BLOCK DOUBLE DIFFERENTIAL DESIGN FOR OFDM WITH CARRIER FREQUENCY OFFSET

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ABSTRACT

Carrier frequency offset (CFO) estimation for orthogonal frequency division multiplexing (OFDM) has caught attention as OFDM systems become widely adopted in recent years. In this paper, we design a novel double differential codec with low computational complexity. Our design bypasses CFO and channel estimation and is easy to be implemented at both transmitter and receiver. It also guarantees full multipath diversity, and reduces the peak-to-average power ratio from the number of subcarriers to the channel order. In addition, it is robust to CFO drifting. The closed form of the performance for our design is derived for OFDM transmissions over frequency-selective channels with CFO. Thorough simulation results corroborate our claims.

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has been widely adopted by wireless and wired communications (e.g., IEEE802.11a, IEEE802.11g in the US, and DAB/DVB, HiperLAN/2 in Europe), because it offers the possibility for high data-rates at low decoding complexity [13]. Relying on multiple orthogonal subcarriers, OFDM schemes turn the frequency-selective channel into a set of parallel flat-fading subchannels. However, the presence of a carrier frequency offset (CFO) destroys the orthogonality among subcarriers, and the resulting intercarrier interference degrades the bit error rate (BER) performance severely [9]. Thus, dealing with CFO is most critical in OFDM systems and has received considerable attention in recent years.

Different methods have been proposed to estimate CFO, e.g. the training-based methods in [10], and the (semi-)blind methods in [7]. However, training-based methods sacrifice bandwidth efficiency on transmitting pilot symbols and restrict the acquisition range to (a couple of) subcarrier spacing [10]. Null subcarrier based semi-blind methods can enlarge the acquisition range with higher complexity [7].

Scalar differential phase shift keying (DPSK) has well documented merits; see e.g., [8]. Differentially encoded OFDM has been adopted in the European DAB standard. Block differential schemes have been designed to collect multipath diversity over frequency-selective channels using OFDM [4]. It is well-known that single differential can only forego channel estimation but not CFO. Since single-differential OFDM systems as in [4] are still sensitive to CFO, CFO estimation based scalar differential design is proposed in [5]. However, the CFO estimator of [5] is limited to one subcarrier spacing acquisition range and it has high complexity while offering no multipath diversity.

Double differential (DD) has been recognized as an effective way to deal with unknown CFO and channels (see e.g., [1, 11]). Scalar DD has been used for OFDM [3] with an irreducible bit-error-rate floor and without any diversity. Multiple symbol block DD is employed for multi-antenna OFDM systems [14] through time-varying channels. It also requires high decoding complexity and does not enable any multipath diversity. In this paper, we propose a novel block double differential (BDD) scheme with low decoding complexity and high performance. Our design bypasses the CFO and channel estimation and is easy to be implemented at both transmitter and receiver. It also guarantees full multipath diversity regardless of the CFO value and channel nulls, and reduces the peak-to-average power ratio from the number of subcarriers to the channel order. In addition, it is robust to CFO drifting. The closed form of the performance for our design is also derived for OFDM transmissions over frequency-selective channels with CFO.

Notation: Upper (lower) bold face letters indicate matrices (column vectors). Superscript $(\cdot)^{\mathcal{H}}$ denotes Hermitian, $(\cdot)^T$ transpose, and $(\cdot)^*$ conjugate. The real and imaginary parts are denoted as $\Re[\cdot]$ and $\Im[\cdot]$; $E[\cdot]$ stands for expectation; diag$[\boldsymbol{x}]$ for a diagonal matrix with $\boldsymbol{x}$ on its main diagonal; and $\boldsymbol{A} \otimes \boldsymbol{B}$ denotes the Kronecker product of matrices $\boldsymbol{A}$ and $\boldsymbol{B}$. For a vector, $\|\cdot\|_2$ denotes the 2-norm. $\boldsymbol{I}_N$ denotes the $N \times N$ identity matrix; and $\boldsymbol{F}_N$ is the normalized $N \times N$ Fast Fourier Transform (FFT) matrix with its (m, n)th element being $N^{-1/2}\exp(-j2\pi mn/N)$.

