BIAS ANALYSIS OF A GAIN/PHASE/DC-OFFSET ESTIMATION TECHNIQUE FOR DIRECT FREQUENCY CONVERSION MODULATORS

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ABSTRACT

This paper studies a least-square-based technique for compensation of gain and phase imbalances and DC-offsets in a direct frequency conversion quadrature modulator. The technique requires the instantaneous power measurement at the transmitter to calculate the gain/phase/DC-offset compensation coefficients. The biases of the estimated coefficients are analyzed when there exist various distortions in the power measurement circuit including circuit noise, quantization error, DC-offset, power detector modeling error, and filtering. Numerical method is used to validate these analyses.

1. INTRODUCTION

Recently in wireless communication industry there has been considerable effort to revolutionize transceiver architectures to reduce cost, physical size, and power consumption. Direct conversion transceiver is an example of such an efficient architecture, which performs frequency conversion between the radio frequency and the baseband in one stage, thus avoiding any use of intermediate frequencies [1],[2]. However, a direct conversion transceiver is susceptible to the gain/phase imbalances between in-phase (I) and quadrature (Q) channels, and to DC voltage offsets in the analog modulator and demodulator circuits. Such gain/phase imbalances are known to degrade the overall communication link performance [3]. The imbalances produce unwanted residual sideband (RSB). Although the performance degradation due to the phase imbalance in the modulator can be largely eliminated by compensating it in a quadrature receiver, the degradation due to the gain imbalance cannot be compensated in the receiver [3]. In addition, the DC-offsets may cause the local oscillator (LO) signal to leak through the modulators. This leakage and the gain/phase imbalances distort the transmitted spectrum, making it difficult to meet the spectrum mask requirement. Therefore, it is desirable to minimize these imbalances and offsets at the transmitter.

Modulators with good balance between the I and Q channels must rely on stringent component specifications that are difficult to achieve as the frequency of operation increases to microwave or mm-wave. In addition, there is always some imbalance in the analog front-end between the I and Q branch amplitudes and phases. A novel pre-compensation technique has been proposed to deal with the gain/phase imbalances and DC-offsets in the quadrature modulator circuit, based on the baseband processing at the transmitter [1] [4]. The technique tries to maximize the LO and RSB suppression by estimating the gain/phase imbalances and DC-offsets using a least-square-based (LS-based) technique from the power measurement at the modulator output and pre-compensating the I and Q signals accordingly. The architecture of the direct transmitter with pre-compensation is shown in Figure 1.

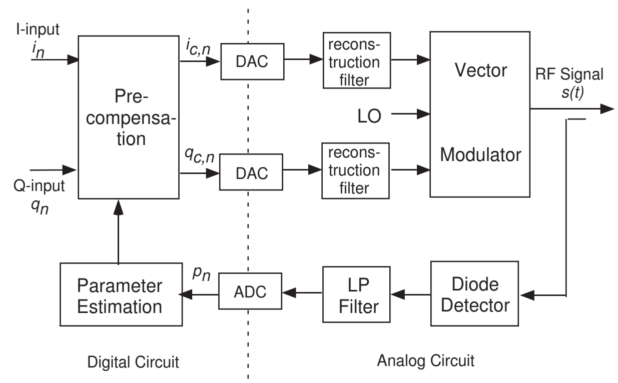

Figure 1. Transmitter model with the pre-compensation of gain/phase imbalances and DC-offsets.

The effects of circuit noise, word length in analog-to-digital converter (ADC), and bandwidth of the power measurement data as well as modeling mismatch in the diode detector on the biases of the compensation coefficients are investigated theoretically. The degradation of the LO and RSB suppression caused by the biases is not discussed here due to space limitation. The analytical results are validated using a numerical method.

The paper is organized as follows: Section 2 introduces the signal model and the pre-compensation technique. Section 3 theoretically analyzes the estimation biases of the gain/phase imbalances and DC-offsets. The numerical results are given in Section 4. The concluding remarks are in Section 5.

