhaustive ML search but its complexity increases exponentially with the channel length, which makes its utilization in high-rate transmissions over multipath channels difficult.

In this paper, we propose a VA in the frequency domain to suppress the ICI, by exploiting the structure of the ICI and null subcarriers, which are originally set in each OFDM symbol to mitigate interferences from/to adjacent OFDM channels. Thanks to the ICI power concentration in OFDM transmissions, the computational complexity of the VA becomes affordable. The efficiency of the proposed equalization is verified by numerical simulations. Under reasonable settings, the VA with a short memory length is shown to outperform linear equalizers, exhibiting a strong robustness to the speed of the channel time variation.

II. SYSTEM MODEL AND PRELIMINARIES

We consider point-to-point wireless Orthogonal Frequency Division Multiplexing (OFDM) transmissions over time- and frequency-selective fading channels. For simplicity, we only deal with one OFDM symbol duration.

Let the number of subcarriers be N. At the transmitter, a serial information data sequence $\{s_0, s_1, \ldots, s_{N-1}\}$ undergoes serial-to-parallel (S/P) conversion to be stacked into one OFDM symbol. Then, an N-points inverse fast

Fourier transform (IFFT) follows to produce the N dimensional data, which is parallel-to-serial (P/S) converted. A cyclic prefix (CP) of length N_{cp} is appended in order to mitigate the multipath effects. The discrete-time baseband equivalent transmitted symbols can be expressed as

$$u(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} s_k e^{j\frac{2\pi kn}{N}}, \quad n \in [-N_{cp}, N-1]. \quad (1)$$

Our discrete-time baseband equivalent FIR channel has maximum order L, and is considered time-varying. We assume that N_{cp} is greater than or equal to the channel order L so that there is no inter-symbol interference (ISI) between OFDM symbols. The received signal is written as

$$y(n) = \sum_{l=0}^{L} h(n,l)u(n-l) + w(n), \quad (2)$$

where $h(n,l)$ is the lth channel tap at time n and $w(n)$ is an additive white Gaussian noise (AWGN) with zero mean and variance σ_w^2. Let us define the channel frequency response at frequency $2\pi k/N$ and at time n as

$$H_k(n) = \sum_{l=0}^{L} h(n,l)e^{-j\frac{2\pi kl}{N}}. \quad (3)$$

At the receiver, we assume perfect timing synchronization. After removing CP, we take an FFT of the received signal to obtain for $k \in [0, N-1]$ that

$$Y_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} y(n)e^{-j\frac{2\pi kn}{N}} = \sum_{n=0}^{N-1} H_{k,n}s_n + W_k, \quad (4)$$

where

$$H_{k,n} = \frac{1}{N} \sum_{m=0}^{N-1} H_k(m)e^{j\frac{2\pi m(n-k)}{N}}, \quad (5)$$

and $W_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} w(n)e^{-j\frac{2\pi kn}{N}}$.

If the channel is time-invariant, then the channel frequency response $H_k(n)$ given by (3) is constant in time n. It follows from (5) that $H_{k,n} = H_k(n)\delta(n-k)$, where $\delta(\cdot)$ stands for Kronecker's delta. In this case, we have $Y_k = H_k(k)s_k + W_k$. There is no inter-carrier interference (ICI) between subcarriers, which enables computationally efficient one-tap frequency-domain equalization at kth subcarrier such that $\hat{s}_k = H_k^{-1}(k)Y_k$, where $\hat{s}_k$ is the output of the equalizer. However, channels are in general time-varying due to the relative motion between the transmitter and the receiver. This generates the ICI term $\sum_{n=0,n\neq k}^{N-1} H_{k,n}s_n$ in the R.H.S. of (4). The one-tap frequency-domain equalizer cannot compensate for the effects of the ICI, resulting in a performance floor that increases with the speed of the channel time variation.

From the received signal in (4), we form a receive vector $Y = [Y_0, Y_1, \ldots, Y_{N-1}]^T$, which is expressed as

$$Y = \boldsymbol{H}s + W, \quad (6)$$

where the (k,n)th entry of the channel matrix $\boldsymbol{H}$ is $H_{k,n}$, $s = [s_0, s_1, \ldots, s_{N-1}]^T$, and $W = [W_0, W_1, \ldots, W_{N-1}]^T$. We assume that the channel impulse response is available at the receiver, e.g., by pilot symbol aided channel estimation [2], [6].