2. SYSTEM MODEL

Consider OFDM transmissions with N subcarriers over an Lth-order frequency-selective fading channel in the presence of unknown carrier frequency offset (CFO). The discrete-time equivalent impulse response vector of the channel is $\boldsymbol{h} = [h_0, \ldots, h_L]^T$. We define the normalized CFO as $\omega_o = 2\pi T_s f_o$, where f_o denotes the physical frequency offset (in Hz), which could be due to Doppler and/or mismatch between transmit-receive oscillators, and T_s is the sampling period. The transmitted symbol at the nth time slot is $\tilde{s}(n)$. In the presence of CFO, the samples at the receive-antenna filter output can be written as:

$$y(n) = e^{j\omega_o n} \sum_{l=0}^{L} h(l)\tilde{s}(n-l) + v(n), \tag{1}$$

where $v(n)$ is zero-mean, white, complex Gaussian noise with variance N_0.

As in traditional OFDM systems, we perform inverse fast Fourier transform (IFFT) and cyclic prefix (CP) insertion operations at the transmitter side. At the receiver, we subtract CP from the data to remove the inter-block interference [13]. We choose that the number of subcarriers $N > L$ and the cyclic prefix length $L_{cp} \geq L$. Without loss of generality, we let $L_{cp} = L$. After CP removal, we can write the matrix-vector counterpart of (1) for the kth OFDM block as [7]:

$$\boldsymbol{x}(k) = e^{j\omega_o (N+L)k} \boldsymbol{D}_{\omega_o} \tilde{\boldsymbol{H}} \boldsymbol{F}_N^{\mathcal{H}} \boldsymbol{s}(k) + \boldsymbol{w}(k), \forall k \in [0, K-1], \tag{2}$$

where the diagonal matrix $\boldsymbol{D}_{\omega_o} := \text{diag}[e^{j\omega_o L}, \ldots, e^{j\omega_o (N+L-1)}]$, $\tilde{\boldsymbol{H}}$ is an $N \times N$ circulant matrix with the first column $[h(0), \ldots, h(L), 0, \ldots, 0]^T$, $\boldsymbol{F}_N^{\mathcal{H}}$ is a normalized N-point IFFT matrix, $\boldsymbol{s}(k) := [s(kN), \ldots, s((k+1)N-1)]^T$ is an information-consisting block, $\boldsymbol{w}(k)$ is the white Gaussian noise vector, and K is the number of blocks each burst.

In Eq. (2), both the channel matrix and the CFO are unknown. Our problem here is to recover the information symbols from $\boldsymbol{x}(k)$.

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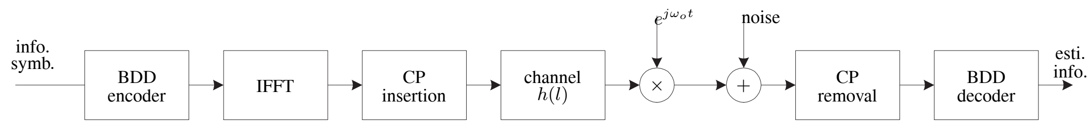

Fig. 1. Block diagram for system model

In the absence of CFO (i.e., $\omega_o = 0$), we can perform FFT operation at the receiver and then the channel matrix becomes

$$\boldsymbol{F}_N \tilde{\boldsymbol{H}} \boldsymbol{F}_N^{\mathcal{H}} = \text{diag}[\tilde{\boldsymbol{h}}], \tag{3}$$

where $\tilde{\boldsymbol{h}} = [\tilde{h}(0), \ldots, \tilde{h}(N-1)]^T$, with $\tilde{h}(n) := \sum_{l=0}^{L} h(l) \exp(-j 2\pi ln/N)$. In this case, if the channel is time invariant for at least two consecutive blocks, a single differential design has been proposed in [4] to bypass channel estimation and at the same time collect multipath diversity. Unlike [7, 5, 10], we estimate neither the channel nor the CFO. Our method is to design a double differential scheme for $\boldsymbol{s}(k)$, so that we can recover the information symbols without knowing or estimating CFO and channels. Following the system block diagram in Fig. 1, we will introduce BDD encoder and decoders.