2. SIGNAL MODEL AND COMPENSATION TECHNIQUE

2.1. Signal Model

Ideally, the I and Q channels of a quadrature communication system are orthogonal to each other. However, due to fabrication imperfections, there always exist gain and phase imbalances that destroy the orthogonality between these two channels, resulting in the unsuppressed image signal. In addition, DC-offsets may also exist, resulting in the LO signal to leak through the modulator.

To represent the various components discussed above, the fol-

[1]US Patent #6,771,709

lowing model of a carrier-modulated signal is adopted

$$s(t) = [i(t) + c_i] \cos(\omega t) - \alpha [q(t) + c_q] \sin(\omega t + \phi) \quad (1)$$

where $i(t)$ and $q(t)$ are the I and Q modulating signals, each having a unity power and independent of each other. ω is the carrier frequency. α and ϕ represent the gain and phase imbalances between I and Q channels, while c_i and c_q are DC-offsets in the I and Q channels, respectively.

By using a trigonometric identity, Eq. (1) can be expressed as

$$s(t) = u(t) \cos(\omega t) - v(t) \sin(\omega t) \quad (2)$$

where $u(t) = [i(t) + c_i] - \alpha[q(t) + c_q] \sin\phi$ and $v(t) = \alpha[q(t) + c_q] \cos\phi$. For $\phi \neq 0$, the I and Q channel signals become correlated, and have different power levels.

2.2. Parameter Estimation And Pre-compensation

An LS-based technique [5] is implemented to estimate the model parameters of Eq. (1) by taking power measurements at the modulator output during the transmission. Denote the instantaneous output power of the modulator using $p(t) = g(u^2(t) + v^2(t))$, where g is gain of the measurement circuit. The sampled power measurement with the sampling interval T_s can be expressed as [4]

$$p_n = \mathbf{a}^{\mathrm{T}} \mathbf{x}_n \quad (3)$$

where $p_n = p(t)|_{t=nT_s}$, the superscript "T" denotes the matrix transpose, $\mathbf{a} = [a_1\ a_2\ a_3\ a_4\ a_5\ a_6]^{\mathrm{T}}$, $\mathbf{x}_n = [x_{1,n}\ x_{2,n}\ x_{3,n}\ x_{4,n}\ x_{5,n}\ x_{6,n}]^{\mathrm{T}}$, $x_{1,n} = i_n^2 = i^2(t)|_{t=nT_s}$, $x_{2,n} = i_n = i(t)|_{t=nT_s}$, $x_{3,n} = i_n q_n = i(t)q(t)|_{t=nT_s}$, $x_{4,n} = q_n = q(t)|_{t=nT_s}$, $x_{5,n} = q_n^2 = q^2(t)|_{t=nT_s}$, $x_{6,n} = 1$, $a_1 = g$, $a_2 = (2c_i - 2\alpha c_q \sin\phi)g$, $a_3 = 2\alpha(\sin\phi)g$, $a_4 = (2\alpha^2 c_q - 2\alpha c_i \sin\phi)g$, $a_5 = \alpha^2 g$, and $a_6 = (c_i^2 + \alpha^2 c_q^2 - 2\alpha c_i c_q \sin\phi)g$.

Instead of finding the model parameters directly, of which the modulator output power is a nonlinear function, we can solve a set of linear equations for a_i's first. This linearization greatly simplifies the procedure, and reduces the problem to estimating the unknown parameter $\mathbf{a}$ given the two observable sets $\mathbf{x}_n$ and p_n ($n = 1,\ 2,\ \cdots,\ N$).

An LS-based technique is used to estimate the 6 a_i parameters, by minimizing the mean square error $J = \sum_{n=1}^{N} \varepsilon_n^2 = \epsilon^{\mathrm{T}} \epsilon$, where $\epsilon = [\varepsilon_1,\ \varepsilon_2,\ \cdots,\ \varepsilon_n]^{\mathrm{T}}$ and $\varepsilon_n = p_n - \mathbf{a}^{\mathrm{T}} \mathbf{x}_n$. The solution can be written as

$$\hat{\mathbf{a}} = (\mathbf{X}^{\mathrm{T}} \mathbf{X})^{-1} \mathbf{X}^{\mathrm{T}} \mathbf{p} \quad (4)$$

where $\hat{\mathbf{a}}$ denotes the estimate of the $\mathbf{a}$, $\mathbf{X} = [\mathbf{x}_1\ \mathbf{x}_2\ \cdots\ \mathbf{x}_n]^{\mathrm{T}}$, and $\mathbf{p} = [p_1\ p_2 \cdots\ p_n]^{\mathrm{T}}$.