For (6), linear block-wise equalizers to suppress the ICI are readily constructed. The zero-forcing (ZF) equalizer outputs $(\boldsymbol{H}^{\mathcal{H}}\boldsymbol{H})^{-1}\boldsymbol{H}^{\mathcal{H}}Y$, where $(\cdot)^{\mathcal{H}}$ denotes the complex conjugate transposition of a matrix. On the other hand, if the receiver knows the symbol variance σ_s^2 and the noise variance σ_w^2, the minimum mean squared error (MMSE) equalizer becomes available, whose output is given by $(\boldsymbol{H}^{\mathcal{H}}\boldsymbol{H}+\sigma_w^2/\sigma_s^2\boldsymbol{I})^{-1}\boldsymbol{H}^{\mathcal{H}}Y$, where $\boldsymbol{I}$ stands for an $N \times N$ identity matrix.

To enhance the performance, the MMSE equalizer with successive interference cancellation was proposed in [2]. One symbol having the largest SNR is hard-detected based on the MMSE equalizer output and then its contribution is subtracted from the received vector. This procedure is repeated until all symbols are detected. The construction of linear equalizers requires $O(N^3)$ computations, which is pretty heavy for large N. A low-complexity MMSE based DFE has also been developed in [3], by utilizing the structure of the channel matrix $\boldsymbol{H}$. But both schemes having decision feedback inevitably suffer from error propagations. In the next section, we will propose a non-linear equalization with affordable complexity, based on maximum likelihood criterion.

III. ICI SUPPRESSION BY VITERBI ALGORITHM

At the receiver, the energy of s_k at the kth subcarrier is leaked only to its neighboring (in a circular fashion) subcarriers [3]. Ignoring the ICI terms which do not significantly affect the kth subcarrier, we assume that the ICI terms come only from $2K$ neighboring subcarriers, i.e.,

A1 $H_{k,m} = 0$ for $k+K < m < N-K+k, k \in [0, K-1]$, for $|k-m| > K, k \in [K, N-K-1]$, and for $k-N+K < m < k-K, k \in [N-K, N-1]$.

The channel matrix is illustrated in Fig. 1, where the entries without dots are assumed to be zero.

We would like to obtain a maximum likelihood equalizer under the assumption A1. Since ICI terms appear in a circular fashion, an exhaustive search is required, which makes its implementation difficult in practice. To avoid the exhaustive search, we will exploit successive null subcarriers, which are usually embedded in every OFDM symbol to mitigate interferences from/to adjacent OFDM channels. For example, IEEE 802.11a standard sets 11 null subcarriers at $k \in [27, 37]$.

Without loss of generality and for notational simplicity, we put two successive null subcarriers at the top and the bottom of s, i.e.,

$$s_k = 0, \quad k \in [0, N_{G_1}-1], k \in [N-N_{G_2}, N-1]. \quad (7)$$

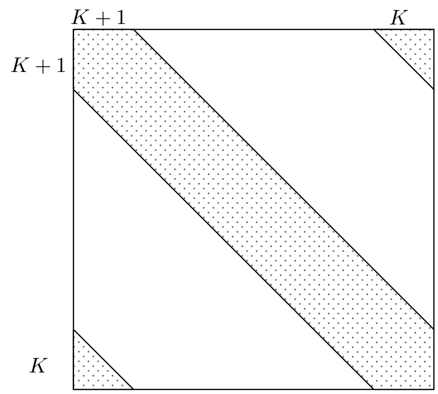

Fig. 1. Channel matrix $\boldsymbol{H}$

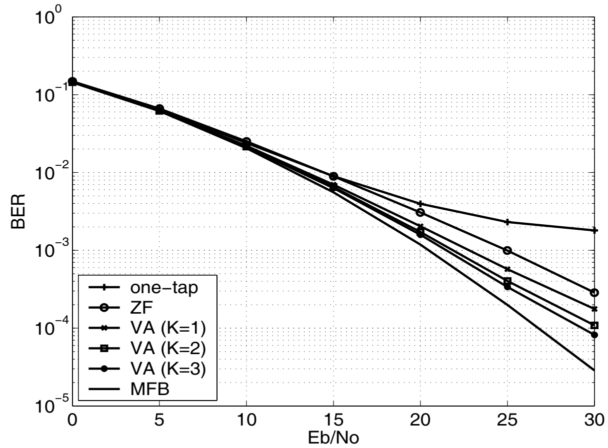

Fig. 2. BER comparison for $f_d T_c = 0.001$

We assume that $K \leq N_{G_1}$ and $K \leq N_{G_2}$. Then, taking (7) into account, we can consider the first N_{G_1} and the last N_{G_2} columns of $\boldsymbol{H}$ as zero vectors. It follows from (4) that

$$Y_k = \sum_{n=-K}^{K} H_{k,n} s_n + W_k. \tag{8}$$

Although the coefficients $H_{k,n}$ vary in k, Y_k depends only on $2K+1$ successive s_n. Thus, a dynamic programming approach can be applied to obtain the optimal maximum likelihood sequence, which results in a Viterbi Algorithm (VA).