3. BLOCK DOUBLE DIFFERENTIAL ENCODER

Suppose the number of subcarriers satisfying $N = (L+1)P$, where P is an integer. The differential encoder is designed as follows:

$$\boldsymbol{u}_k = \boldsymbol{F}_P \boldsymbol{G}(k) \boldsymbol{F}_P^{\mathcal{H}} \boldsymbol{u}_{k-1}, \text{ and } \boldsymbol{G}(k) = \boldsymbol{C}(k)\boldsymbol{G}(k-1), \tag{4}$$

where $\boldsymbol{F}_P$ denotes a normalized P-point FFT matrix, $\boldsymbol{C}(k)$ and $\boldsymbol{G}(k)$ are $P \times P$ diagonal matrices with PSK symbols on the main diagonal, and $\boldsymbol{C}(k)$ contains the information to be transmitted. The initial values are chosen such as:

$$\boldsymbol{u}_0 = \boldsymbol{u}_1 = \sqrt{P}[1\ \boldsymbol{0}_{1\times(P-1)}]^T; \text{ and } \boldsymbol{G}(1) = \boldsymbol{I}_P.$$

The kth transmitted OFDM block is designed as

$$\boldsymbol{s}(k) = \boldsymbol{1}_{L+1} \otimes \boldsymbol{u}_k \ \Leftrightarrow\ \boldsymbol{D}_s(k) = \boldsymbol{I}_{L+1} \otimes \text{diag}[\boldsymbol{u}_k], \tag{5}$$

where $\boldsymbol{1}_{L+1}$ denotes a vector with all elements as one. Note that here we do not design both differential steps as diagonal. However, $\boldsymbol{F}_P^{\mathcal{H}} \boldsymbol{G}(k) \boldsymbol{F}_P$ is still a unitary matrix, which means that it will not cause any "blow-up" or "diminish" problem as time goes on. In the following, we will show how this BDD design works for OFDM systems with unknown CFO.

The frequency channel response vector $\tilde{\boldsymbol{h}}$ in (3) can be rewritten as $\tilde{\boldsymbol{h}} = \sqrt{N}\boldsymbol{F}_{0:L}\boldsymbol{h}$, where $\boldsymbol{F}_{0:L}$ is the first $L+1$ columns of $\boldsymbol{F}_N$. Based on (5), we prove that (see [6, Appendix A] for a proof):

$$\sqrt{N}\boldsymbol{F}_N^{\mathcal{H}} \boldsymbol{D}_s(k) \boldsymbol{F}_{0:L} = \sqrt{L+1}(\boldsymbol{F}_P^{\mathcal{H}} \boldsymbol{u}_k) \otimes \boldsymbol{I}_{L+1}. \tag{6}$$

Plugging (5), (6) into (2), the input-output relationship becomes

$$\boldsymbol{x}(k) = e^{j\omega_o(N+L)k}\sqrt{L+1}\boldsymbol{D}_{\omega_o}\left((\boldsymbol{F}_P^{\mathcal{H}}\boldsymbol{u}_k) \otimes \boldsymbol{I}_{L+1}\right)\boldsymbol{h} + \boldsymbol{w}(k).$$

Recalling the encoder structure in (4) and using the property that $(\boldsymbol{AC}) \otimes (\boldsymbol{BD}) = (\boldsymbol{A} \otimes \boldsymbol{B})(\boldsymbol{C} \otimes \boldsymbol{D})$ with matched matrix sizes, we have that

$$\begin{aligned}\boldsymbol{x}(k) = {} & e^{j\omega_o(N+L)k}\sqrt{L+1}\,(\boldsymbol{G}(k) \otimes \boldsymbol{I}_{L+1})\,\boldsymbol{D}_{\omega_o} \\ & \left((\boldsymbol{F}_P^{\mathcal{H}}\boldsymbol{u}_{k-1}) \otimes \boldsymbol{I}_{L+1}\right)\boldsymbol{h} + \boldsymbol{w}(k),\end{aligned} \tag{7}$$

where we interchange matrices $\boldsymbol{D}_{\omega_o}$ with $\boldsymbol{G}(k) \otimes \boldsymbol{I}_{L+1}$ because both of them are diagonal matrices. In the absence of noise, we have the recursive equation as:

$$\boldsymbol{x}(k) = e^{j\omega_o(N+L)}\,(\boldsymbol{G}(k) \otimes \boldsymbol{I}_{L+1})\,\boldsymbol{x}(k-1).$$