Once a_i's are obtained, we can derive the gain and phase imbalance estimates as

$$\hat{\alpha} = \sqrt{\hat{a}_5/\hat{a}_1} \text{and} \hat{\phi} = \sin^{-1}\left(\frac{-\hat{a}_3}{2\sqrt{\hat{a}_1 \hat{a}_5}}\right)$$

The DC-offsets c_i and c_q can be computed as follows:

$$\begin{bmatrix} \hat{c}_i \\ \hat{c}_q \end{bmatrix} = \begin{bmatrix} 2\hat{a}_1 & -2\hat{a}_1 \hat{\alpha} \sin\hat{\phi} \\ -2\hat{a}_1 \hat{\alpha} \sin\hat{\phi} & 2\hat{a}_1 \hat{\alpha}^2 \end{bmatrix}^{-1} \begin{bmatrix} \hat{a}_2 \\ \hat{a}_4 \end{bmatrix}$$

Having obtained the estimates of the gain/phase imbalances and DC-offsets, we can pre-compensate the I and Q channel signals as follows:

$$\begin{bmatrix} i_{c,n} \\ q_{c,n} \end{bmatrix} = \begin{bmatrix} 1 & -\alpha \sin\phi \\ 0 & \alpha \cos\phi \end{bmatrix}^{-1} \begin{bmatrix} u_n \\ v_n \end{bmatrix} - \begin{bmatrix} c_i \\ c_q \end{bmatrix} \quad (5)$$

3. THEORETICAL ANALYSIS

The power measurement circuit includes a coupler, a diode detector, a low pass filter, and an ADC. Distortions may occur to the power measurement, such as quantization error, nonlinearity, filtering, and noise. The power measurement signal can be expressed as

$$p_n' = p_n + \Delta p_n \quad (6)$$

where Δp_n represents the measurement error. Using this measurement model, the LS-based solution for the gain/phase/DC-offset coefficients becomes

$$\begin{aligned} \hat{\mathbf{a}}_\Delta &= (\mathbf{X}^{\mathrm{T}} \mathbf{X})^{-1} \mathbf{X}^{\mathrm{T}} \mathbf{p}' \\ &= (\mathbf{X}^{\mathrm{T}} \mathbf{X})^{-1} \mathbf{X}^{\mathrm{T}} \mathbf{p} + (\mathbf{X}^{\mathrm{T}} \mathbf{X})^{-1} \mathbf{X}^{\mathrm{T}} \Delta\mathbf{p} \quad (7) \end{aligned}$$

where $\mathbf{p}' = [p_1'\ p_2'\ \cdots\ p_N']^{\mathrm{T}}$ and $\Delta\mathbf{p} = [\Delta p_1\ \Delta p_2\ \cdots\ \Delta p_N]^{\mathrm{T}}$. As $\mathbf{p}$ does not include any measurement error, the first term in Eq. (7) is the true solution of $\mathbf{a}$. That is, $(\mathbf{X}^{\mathrm{T}} \mathbf{X})^{-1} \mathbf{X}^{\mathrm{T}} \mathbf{p} = \mathbf{a}$. Eq. (7) can be written as

$$\begin{aligned} \hat{\mathbf{a}}_\Delta &= \mathbf{a} + (\mathbf{X}^{\mathrm{T}} \mathbf{X})^{-1} \mathbf{X} \Delta\mathbf{p} \quad (8) \\ &\triangleq \mathbf{a} + \Phi_N^{-1} \Gamma_{\Delta,N} \quad (9) \end{aligned}$$

where $\Phi_N = \frac{1}{N} \mathbf{X}^{\mathrm{T}} \mathbf{X}$ and $\Gamma_{\Delta,N} = \frac{1}{N} \mathbf{X} \Delta\mathbf{p}$.

