PEAK-TO-AVERAGE POWER RATIO REDUCTION IN OFDM WITH BLIND SELECTED PILOT TONE MODULATION

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ABSTRACT

Orthogonal Frequency Division Multiplexing (OFDM) is a spectrally efficient multicarrier modulation technique for high speed data transmission over multipath fading channels. However, the price paid for the high spectral efficiency is low power efficiency. OFDM signals suffer from high peak-to-average power ratios (PARs) which lead to power inefficiency in the RF portion of the transmitter. Selected mapping (SLM) is a promising distortionless technique to reduce the PAR of an OFDM signal. In this paper, we propose a novel technique which links the index of the phase rotation sequence in SLM to the location of the pilot tones that are used to estimate the channel. Each pilot tone location - phase sequence selection can lead to a different PAR value for the time-domain OFDM signal, and the signal with the lowest PAR value is transmitted. Our proposed method is "blind" in the sense that the "optimum" pilot tone location - phase sequence index is not transmitted as side information. We describe a novel technique to blindly detect the optimum index at the receiver by taking advantage of the disparity between the pilot tone and information signal powers. PAR reduction performance as well as BER performance of the proposed method in frequency selective fading channels are illustrated.

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is a spectrally efficient multicarrier modulation technique for high speed data transmission over multipath fading channels. However, the price paid for the high spectral efficiency is low power efficiency.

For frequency selective block fading channels, the channel state information (CSI) can be acquired by modulating pilot tones onto predetermined sub-carriers; this is called pilot tone assisted modulation (PTAM) [1]. OFDM signals suffer from high amplitude fluctuations; i.e., large peak-to-average power ratios (PARs). Large PARs require significant back-off of the average operating power of a RF power amplifier (PA) if the signal is to be linearly amplified. Power inefficiency leads to low battery life for the mobile user and high operating cost for the base station.

Denote by $\{X_l[k]\}_{k=0}^{N-1}$ the *l*th block of the frequency domain OFDM signal drawn from a known constellation, where *N* is the number of sub-carriers. For the rest of the paper, we will drop the block index *l* for notational simplicity, since OFDM can be free of inter-block interference with proper use of the cyclic prefix. The complex baseband OFDM signal can be written as

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X[k] \ e^{j2\pi kt/T_s}, \quad 0 \le t \le T_s, \qquad (1)$$

where T_s is the OFDM symbol period and $j = \sqrt{-1}$. The symbol-wise PAR of x(t) is defined as [2]

$$PAR(x(t)) = \frac{\mathcal{P}_{max}}{\mathcal{P}_{av}},$$
(2)

where $\mathcal{P}_{\max} = \max_{0 \le t \le T_s} |x(t)|^2$ is the peak power, $P_{av} =$

 $\frac{1}{T_s}\int_0^{T_s}|x(t)|^2dt$ is the average power of the OFDM symbol. The topic of PAR reduction has attracted a lot of attention in the past decade. Proposed techniques include (i) distortionless PAR reduction, such as coding, tone reservation, tone injection, selected mapping and partial transmit sequence; (ii) PAR reduction with distortion, such as deliberate clipping, transmit filtering, companding approaches etc.; and (iii) various combinations of the above (see [3–6] and reference therein). These methods entail different PAR reduction capability - complexity - information rate - distortion tradeoffs.

We are interested in the selected mapping (SLM) approach which was first proposed by Bauml, Fischer and Huber in [4]. SLM has a relatively low complexity, is distortionless, and is an effective PAR reduction method. Denote by $\phi_k^{(m)}$, $0 \le k \le N-1$, $0 \le m \le M-1$, a set of M (random) phase sequences. In SLM, we first rotate the phases of X[k] as

$$Z^{(m)}[k] = X[k]e^{j\phi_k^{(m)}}.$$
(3)

It is clear that $Z^{(m)}[k]$ and X[k] contain the same information, but their time-domain counterparts $z^{(m)}(t)$ and x(t)can have very different PAR values. From the M candidate $z^{(m)}(t)$ signals, $z^{(\bar{m})}(t)$, which has the lowest PAR, is transmitted. The index \bar{m} (log₂ M bits) may be transmitted as side information, which is of critical importance to the receiver for decoding and is generally protected by channel coding [4].