The information matrix $\boldsymbol{C}(k)$ can thus be estimated from

$$\boldsymbol{D}_x(k)\boldsymbol{D}_x^{\mathcal{H}}(k-1) = (\boldsymbol{C}(k) \otimes \boldsymbol{I}_{L+1})\,\boldsymbol{D}_x(k-1)\boldsymbol{D}_x^{\mathcal{H}}(k-2), \tag{8}$$

where $\boldsymbol{D}_x(k) = \text{diag}[\boldsymbol{x}(k)]$. This enables the double-differential design to bypass both channel and CFO estimation for OFDM systems. Eq. (8) provides a "heuristic" decoder for our BDD encoder in (4). In the following, we will derive an optimal decoder, which turns out to be equivalent to this "heuristic" decoder.

4. DOUBLE DIFFERENTIAL DECODER

In this section, we will derive the optimal decoder and the corresponding performance analysis for the BDD design.

4.1. Decoder Design

Observing (8), we obtain that to estimate $\boldsymbol{C}(k)$, we need observations $\boldsymbol{x}(k)$, $\boldsymbol{x}(k-1)$, and $\boldsymbol{x}(k-2)$. Define $\tilde{\boldsymbol{x}}(k) = [\boldsymbol{x}^T(k), \boldsymbol{x}^T(k-1), \boldsymbol{x}^T(k-2)]^T$ and $\boldsymbol{z}_k = e^{j\omega_o(N+L)k}\sqrt{L+1}\boldsymbol{D}_{\omega_o}\left((\boldsymbol{F}_P^{\mathcal{H}}\boldsymbol{u}_k) \otimes \boldsymbol{I}_{L+1}\right)\boldsymbol{h}$ [c.f. (7)]. It is ready to verify that

$$\boldsymbol{z}_k = e^{j\omega_o(N+L)}\,(\boldsymbol{G}(k) \otimes \boldsymbol{I}_{L+1})\,\boldsymbol{z}_{k-1}.$$

The maximum likelihood (ML) estimator of $\boldsymbol{C}(k)$ is given as

$$\hat{\boldsymbol{C}}(k) = \arg\min_{\boldsymbol{C}\in\boldsymbol{\mathcal{V}},\boldsymbol{z}_{k-1},\omega_o} \left\| \tilde{\boldsymbol{x}}(k) - \begin{bmatrix} e^{j\omega_o(N+L)}[\boldsymbol{C}\boldsymbol{G}(k-1) \otimes \boldsymbol{I}_{L+1}] \\ \boldsymbol{I}_N \\ e^{-j\omega_o(N+L)}[\boldsymbol{G}^{\mathcal{H}}(k-1) \otimes \boldsymbol{I}_{L+1}] \end{bmatrix} \boldsymbol{z}_{k-1} \right\|_2^2. \tag{9}$$

Note that this ML estimator is conditioned on $\boldsymbol{z}_{k-1}$, and ω_o. We show that (9) is equivalent to the following scalar decoder [6]:

$$\begin{aligned}\hat{c}_p(k) = {} & \arg\max_{c_p\in\mathcal{V},\omega_o} \sum_{l=0}^{L} |e^{j\omega_o(N+L)} g_p(k-1) c_p x^*_{p(L+1)+l}(k) \\ & + x^*_{p(L+1)+l}(k-1) + e^{-j\omega_o(N+L)} g_p^*(k-1) x^*_{p(L+1)+l}(k-2)|^2,\end{aligned} \tag{10}$$

where $g_p(k)$ denotes the pth element on the main diagonal of the matrix $\boldsymbol{G}(k)$, similarly, $\hat{c}_p(k)$ is for $\hat{\boldsymbol{C}}(k)$, and $x_p(k)$ stands for the pth element of $\boldsymbol{x}(k)$. Note that in this scalar case, the cardinality of $\mathcal{V}$ is only $2^{R(L+1)}$, if the transmission rate is R bits per symbol.