Let us make a few assumptions about the characteristics of the I and Q signals, which can be easily justified for most of the quadrature modulation communication systems:

1. The i_n, q_n, and p_n' are stationary ergodic random processes.
2. The probability distribution functions (PDFs) of I and Q signals, i_n and q_n, are symmetric. That is, $f(-i, q) = f(i, q) = f(i, -q) = f(-i, -q)$, where $f(i, q)$ is the joint PDF of the I and Q signals. This can be translated to

   $$E[i_n^{k_1} q_n^{k_2}] = 0 \text{ for } k_1 \text{ or } k_2 \text{ is odd} \quad (10)$$

   where $E[\cdot]$ denotes the mathematical expectation operator.
3. The I and Q signals are identically distributed.

Given the above assumptions, the bias can be obtained by studying the behavior of $\Phi_N^{-1} \Gamma_{\Delta,N}$ when N approaches ∞. It can be shown that

$$\underline{\Phi} = \lim_{N \to \infty} \Phi_N = \begin{bmatrix} \mu_4 & 0 & 0 & 0 & \mu_{2,2} & \mu_2 \\ 0 & \mu_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu_{2,2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu_2 & 0 & 0 \\ \mu_{2,2} & 0 & 0 & 0 & \mu_4 & \mu_2 \\ \mu_2 & 0 & 0 & 0 & \mu_2 & 1 \end{bmatrix} \quad (11)$$

where $\mu_k = \mathrm{E}[i_n^k] = \mathrm{E}[q_n^k]$ and $\mu_{k_1,k_2} = \mathrm{E}[i_n^{k_1} q_n^{k_2}]$. The inverse of $\underline{\Phi}$ can be expressed as

$$\underline{\Phi}^{-1} = \frac{1}{\Lambda} \begin{bmatrix} \mu_4 - \mu_2^2 & 0 & 0 & 0 & \mu_2^2 - \mu_{2,2} & \mu_{2,2}\mu_2 - \mu_2\mu_4 \\ 0 & \frac{\Lambda}{\mu_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\Lambda}{\mu_{2,2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\Lambda}{\mu_2} & 0 & 0 \\ \mu_2^2 - \mu_{2,2} & 0 & 0 & 0 & \mu_4 - \mu_2^2 & \mu_{2,2}\mu_2 - \mu_4\mu_2 \\ \mu_{2,2}\mu_2 - \mu_2\mu_4 & 0 & 0 & 0 & \mu_{2,2}\mu_2 - \mu_4\mu_2 & \mu_4^2 - \mu_{2,2}^2 \end{bmatrix}$$

where $\Lambda = \mu_4^2 + 2\mu_4\mu_2^2 - 2\mu_{2,2}\mu_2^2 + \mu_{2,2}^2$.

The behaviors of the $\Gamma_{\Delta,N}$ are different for various measurement distortions, and they are analyzed in the following subsections.

3.1. Circuit Noise and Quantization Error

The circuit noise consists mainly of the thermal noise, which is generally modeled as a zero mean white Gaussian process. The quantization error is the difference between the true value of the analog sample and its corresponding quantized sample at the ADC output, and is generally modeled as a uniformly distributed white process. The mean of quantization error is zero for quantizers that round the sample value to the nearest quantization level. Let w_n denote the sum of path circuit and quantization error. $\Gamma_{\Delta,N}$ becomes

$$\Gamma_{\Delta,N} = \frac{1}{N}\sum_{n=1}^{N}\left[i_n^2 w_n \;\; i_n w_n \;\; i_n q_n w_n \;\; q_n w_n \;\; q_n^2 w_n \;\; w_n\right]^{\mathrm{T}} \quad (12)$$

When $N \to \infty$, it becomes

$$\underline{\Gamma}_{\Delta} = \lim_{N\to\infty}\Gamma_{\Delta,N} = [\mu_2 m_w \;\; 0 \;\; 0 \;\; 0 \;\; \mu_2 m_w \;\; m_w]^{\mathrm{T}} = \mathbf{0}_{6\times 1} \quad (13)$$

where m_w is the mean of w_n, and $\mathbf{0}_{6\times 1}$ is a 6-by-1 matrix of zeros.