To avoid the information rate loss caused by the transmission of the optimum index \bar{m} , a few blind SLM schemes have been proposed. In [5], a scrambling technique was described. A $\log_2 M$ -bit binary label is inserted as prefix to the frequency-domain OFDM signal and passed through a scrambler. Since the selected label is used in the receiver implicitly during descrambling, an erroneous reception of the label bits does not affect the error performance. In [6], a blind SLM receiver was proposed by employing a maximum likelyhood (ML) decoder, which avoids the transmission of any side information.

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This paper combines the merits of PTAM and SLM, and proposes a novel joint channel estimation and PAR reduction scheme. Instead of fixing the pilot tone locations as in conventional PTAM, we try different pilot tone locations, and synchronize the movement of the pilot tones with the choice of the phase rotation sequence. The pilot tone / phase sequence combination that results in the lowest PAR of the time-domain signal is used for transmission. However, the optimum index is not transmitted as side information in order to maintain the information rate. At the receiver, by taking advantage of the disparity between the pilot tone and information signal powers, we can blindly detect the optimum index by resorting to simple time averages.

2. REVIEW OF PTAM-OFDM

In a PTAM-OFDM system, P pilot tones are inserted in the frequency domain in order to acquire the CSI; $P \ge L$ is assumed where L is the length of the finite impulse response (FIR) channel. The transmitted frequency domain signal can be described by

$$X[k] = \begin{cases} B[k], & k \in \mathbf{\Omega}_0\\ S[k], & k \in \mathbf{\Omega}_0^{\perp} \end{cases}, \quad 0 \le k \le N - 1, \qquad (4)$$

where Ω_0 is the set of the *P* pilot tone indices in ascending order, Ω_0^{\perp} denotes the complement of Ω_0 (i.e., the set of N-P information sub-symbol indices in ascending order), $\{B[k]\}_{k\in\Omega_0}$ are the pilot tones, and $\{S[k]\}_{k\in\Omega_0^{\perp}}$ are the frequency-domain information sub-symbols.

According to [1], the optimal way to place the pilot tones is to modulate P = L pilot tones with equal power onto equi-spaced sub-carriers. For simplicity, let us assume that the number of sub-carriers N is an integer multiple of P; i.e., R = N/P is an integer. Next define a set of P equispaced pilot tone indices as

$$\mathbf{\Omega}_0 \triangleq \left\{ k_i \mid k_i = iR + \tau_0, 0 \le i \le P - 1, 0 \le \tau_0 \le R - 1 \right\},$$
(5)

which can be characterized by the delay τ_0 alone.

After taking the IFFT of $\tilde{X}[k]$, a length G cyclic prefix is padded onto the time-domain OFDM symbol x[n] to yield $\tilde{x}[n] = x[n + N - G]_N$, $0 \le n \le N + G - 1$, where $[n]_N$ is the residue of n divided by N. The time-domain signal $\tilde{x}[n]$ is then transmitted through the channel.

We consider a frequency selective block fading channel, which is modeled by a time-invariant (over a block of N + G samples) FIR filter h[n]. The received signal is $\tilde{y}[n] = \tilde{x}[n] * h[n] + v[n]$, $0 \le n \le N + G - 1$, where * denotes linear convolution; h[n] is the impulse response that is the convolution of the transmit filter, the frequency selective channel, and the receive filter; and v[n] is the zero-mean additive noise. After removing the cyclic prefix and taking the FFT on the resulting $y[n] = \tilde{y}[n+G]$, $0 \le n \le N-1$, we obtain a set of N linear equations in the frequency domain

$$Y[k] = X[k]H[k] + V[k], \quad 0 \le k \le N - 1, \tag{6}$$

where $Y[k] = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} y[n] e^{-j2\pi kn/N}$, V[k] is defined similarly as Y[k], and $H[k] = \sum_{n=0}^{L-1} h[n] e^{-j2\pi kn/N}$. Since X[k] = B[k] for $k \in \Omega_0$, we obtain from (6)