However, based on the scalar detector in Eq. (10), we still cannot perform ML estimation, because it depends on ω_o, which is unknown. In this case, we can use the "heuristic" decoder as the ones in [11, 1], or similarly the one provided by (8). However, the question is whether this "heuristic" decoder is optimal. Recently, [12] has answered this question. It has been shown that the ML decoder in (10) is equivalent to the following decoder which is a scalar counterpart of the one provided by (8) (see [12, Appendix] for a proof):

$$\begin{aligned}\hat{c}_p(k) = {} & \arg\max_{c_p\in\mathcal{V}} \sum_{l=0}^{L} \Re[c_p x^*_{p(L+1)+l}(k) x^2_{p(L+1)+l}(k-1) \\ & x^*_{p(L+1)+l}(k-2)],\ \forall p \in [0, P-1].\end{aligned} \tag{11}$$

The decoding complexity in (11) is with the order of $2^{R(L+1)}$. It does not depend on the number of subcarriers.

4.2. Performance Analysis

In Section 4.1, we derive (11) according to the standard ML decoding procedure to show that the estimator in (11) is optimal. In the following, we provide another way to derive the same estimator. This new way will help us to analyze the performance of the decoder.

Define $\tilde{\boldsymbol{u}}_k := \boldsymbol{F}_P^{\mathcal{H}} \boldsymbol{u}_k$. From (4), we have $\tilde{\boldsymbol{u}}_k = \boldsymbol{G}(k)\tilde{\boldsymbol{u}}_{k-1}$. Starting from (7), we can write the scalar input-output relationship for the $(p(L+1)+l)$th element as:

$$x_{p(L+1)+l}(k) = e^{j\omega_o(N+L)} g_p(k) x_{p(L+1)+l}(k-1) + \tilde{w}_{p(L+1)+l}(l), \quad (12)$$

where $\tilde{u}_p(k)$ is the pth element of $\tilde{\boldsymbol{u}}_k$, and $\tilde{w}_{p(L+1)+l}(l) = w_{p(L+1)+l}(l) - e^{j\omega_o(N+L)} g_p(k) w_{p(L+1)+l}(k-1)$. Using (12), we can verify that

$$\begin{aligned} & x_{p(L+1)+l}(k) x^*_{p(L+1)+l}(k-1) \\ = \ & c_p(k) x_{p(L+1)+l}(k-1) x^*_{p(L+1)+l}(k-2) + \eta_{p(L+1)+l}(k), \end{aligned} \quad (13)$$

where the equivalent noise at time-slot $n = p(L+1)+l$ is

$$\begin{aligned} \eta_n(k) = & e^{-j\omega_o(N+L)} g_p^*(k-1) x_n^*(k-2) \tilde{w}_n(k) \\ & + e^{j\omega_o(N+L)} g_p(k) x_n(k-1) \tilde{w}_n^*(k-1) + \tilde{w}_n^*(k-1)\tilde{w}_n(k). \end{aligned} \quad (14)$$

At high signal-to-noise ratio (SNR), $\eta_n(k)$ is approximately Gaussian distributed, because the high order noise terms as the last term in (14) can be ignored [8]. The ML estimator based on (13) is the same as the one in (10). In the following, we will use (13) to analyze the performance of our system.

Note that $w_{p(L+1)+l}(k)$'s are Gaussian distributed with zero mean and they are statistically independent for different k, l, and p. If $E[g_p(k)] = 0$, then we can verify that

$$\begin{aligned} & E[\eta_{p(L+1)+l}(k)] = 0, \text{ and} \\ & E[|\eta_{p(L+1)+l}(k)|^2] = ((L+1)|h(l)|^2 + N_0) 4N_0 + 4N_0^2. \end{aligned}$$

In (13), we notice that $c_p(k)$ does not depend on the index l. In other words, there are $L+1$ observations for the same unknown. Given one realization of frequency-selective channel, the symbol error rate (SER) of $c_p(k)$ drawn from $2^{R(L+1)}$-ary PSK constellation is [8, p.270]

$$P_e(\boldsymbol{h}) \approx 2Q\left(\sqrt{\frac{\sum_{l=0}^{L}|h(l)|^2}{2N_0}} \sin(\pi 2^{-R(L+1)})\right). \quad (15)$$