Suppose that the constellation is drawn from a finite alphabet of size N_A. Let us define the state $\mathcal{S}_k$ as $\mathcal{S}_k = [s_{k+K-1}, s_{k+K-2}, \ldots, s_{k-K}]$. Then, the number of states is N_A^{2K}. We denote the estimate of s_n as $\hat{s}_n$ and define the estimate of Y_k as

$$\hat{Y}_k = \sum_{n=-K}^{K} H_{k,n} \hat{s}_n. \tag{9}$$

To obtain all possible $\hat{Y}_k$, $(2K+1)N_A^{2K+1}$ computations are required.

Let the probability density function of the received sequence $\{Y_0, Y_1, \ldots, Y_k\}$ conditioned on the transmitted sequence $\{s_0, s_1, \ldots, s_{k+K}\}$ be $p(Y_0, Y_1, \ldots, Y_k | s_0, s_1, \ldots, s_{k+K})$. Under the assumption A1, since the noise is AWGN, the log-likelihood function at k, defined as

$$\mathcal{L}_k := \log p(Y_0, Y_1, \ldots, Y_k | s_0, s_1, \ldots, s_{k+K}),$$

can be factored into

$$\mathcal{L}_k = \mathcal{L}_{k-1} - \left| Y_k - \hat{Y}_k \right|^2. \tag{10}$$

We compute for every state, the path metric $|Y_k - \hat{Y}_k|^2$ of N_A possible state transitions, which requires N_A^{2K+1} computations. Then, we select a state transition, or equivalently a *surviving path* in the state-diagram trellis, that has the maximum log-likelihood for that state, to obtain candidates of the maximum likelihood sequence. In total, our algorithm requires $O[2(K+1)N_A^{2K+1}]$ computations per information symbol.

Suppose a QPSK constellation, i.e., $N_A = 4$, and take $K = 2$. Let N be 64. Then, the complexity per information symbol of our algorithm is about $2 \cdot 3 \cdot 4^5 = 1.5 \cdot 2^{12}$, which is comparable with the complexity $O(64^3/64) = O(2^{12})$ of the linear equalizers. The complexity of our algorithm depends on the constellation size N_A and the value of K, while the complexities of linear equalizers depend on the number of subcarriers N. As shown by simulations, even for rapidly changing channels, small K is enough to achieve moderate performance. Thus, for small N_A and large N, our non-linear equalization becomes more computationally efficient than linear equalizations.

IV. NUMERICAL EXAMPLES

We test the proposed VA with $K = 1, 2, 3$ for different maximum Doppler frequencies. Each OFDM symbol has 64 subcarriers with 4 successive null subcarriers at the frequency edges. Of 64 subcarriers, 56 are used to carry information data.

We generate 10^3 Rayleigh channels, having 8 complex zero-mean Gaussian taps with identical power profile. Channel taps are independent of each other and fade according to the Jakes fading model [7], where each tap is generated as in [8]. The length of the cyclic prefix is 8 so that there is no ISI.

BPSK constellation is adopted. We compare BER of VA with BERs of one-tap equalizer and ZF equalizer. For $K = 1, 2, 3$, our VA equalizer requires about $4 \cdot 2^3 = 32$, $6 \cdot 2^5 = 192$, and $8 \cdot 2^7 = 1024$ computations