Since X[k] = B[k] for $k \in \Omega_0$, we obtain from (6) an estimate of H[k] at the P points of Ω_0 : $\hat{H}[k] = Y[k]/B[k]$, $k \in \Omega_0$. Since H[k] is constrained by P parameters, $\{h[n]\}_{n=0}^{P-1}$, we can then solve for H[k] at any k. Afterwards, the information sub-symbols can be estimated as $\hat{S}[k] = Y[k]/\hat{H}[k]$, $k \in \Omega_0^{\perp}$, which are then decoded to yield the $\bar{S}[k]$ estimates belonging to the known constellations.

3. PROPOSED TECHNIQUE - BLIND SELECTED PILOT TONE MODULATION

In this section, we describe our proposed blind selected pilot tone modulation (BSPTM) technique which is a combination of channel sounding and effective PAR reduction, at a very low computational cost.

3.1. Disparity in the pilot and information signal powers

We first point out an interesting feature of PTAM: the pilot tones generally have stronger average power than the information sub-symbols – this forms the basis of our BSPTM approach.

Let us denote by β , the power allocation factor, which is the ratio between the power allocated to all the pilot tones and the total transmitted power; i.e., $\beta = P\sigma_p^2/(P\sigma_p^2 + (N-P)\sigma_s^2)$, where $\sigma_p^2 = 1/P\sum_{k\in\Omega_0}|B[k]|^2$ is the average power of the pilot tones and σ_s^2 is the variance of S[k]. If B[k] is equi-powered as suggested by [1], $|B[k]|^2 = \sigma_p^2$, $\forall k \in \Omega_0$. It follows from the definition of β ,

$$\frac{\sigma_p^2}{\sigma_s^2} = \frac{\beta(N-P)}{(1-\beta)P} = \frac{\frac{N}{P}-1}{\frac{1}{\beta}-1},\tag{7}$$

which depends on the N/P ratio and β .

In [1], the optimal β was determined as

$$\beta = 1 - \frac{1}{1 + \sqrt{1/(\frac{N}{P} - 1)}},\tag{8}$$

by minimizing the mean-squared error in the source estimates $\hat{S}[k]$, $k \in \mathbf{\Omega}_0^{\perp}$. Substituting the optimum β value of (8) into (7), we find $\sigma_p^2/\sigma_s^2 = \sqrt{N/P - 1}$, which depends on N/P only. Since $N \gg P$, the pilot tones have a much stronger power than the information sub-symbols. For example, for $P \leq 16$ and $N \geq 160$, we have $\sigma_p^2/\sigma_s^2 \geq 3$. On the other hand, if $P \leq 8$ and $N \geq 296$, we have $\sigma_p^2/\sigma_s^2 \geq 6$. Both are realistic scenarios. We will describe later how the $\sigma_p^2/\sigma_s^2 \gg 1$ relationship helps us to detect the pilot tone location parameter τ_0 .

3.2. PAR Reduction by BSPTM

According to [1], as long as the pilot tones are equi-powered and equi-spaced, channel estimation performance is not affected. Therefore, instead of using a pre-selected τ_0 , we can try different delays (positions) for the pilot tones. The novelty of our approach is to tie the location of the pilot tones to the different phase rotation sequences. This enables PAR reduction without the transmission of side information.

Recall the conventional SLM as described in (3), where the phase sequences are indexed by m. Let us use the same m to index the M candidate delays for the pilot tones; i.e., $\boldsymbol{\tau} \triangleq \{\tau_0, \tau_1, \ldots, \tau_m, \ldots, \tau_{M-1}\}.$

The maximum number of distinct pilot tone locations is R = N/P, in which case $\{\tau_0 = 0, \tau_1 = 1, \ldots, \tau_{M-1} = R-1\}$. However, since R can be quite large and we do not usually need M greater than, say 8, there is some flexibility in defining $\boldsymbol{\tau}$. For example, if R = 8 and M = 4, we can have $\{\tau_0 = 0, \tau_1 = 2, \tau_2 = 4, \tau_3 = 6\}$ or $\{\tau_0 = 0, \tau_1 = 1, \tau_2 = 3, \tau_3 = 7\}$, and so on. The particular choice for $\boldsymbol{\tau}$ is not critically important; however, both the transmitter and the receiver should use the same convention for $\boldsymbol{\tau}$.