The average SER over the channel realizations is given as

$$P_E = E_h[P_e(\boldsymbol{h})] \approx 2\int_0^\infty Q\left(\sqrt{\frac{\beta}{2N_0}} \sin(\pi 2^{-R(L+1)})\right) p(\beta) d\beta, \quad (16)$$

where $\beta = \sum_{l=0}^{L} |h(l)|^2$ and $p(\beta)$ is the probability distribution function (pdf) of β. When the channel taps are identically independent distributed complex Gaussian with zero mean and variance $1/(L+1)$, the pdf of β can be viewed as a $\mathcal{X}_2$-distribution with $2(L+1)$ degrees of freedom. The closed form for the average SER in (16) is given as [6]

$$P_E \approx \frac{\Gamma(L+3/2)(L+1)^{2L+1}}{\sqrt{\pi}\Gamma(L+2)} \left(\frac{\sin^2\left(\pi 2^{-R(L+1)}\right)}{2N_0}\right)^{-(L+1)}, \quad (17)$$

where $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$. Eq. (17) shows that the diversity order and the coding gain for the BDD design are:

$$\begin{aligned} & G_d = L+1, \\ \text{and } & G_c = \frac{\sin^2\left(\pi 2^{-R(L+1)}\right)}{2(L+1)^2} \left(\frac{\Gamma(L+3/2)}{\sqrt{\pi}(L+1)\Gamma(L+2)}\right)^{-1/(L+1)} \end{aligned} \quad (18)$$

Thus, we can see that as L increases, diversity increases while coding gain decreases. The Kronecker product operation in (5) essentially corresponds to the repetition coding over flat-fading subchannels to collect diversity. However, this operation reduces coding gain or transmission efficiency. Improved schemes with large coding gains will be one of the future topics. Some other properties of our BDD scheme are summarized in the following remarks:

Remark 1: Just as single differential schemes are robust to slow channel time variation, our double differential scheme is robust to slow CFO variation (drifting). Observing (11), we know that if the CFO changes slow enough that it is approximately constant within three OFDM blocks, the ML detector in (11) is still effective. This is an advantage of double differential design which has not been addressed in the literature.

Remark 2: The double differential design in (4) with PSK modulation reduces the peak-to-average power ratio of OFDM systems from N to $L+1$ [6].

5. SIMULATION RESULTS

In this section, we design some tests to demonstrate the performance of the proposed BDD scheme. In all simulations except specially mentioned, we adopt the system with carrier frequency $f_c = 5.2$ GHz, sampling period $T_s = 0.0625$ μs, number of subcarriers $N = 64$, and thus the OFDM symbol period is 4 μs. The physical CFO is chosen as 0.1% of the carrier frequency. Therefore, the normalized CFO is $\omega_o = 2\pi T_s f_o = 0.65\pi$. Note that this is out of the acquisition range of the CFO estimators in e.g., [5]. The SNR is defined as symbol energy versus noise power ratio. The transmission rate is $R = 1$ bit per symbol. The channel taps are independently generated by complex Gaussian distribution with zero mean and variance satisfying the normalized exponential power delay profile. The number of blocks is $K = 10$.

Test case 1 (multipath diversity): The purpose of this test is to simulate the performance of the BDD design and verify the performance claim in Section 4. We compare the performance over flat fading ($L = 0$) and two-path ($L = 1$) channels. The encoder used is in (4), and the decoder is given in (11). When $L = 0$, the constellation $\mathcal{V}$ is binary PSK, and when $L = 1$, the constellation $\mathcal{V}$ is quadrature PSK. The average SERs are depicted in Figure 2. We observe that the diversity order is indeed $L+1$. When SNR is low (in our case less than 12dB), the BDD design has better performance in flat fading channels, mainly because Euclidean distance also plays an important role to determine the performance. In the same figure, we also plot the analytical result in (17). It has been shown that at high SNR, our simulated result is quite consistent to the theoretical result.

Test case 2 (performance comparisons): In this test case, we will compare our BDD with single differential (SD) OFDM (without any CFO estimator) and with the method in [5]; i.e., a method using SD encoder and decoder with a CFO estimator. To keep the same transmission rate, we choose 5 blocks for one burst as in [5]. The CFO estimator is given in [5, Eq. 18]. We plot average SER versus CFO with fixed SNR. Observing Fig. 3, we notice that when SNR is 20dB, our BDD is the best among three for any CFO. When CFO is extremely small (e.g., $\omega_o < 0.0025$), SD without any CFO estimation outperforms the one in [5], while within a certain range (approximately until $\omega_o = \pi/64 \approx 0.05$), the method in [5] is better than SD without CFO estimation. Because as ω_o increases, periodically $(N+L)\omega_o$ becomes a multiple of 2π, SD's performance is equivalent to zero CFO case.