Then we have $\hat{\mathbf{a}}_{\Delta} = \mathbf{a}$. In other words, in the presence of circuit noise and quantization error, the LS estimates of the gain/phase/DC-offset coefficients are unbiased.

3.2. DC-offset

The diode detector output can be either negative or positive depending on its output polarity. To maximize the dynamic range of the ADC, a nominally fixed biasing voltage is added to the power measurement. In addition, there may be additional small but unknown DC-offsets in various parts of the power measurement circuit.

To model the DC-offsets, the power measurement can be written as $p_n' = p_n + c$, where c is the total DC-offset. The $\Gamma_{\Delta,N}$ becomes

$$\Gamma_{\Delta,N} = \frac{c}{N}\left[\sum_{n=1}^{N} i_n^2 \;\; \sum_{n=1}^{N} i_n \;\; \sum_{n=1}^{N} i_n q_n \;\; \sum_{n=1}^{N} q_n \;\; \sum_{n=1}^{N} q_n^2 \;\; N\right]^{\mathrm{T}} \quad (14)$$

When $N \to \infty$, it becomes

$$\underline{\Gamma}_{\Delta} = \lim_{N\to\infty}\Gamma_{\Delta,N} = c\,[\mu_2 \;\; 0 \;\; 0 \;\; 0 \;\; \mu_2 \;\; 1]^{\mathrm{T}} \quad (15)$$

It can be shown that

$$\begin{aligned}\hat{\mathbf{a}}_{\Delta} &= \underline{\Phi}^{-1}\underline{\Gamma}_{\Delta} \\ &= \left[0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; c\frac{2(\mu_{2,2}\mu_2^2 - \mu_4\mu_2^2) + \mu_4^2 - \mu_{2,2}^2}{\Lambda}\right]^{\mathrm{T}} \end{aligned} \quad (16)$$

From Eq. (16), it can be seen that the DC-offset in the power measurement circuits only bring a bias to the estimation of a_6. However, the gain/phase/DC-offset estimation is independent of a_6 while translating $\hat{\mathbf{a}}$ to $\hat{\alpha}$, $\hat{\phi}$, $\hat{c}_i$, and $\hat{c}_q$. This means that the DC-offset in the power measurement does not affect the estimation of gain/phase imbalances and DC-offsets.

3.3. Modeling Mismatch

The detector diode is assumed to be an ideal square-law device. That is, the diode output voltage is proportional to the square of its input envelope. The objective here is to analyze the effect of diode modeling mismatch on the accuracy of the estimated coefficients.

Suppose the diode is modeled by $y = f(|x|)$, where y denotes the output of the detector diode and $|x|$ the amplitude of input x. As p_n is the ideal square-law diode output, we have $p_n = c_1|x_n|^2$, where c_1 is constant. Thus the actual output can be given by

$$p_n' = f(|x_n|) = f(\sqrt{p_n/c_1}) \quad (17)$$

An truncated Taylor expansion of M-order, which approximates the nonlinearity in Eq. (17), can be described as

$$p_n' = \sum_{m=1}^{M}\gamma_m p_n^m = \gamma_1\left[p_n + \sum_{m=2}^{M}\frac{\gamma_m}{\gamma_1}p_n^m\right] \quad (18)$$

Without loss of generality, let $\gamma_1 = 1$. Eq. (18) becomes

$$p_n' = p_n + \sum_{m=2}^{M}\gamma_m p_n^m \quad (19)$$

The difference between the actual diode output and ideal output can be expressed as

$$\Delta p_n = \sum_{m=2}^{M}\gamma_m p_n^m = \sum_{m=2}^{M}\gamma_m\left(\sum_{j=1}^{6}a_j x_{j,n}\right)^m \quad (20)$$