The mth PTAM-OFDM signal is given by

$$X^{(m)}[k] = \begin{cases} B[k], \, k \in \mathbf{\Omega}_m, \\ S[k], \, k \in \mathbf{\Omega}_m^{\perp}, \\ 0 \le k \le N-1, \, 0 \le m \le M-1, \end{cases}$$

where Ω_m is characterized by τ_m similar to the way that Ω_0 is characterized by τ_0 .

Next perform the phase rotations,

$$Z^{(m)}[k] = X^{(m)}[k]e^{j\phi_k^{(m)}}, \quad 0 \le m \le M - 1.$$
(9)

We emphasize that the m in $X^{(m)}[k]$ and the m in $\phi_k^{(m)}$ are the same, a key feature of our algorithm. Similar to SLM, $z^{(m)}(t)$ and PAR $\left(z^{(m)}(t)\right)$ are evaluated and $z^{(\bar{m})}(t)$, which has the lowest PAR among $\{z^{(m)}(t)\}_{m=0}^{M-1}$, is transmitted. In other words, the optimum pilot tone location phase sequence index is $\bar{m} = \underset{0 \le m \le M-1}{\operatorname{argmin}} \left\{ \operatorname{PAR} \left(z^{(m)}(t) \right) \right\}$. We will demonstrate the PAR reducing capability of BSPTM in Section 4.

3.3. Blind Detection of $\tau_{\bar{m}}$

At the receiver, we need to determine $\tau_{\bar{m}}$, or equivalently, the optimum index \bar{m} . Let us replace the X[k] in (6) by the $Z^{(\bar{m})}[k]$ of (9) and write:

$$Y[k] = Z^{(\bar{m})}[k]H[k] + V[k] = \begin{cases} B[k]e^{j\phi_{k}^{(\bar{m})}}H[k] + V[k], & k \in \Omega_{\bar{m}}, \\ S[k]e^{j\phi_{k}^{(\bar{m})}}H[k] + V[k], & k \in \Omega_{\bar{m}}^{\perp}. \end{cases}$$
(10)

Our task here is to detect $\tau_{\bar{m}}$ from $\{Y[k]\}_{k=0}^{N-1}$, knowing the candidate set of all possible locations; i.e., τ .

- We utilize the following facts in our discussions next:
- 1. V[k] is zero-mean and uncorrelated with S[k],
- 2. $|B[k]|^2 = \sigma_p^2$ is constant $\forall k \in \mathbf{\Omega}_{\bar{m}}$,
- 3. the phase rotation does not affect the power of the subsymbols; i.e., $|X^{(m)}[k]|^2 = |X[k]|^2, \forall m, k.$

We also recall the following notations: $\sigma_s^2 = E[|S[k]|^2]$ and $\sigma_v^2 = E[|V[k]|^2]$. For a given block of data, we can treat H[k] as deterministic. It follows from (10) that

$$E[|Y[k]|^{2}] = \begin{cases} \sigma_{p}^{2} |H[k]|^{2} + \sigma_{v}^{2}, & k \in \Omega_{\bar{m}}, \\ \sigma_{s}^{2} |H[k]|^{2} + \sigma_{v}^{2}, & k \in \Omega_{\bar{m}}^{\perp}. \end{cases}$$
(11)