Test case 3 (CFO drifting robustness): As mentioned in Remark 1, we expect that our BDD scheme is robust to CFO drifting relative to

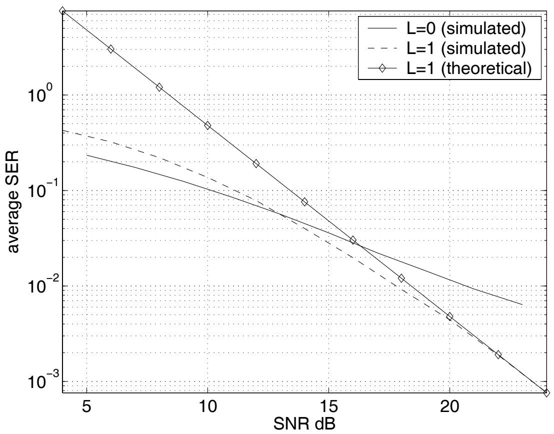

Fig. 2. Average SER with different channel orders

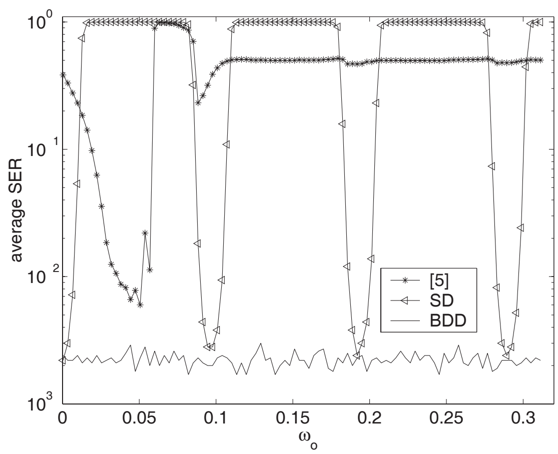

Fig. 3. Average SER versus CFO (SNR=20dB)

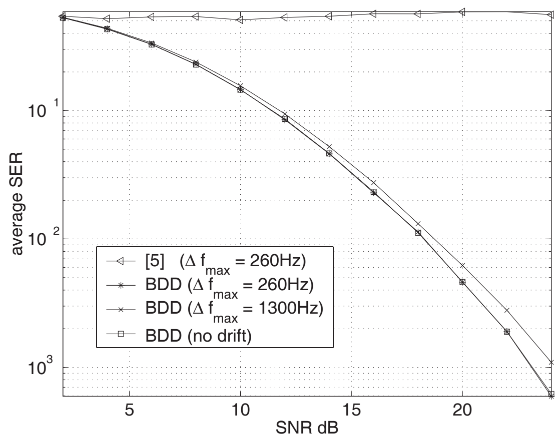

Fig. 4. Average SER with different fast and slow CFO drifting

other training or blind methods. In this test case, we use simulation to verify our claim.

The CFO drifting is modelled as a random process. Suppose the CFO drifting is zero within each OFDM block. But from block to block, it drifts randomly with a small step. Suppose the CFO at the kth block is $f_o(k)$ Hz. For the $(k+1)$st block, the CFO becomes

$$f_o(k+1) = f_o(k) + \Delta f(k),$$

where $\Delta f(k)$ is drawn uniformly from $[-\Delta f_{\max}, \Delta f_{\max}]$. The performance with different $\Delta f_{\max}$ is given in Figure 4. Even with maximum CFO drifting 0.25 Hz pMHz, our double differential scheme still works well. As CFO drifting further increases, our scheme performance decreases slowly. As a comparison, we also perform the method in [5] with $\omega_o = 0.0141\pi = 0.45$ subcarrier spacing. We notice that even 0.05 Hz pMHz (0.1% subcarrier spacing) maximum drift hurts the performance tremendously. This shows another unique advantage of our BDD design.

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