Substituting Eq. (20) into Eq. (9) yields the expression of $\Gamma_{\Delta,N}$. As $N \to \infty$, $\Gamma_{\Delta,N}$ becomes Eq. (21)

The first 5 elements of $\hat{\mathbf{a}}_{\Delta} = \underline{\Phi}^{-1}\underline{\Gamma}_{\Delta}$ are not zero. The biases of the gain/phase imbalance and DC-offset estimates exist if the diode operates at a region deviating from square-law characteristic. The biases depend on the deviation of diode detector, as well as the statistical characteristic of the I&Q signals.

3.4. Filtering Effect

The filtering effect can be described as

$$p_n' = \sum_{r=-L_1}^{L_2}\beta_r p(n-r) = \beta_0\left[p_n + \sum_{\substack{r=-L_1\\ r\neq 0}}^{L_2}\frac{\beta_r}{\beta_0}p(n-r)\right] \quad (22)$$

Substituting Eq. (22) into Eq. (9) with $\beta_0 = 1$ yields $\underline{\Gamma}_{\Delta} =$

$$\left[\sum_{\substack{r=-L_1\\ r\neq 0}}^{L_2}\sum_{i=1}^{6}\beta_r a_i\mu_{i,1}(r) \quad \sum_{\substack{r=-L_1\\ r\neq 0}}^{L_2}\sum_{i=1}^{6}\beta_r a_i\mu_{i,2}(r) \quad \sum_{\substack{r=-L_1\\ r\neq 0}}^{L_2}\sum_{i=1}^{6}\beta_r a_i\mu_{i,3}(r) \quad \sum_{\substack{r=-L_1\\ r\neq 0}}^{L_2}\sum_{i=1}^{6}\beta_r a_i\mu_{i,4}(r) \quad \sum_{\substack{r=-L_1\\ r\neq 0}}^{L_2}\sum_{i=1}^{6}\beta_r a_i\mu_{i,5}(r) \quad \sum_{\substack{r=-L_1\\ r\neq 0}}^{L_2}\sum_{i=1}^{6}\beta_r a_i\mu_{i,6}(r)\right]^{\mathrm{T}}$$

where $\mu_{i,k}(r) = \mathrm{E}[x_{i,n-r}x_{k,n}]$. Similar to the modeling mismatch of the diode detector, the filtering also introduces the biases

$$\Gamma_\Delta = \begin{bmatrix} \sum_{m=2}^{M} \sum_{j_1+j_2+j_3+j_4+j_5+j_6=m} \frac{\gamma_m m!}{j_1!j_2!j_3!j_4!j_5!j_6!} a_1^{j_1} a_2^{j_2} a_3^{j_3} a_4^{j_4} a_5^{j_5} a_6^{j_6} \mu_{2j_1+j_2+j_3+2,2j_5+j_4+j_3} \\ \sum_{m=2}^{M} \sum_{j_1+j_2+j_3+j_4+j_5+j_6=m} \frac{\gamma_m m!}{j_1!j_2!j_3!j_4!j_5!j_6!} a_1^{j_1} a_2^{j_2} a_3^{j_3} a_4^{j_4} a_5^{j_5} a_6^{j_6} \mu_{2j_1+j_2+j_3+1,2j_5+j_4+j_3} \\ \sum_{m=2}^{M} \sum_{j_1+j_2+j_3+j_4+j_5+j_6=m} \frac{\gamma_m m!}{j_1!j_2!j_3!j_4!j_5!j_6!} a_1^{j_1} a_2^{j_2} a_3^{j_3} a_4^{j_4} a_5^{j_5} a_6^{j_6} \mu_{2j_1+j_2+j_3+1,2j_5+j_4+j_3+1} \\ \sum_{m=2}^{M} \sum_{j_1+j_2+j_3+j_4+j_5+j_6=m} \frac{\gamma_m m!}{j_1!j_2!j_3!j_4!j_5!j_6!} a_1^{j_1} a_2^{j_2} a_3^{j_3} a_4^{j_4} a_5^{j_5} a_6^{j_6} \mu_{2j_1+j_2+j_3,2j_5+j_4+j_3+1} \\ \sum_{m=2}^{M} \sum_{j_1+j_2+j_3+j_4+j_5+j_6=m} \frac{\gamma_m m!}{j_1!j_2!j_3!j_4!j_5!j_6!} a_1^{j_1} a_2^{j_2} a_3^{j_3} a_4^{j_4} a_5^{j_5} a_6^{j_6} \mu_{2j_1+j_2+j_3,2j_5+j_4+j_3+2} \\ \sum_{m=2}^{M} \sum_{j_1+j_2+j_3+j_4+j_5+j_6=m} \frac{\gamma_m m!}{j_1!j_2!j_3!j_4!j_5!j_6!} a_1^{j_1} a_2^{j_2} a_3^{j_3} a_4^{j_4} a_5^{j_5} a_6^{j_6} \mu_{2j_1+j_2+j_3,2j_5+j_4+j_3} \end{bmatrix} \quad (21)$$