Next, let us write k = iR + r, where $0 \le i \le P - 1$ and $0 \le r \le R - 1$, and denote by $Y_i[r] = Y[iR + r]$ the *i*th sub-record (of length-*R*) of Y[k]. Denote also $\varepsilon_h(r) = \frac{1}{P} \sum_{i=0}^{P-1} |H[iR + r]|^2$. Then we will have

$$\rho_r = \frac{1}{P} E\left[\sum_{i=0}^{P-1} |Y_i[r]|^2\right] = \begin{cases} \sigma_p^2 \ \varepsilon_h(r) + \sigma_v^2, & r = \tau_{\bar{m}}, \\ \sigma_s^2 \ \varepsilon_h(r) + \sigma_v^2, & r \neq \tau_{\bar{m}}. \end{cases}$$

Even though |H[k]| can fluctuate greatly, as long as $\varepsilon_h(r)$ is relatively flat over the M points in τ , we will observe that ρ_r peaks at $r = \tau_{\bar{m}}$.

In practice, we estimate ρ_r as $\hat{\rho}_r = \frac{1}{P} \sum_{i=0}^{P-1} |Y_i[r]|^2$, which is the synchronized average of the power of $Y_i[r]$.

In summary, the optimum pilot location is found as

$$\hat{\tau}_{\bar{m}} = \operatorname*{argmax}_{r \in \boldsymbol{\tau}} \left\{ \hat{\rho}_r \right\}.$$
(12)

4. SIMULATIONS

In our numerical examples, the number of sub-carriers is N = 128, the length of the FIR channel is L = 4, the number of pilot tones is P = L = 4, and the power allocation factor β is determined by (8) to be 0.15. The N-P information sub-symbols were independently drawn from a QPSK constellation. The signal-to-noise ratio (SNR) is defined as SNR = \mathcal{P}_{dc}/N_0 , where \mathcal{P}_{dc} is the DC power consumed by the PA and $N_0 = 2\sigma_v^2$ is the power spectral density of the additive noise.

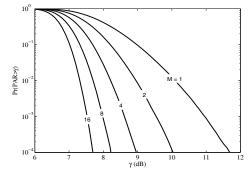


Figure 1. CCDF of the PAR of the BSPTM signals.

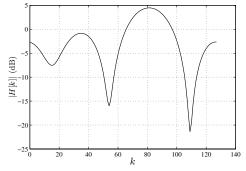


Figure 2. Magnitude response of the channel.

4.1. PAR Reduction Performance

In this example, we approximate the continuous-time PAR of (2) by evaluating the discrete-time PAR of the 4-time oversampled OFDM signals [3]. 10^6 Monte Carlo trials were conducted. Fig. 1 shows the complementary cumulative distribution functions (CCDFs) of the PAR of the transmitted signal $z^{(\bar{m})}(t)$ with different number of selections, M, and M = 1 corresponds to the original PTAM-OFDM case. We observe that when M = 8, the proposed algorithm can achieve 3.5 dB of PAR reduction (compared with the M = 1 case) at the CCDF level of 10^{-4} . It is evident from Fig. 1 that the larger the M, the smaller the resulting PAR. On the other hand, the computational complexity increases as M increases; there is also a diminishing return in the PAR reduction capability as M further increases. As a rule of thumb, we recommend to use min $\{R, 4\} \leq M \leq \min\{R, 8\}$.

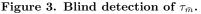
4.2. Blind Detection of $\tau_{\bar{m}}$ (or \bar{m})

We conducted a numerical example to illustrate the blind detection of τ_p from $|Y[k]|^2$. In this example, SNR= 0 dB, R = 32, $\tau = \{0, 4, 8, 12, 16, 20, 24, 28\}$, and thus M = 8. Fig. 2 shows |H[k]| vs. k for one realization of the Rayleigh fading channel, which exhibited several deep nulls. For the given OFDM symbol, the 6th mapped signal had the lowest PAR value among $z^{(m)}(t)$, $0 \le m \le 7$. Therefore, the selected pilot tone location was $\tau_6 = 24$ and $z^{(6)}(t)$ was transmitted. At the receiver, $|Y[k]|^2$ is first calculated. In Fig. 3, for each sub-record $|Y_i[r]|^2$, we use circles to indicate the values corresponding to the M candidate locations $r \in \tau$. From the $\hat{\rho}_r$ plot, we found $\hat{\tau}_{\bar{m}} = 24$ (or equivalently, $\hat{m} = 6$), which was indeed the true $\tau_{\bar{m}}$ that was used for transmission.