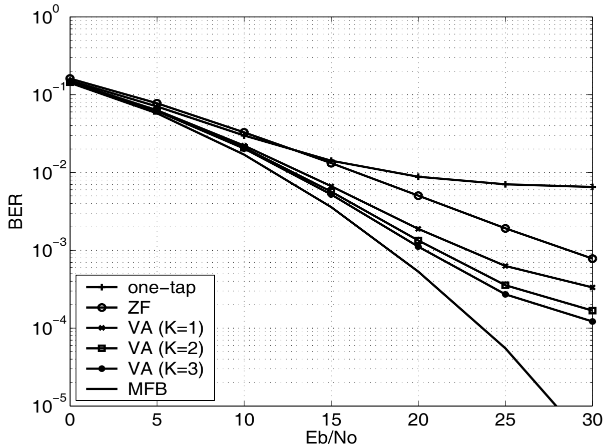

Fig. 3. BER comparison for $f_d T_c = 0.002$

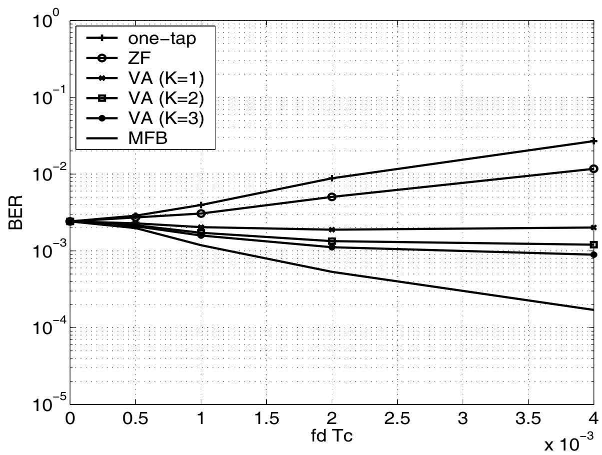

Fig. 4. BER comparison against the maximum Doppler frequency at a fixed $E_b/N_o = 20$dB

per information symbol, while the ZF equalizer requires $O(4096)$ computations per information symbol.

For $f_d T_c = 0.001$, where f_d and T_c respectively denote the maximum Doppler frequency and the chip duration time, BER performance of our tested systems are illustrated in Fig. 2. The matched filter bound (MFB) [3] is also plotted, which is the best performance if ICI is completely canceled without noise enhancement.

One-tap equalizer exhibits a performance floor at high SNR due to ICI, since it does not take the ICI into account at all. The ZF equalizer does not have a performance floor and outperforms the one-tap equalizer. Even with $K = 1$, VA has better performance than ZF equalizer. For VA, no significant improvement can be seen in increasing from $K = 2$ to $K = 3$. This implies that most of the ICI on each subcarrier comes from its four neighboring subcarriers.

Fig. 3 depicts BER performances for relatively fast fading channels with $f_d T_c = 0.002$. Compared with the former case, the performances of one-tap equalizer and ZF equalizer degrade, while the performance of VA remains more or less unchanged. The VA with $K = 1$, whose computational complexity per information symbol is just 32, significantly outperforms the ZF equalizer.

To see the impact of the maximum Doppler frequency on BER performance, Fig. 4 shows BERs for different maximum Doppler frequencies, $(0, 0.0005, 0.001, 0.002, 0.004)$, at a fixed $E_b/N_0 = 20$dB. Increase in the channel time variation has two major effects: i) More ICI power arises, which degrades the BER performances of one-tap equalizer and ZF equalizer. ii) More time diversity gain becomes available, as suggested by the MFB, which enhances BER performance if ICI is suppressed appropriately. As can be seen in Fig. 4, VA is robust to the maximum Doppler frequency, balancing the two contradicting Doppler frequency effects. Up to $f_d T_c = 0.004$, the VA even with $K = 1$ achieves the same performance without ICI, i.e., $f_d = 0$. This highlights the robustness of VA to the channel time variation.

V. REFERENCES

[1] W. G. Jeon, K. H. Chang, and Y. S. Cho, “An equalization technique for orthogonal frequency-division multiplexing systems in time-variant multipath channels,” *IEEE Transactions on Communications*, vol. 47, no. 1, pp. 27–32, January 1999.

[2] Y.-S. Choi, P. J. Voltz, and F. A. Cassara, “On channel estimation and detection for multicarrier signals in fast and selective Rayleigh fading channels,” *IEEE Transactions on Communications*, vol. 49, no. 8, pp. 1375–1387, August 2001.

[3] X. Cai and G. B. Giannakis, “Bounding performance and suppressing inter-carrier interference in wireless mobile OFDM,” *IEEE Transactions on Communications*, vol. 51, no. 12, pp. 2047–2056, December 2003.

[4] J. Proakis, *Digital Communications*, McGraw-Hill, New York, fourth edition, 2001.

[5] G. D. Forney, Jr., “Maximum-likelihood sequence estimation of digital sequences in the presence of intersymbol interference,” *IEEE Transactions on Information Theory*, vol. 18, no. 3, pp. 363–378, May 1972.

[6] Y. (G.) Li, “Pilot-symbol-aided channel estimation for OFDM in wireless systems,” *IEEE Transactions on Vehicular Technology*, vol. 49, no. 4, pp. 1207–1215, July 2000.

[7] W. A. Jakes, *Microwave Mobile Communications*, John Wiley and Sons, New York, 1974.

[8] J. K. Cavers, *Mobile Channel Characteristics*, Kluwer Academic Publishers, 2000.