of the gain/phase imbalance and DC-offset estimates. The biases depend on the filter coefficients and the statistical characteristic of the I& Q signals.

4. NUMERICAL RESULTS

Numerical method is used to validate the bias analyses of the LS-based technique for the gain/phase/DC-offset compensation. The following values are assumed: $\alpha = 0.95$, $\phi = 5^o$, $c_i = -0.05$ and $c_q = 0.05$. They represent typical values of component deviation in the modulator. The modulation is 8-PSK. A raised cosine filter with a rolloff factor of 0.25 is used as the pulse shaping function. The following five cases are studied.

- Case I: Circuit noise is AWGN with SNR =10dB;
- Case II: The power measurement data are sampled using 6-bit ADC;
- Case III: The DC-offset in the power measurement is assumed to be 0.4;
- Case IV: The diode operates at a non-square-law region and can be modeled as $p_n' = p_n + 0.2p_n^2 + 0.1p_n^3$;
- Case V: The coefficients of the LP filter are [-0.06,0.00,0.10,-0.21,0.30,1.00, 0.30,-0.21,0.10,0.00,-0.06].

The numerical results of the estimation biases are obtained by averaging 50 LS-based solutions with $N = 100$. To get the theoretical biases, 10000 samples are used to calculate the correlation of the I&Q signal. The results are shown in Table 1.

It follows that the theoretical analyses are consistent with the numerical results. We can also see that the LS-based technique is quite robust to the distortions in the power measurement circuit.

Table 1. Theoretical and numerical results of estimate biases

Case	$\alpha(\hat{\alpha})$	$\phi(\hat{\phi})$	$c_i(\hat{c_i})$	$c_q(\hat{c_q})$
I	0 (7.0e-4)	0 (-8.2e-2)	0 (7.5e-4)	0 (-9.3e-4)
II	0 (-1.3e-4)	0 (2.6e-3)	0 (-1.7e-5)	0 (1.5e-5)
III	0 (0)	0 (0)	0 (0)	0 (0)
IV	-4.5e-4 (-4.6e-4)	0.192 (0.184)	-2.2e-4 (-2.3e-4)	-1.0e-4 (-1.1e-4)
V	-3.4e-2 (-3.7e-2)	4.12 (4.11)	-2.6e-2 (-2.8e-2)	2.6e-2 (2.67e-2)

5. CONCLUSIONS

This paper analyzed the estimation bias of a least-square-based (LS-based) technique to compensate the gain and phase imbalances and DC-offsets in a quadrature modulator. Various distortions in the power measurement circuit were discussed: noise, quantization error, DC-offset, power detector modeling error, and filtering effect. It was found that the noise, quantization error, and DC-offset did not affect the estimation bias. The modeling error and filtering introduced a small estimation bias due to the correlation of I&Q signals. Numerical method was used to validate the analysis. The results in this paper can help system designers to specify the requirement of the power measurement circuit.

6. REFERENCES

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