4.3. Comparison with Ref. [6] on detecting \bar{m}

In this example, we shall compare the performance of the proposed BSPTM method and that of [6] in the presence of Rayleigh fading. The simulation parameters were the

Table 1. Error rates in detecting \bar{m} . 5 dB10 dB 20 dB SNR $0 \, \mathrm{dB}$ Ref. |6 35.28%5.02%0.46%0.02%BSP 0.09%TM1.42'0% 0% $Y_{2}[r]|^{2}$ $Y_{3}[r]|^{2}$ ŷ,



same as in the previous example, and 10^5 Monte Carlo trials were conducted. Note that the ML decoder of [6] needs the CSI in order to detect the optimum phase sequence index \bar{m} , but BSPTM does not. Table 1 compares the error rates in detecting \bar{m} for the method of [6] and our proposed BSPTM technique. We point out that for the detection of \bar{m} , we had assumed perfect CSI for the method of [6] but no knowledge of the CSI for BSPTM. Despite of this favorable setup for [6], BSPTM is still more robust.

Moreover, the ML decoder of [6] has a higher computational complexity than BSPTM. For example, if $\{X[k]\}$ are drawn from the 16QAM constellation, the ML decoder requires 16MN magnitude-squared $(|\cdot|^2)$ operations, whereas BSPTM only needs N of them.

4.4. BER Performance

We compare next, the BER performance of BSPTM-OFDM with that of PTAM-OFDM for N = 128, P = 4, $\beta = 0.15$, and M = 8. The receiver consists of a zero-forcing equalizer and a suboptimal but simple hard-decision decoder [1]. Following [1], we tried two types of channels: a fixed FIR channel with tap coefficients $\mathbf{h} = [-0.0471 + 0.0458j, -0.7600 + 0.0633j, 0.5488 - 0.1963j, -0.2649 + 0.0646j]^T$, and a Rayleigh fading channel with i.i.d. complex Gaussian taps. The BER was evaluated by averaging over 10^5 Monte Carlo trials.

Fig. 4 shows the BER performance of the proposed BSPTM technique and that of PTAM-OFDM for the fixed channel case. Fig. 5 shows a similar comparison for the Rayleigh fading case. We can see from both figures that the PTAM-OFDM performance is only 1-2 dB away from the known channel case, which can serve as a benchmark. However, our proposed BSPTM-OFDM offers even better BER performance, which approaches the performance of the known channel case for both the fixed and the Rayleigh fading channels. Such superior performance is possible, since we have taken advantage of the reduction in the PAR to boost the average transmission power for the same DC power. Specifically, we kept the peak power of an OFDM block fixed, but adjusted the average power according to the actual PAR value of the block – this linear scaling approach [2] ensures the most efficient utilization of the PA; in other words, we made the average transmit power to be $\propto 1/PAR$ [2]. Eventually, the benefit of PAR reduction is realized as decrease in the BER.

5. CONCLUSIONS

Combining the frameworks of pilot tone assisted modulation (PTAM) for OFDM and selected mapping (SLM), we proposed a novel joint channel estimation and PAR reduction

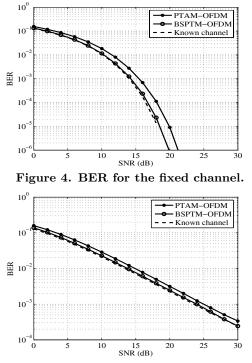


Figure 5. BER for the Rayleigh channels.

scheme: blind selected pilot tone modulation (BSPTM). The index for SLM is carried by the location of the pilot tones, which can be blindly detected at the receiver by capitalizing on the average power difference between the pilot tones and the information signal. Since no side information needs to be transmitted, the proposed method is both power efficient and bandwidth efficient. Simulation results demonstrate the PAR reducing capability and the robustness of BSPTM-OFDM over frequency selective fading channels in the presence of additive noise.

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