ONE-SHOT SUBSPACE BASED METHOD FOR BLIND CFO ESTIMATION FOR OFDM

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ABSTRACT

In this paper, we propose a novel subspace based approach for blind carrier frequency offset estimation in OFDM. Correlation in squared spectrum of the channel is exploited and low rank signal model is thereby obtained without virtual subcarriers. The proposed estimator accomplishes frequency synchronization with a single OFDM block. No extensive time averaging is needed, which makes the approach very attractive for time and frequency selective channels where the offset may be time varying. The method is statistically very efficient since close to optimal performance is achieved with respect to the Cramér-Rao bound with a single block.

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is a powerful technique to handle impairments of wireless communication media such as multipath propagation, with simplified receiver design. OFDM is a viable candidate for future 4G wireless communications standards. One of the main drawbacks of OFDM is its high sensitivity to carrier frequency offsets (CFO) caused by the oscillator inaccuracies and the Doppler shift due to mobility, giving rise to inter-carrier interference (ICI). Therefore, frequency offset estimation must be accomplished with high fidelity.

In this paper we derive the method for real constant modulus modulation schemes, due to lack of space. Complex modulations require some additional manipulations. The method is blind since it does not require a priori knowledge of the transmitted data or the channel. The proposed one-shot frequency offset estimator needs only a single OFDM block to work with, unlike the majority of blind techniques [1, 3, 6, 7, 8] which do almost always require extensive time averaging.

The key idea is exploit correlation among OFDM subcarriers and more specifically in squared channel spectrum. Novelty of the method relies on the fact that low rank signal model may be derived without any virtual subcarriers. Existing subspace CFO estimation methods for OFDM commonly require virtual subcarriers in order to use the low rank signal model [2].

Performance comparison with the Cramér-Rao bound (CRB) for the blind CFO estimation problem in OFDM [1] demonstrate the high accuracy of the proposed method over broad range of signal-to-noise ratios (SNR). CRB is almost reached with a single OFDM block. Close to optimal performance is achieved compared to existing blind [2, 6, 7, 8], semi-blind [5] and even pilot-aided CFO estimators [9].

The rest of the paper is organized as follows. The system model is briefly described next. Frequency domain correlation in OFDM is studied in Section 3. Then, Section 4 presents the blind CFO estimation algorithm. Simulation results using realistic channel model and different noise levels are presented in Section 5. Finally, Section 6 concludes the paper.

2. SYSTEM MODEL

We use a general OFDM transmission model from [8]. Let $\mathbf{a}(k) = [a_0(k), \ldots, a_{N-1}(k)]^T$ be the $N \times 1$ symbol vector at time instance k. We assume unit energy symbol constellations are used, i.e. $|a_i(k)|^2 = 1$, $i = 0, \ldots, N-1$. The received OFDM $N \times 1$ signal block in time domain after cyclic prefix removal, including the frequency offset, is expressed as

$$\mathbf{z}(k) = \mathbf{C}_\epsilon \mathbf{F}^H \mathbf{D}_{\tilde{\mathbf{h}}}(k)\mathbf{a}(k) + \mathbf{w}(k), \tag{1}$$

where $\mathbf{F} = \left\{ \frac{1}{\sqrt{N}} \exp\left(-j\frac{2\pi kl}{N}\right)\right\}_{k,l=0,\ldots,N-1}$ is the $N \times N$ discrete Fourier transform (DFT) matrix, ^{H} denotes the Hermitian transpose and N is the total number of subcarriers. The diagonal matrix $\mathbf{C}_\epsilon$ introduces the frequency offset and is defined as

$$\mathbf{C}_\epsilon = \exp\left(j\frac{2\pi L\epsilon}{N}\right) \cdot \mathrm{diag}\left\{1, \ldots, \exp\left(j\frac{2\pi(N-1)\epsilon}{N}\right)\right\} \tag{2}$$

where L is the length of the cyclic prefix ($L < N$). The length of the whole OFDM block is $P = N + L$. The quantity $\epsilon \in [0, 1)$ is referred to as normalized frequency offset (wrt. intercarrier spacing). The diagonal matrix $\mathbf{D}_{\tilde{\mathbf{h}}}(k)$ of size $N \times N$ in (1) contains the channel frequency response $\tilde{\mathbf{h}}(k) = [\tilde{h}_1, \ldots, \tilde{h}_N]^T$ at time instance k on its main diagonal. The complex noise term $\mathbf{w}$ is assumed to be zero-mean proper complex Gaussian. The signal and noise processes are assumed to be mutually independent, and i.i.d. over time.

Given an estimate μ of the true value ϵ, CFO compensation may be performed at the receiver as

$$\mathbf{u}_\mu(k) = \mathbf{F}\mathbf{C}_\mu^* \mathbf{z}(k), \tag{3}$$

where $\mathbf{u}_\mu$ is the $N \times 1$ vector in frequency domain obtained after compensation for the offset followed by DFT. The matrix $\mathbf{C}_\mu$ has the same structure as in (2) and * denotes the complex conjugate.

3. FREQUENCY DOMAIN CORRELATION

3.1. Correlation in channel frequency response

The channel is assumed to be block fading and to have a maximum of L_h taps, hence it is frequency selective. The length of the cyclic prefix is set as $L \geq L_h$ in order to avoid inter-block interference. A key idea in OFDM transmission is the frequency correlation of the channel among subcarriers induced by the DFT. Let $\mathbf{h}(k)$ be the $L_h \times 1$ channel impulse response in time domain corresponding to the $N \times 1$ channel frequency response vector $\tilde{\mathbf{h}}(k)$. Since $L_h < N$, vectors $\mathbf{h}(k)$ and $\tilde{\mathbf{h}}(k)$ are related by an N point DFT as $\tilde{\mathbf{h}}(k) = \sqrt{N}\mathbf{F}_{\{:,1:L_h\}}\mathbf{h}(k)$, where the matrix $\mathbf{F}_{\{:,1:L_h\}}$ is made from the L_h first columns of the DFT matrix $\mathbf{F}$. Recalling that the $L_h \times 1$ vector $\mathbf{h}(k)$ may be obtained from $\tilde{\mathbf{h}}(k)$ via Inverse Discrete Fourier (IDFT) transform as $\mathbf{h}(k) = \frac{1}{\sqrt{N}}\mathbf{F}^H_{\{:,1:L_h\}}\tilde{\mathbf{h}}(k)$, the following relationship may be established:

$$\tilde{\mathbf{h}}(k) \quad = \quad \mathbf{F}_{\{:,1:L_h\}}\mathbf{F}^H_{\{:,1:L_h\}}\tilde{\mathbf{h}}(k) \quad = \quad \mathbf{A}\tilde{\mathbf{h}}(k), \qquad (4)$$

where the DFT/IDFT pair is denoted by $\mathbf{A} = \mathbf{F}_{\{:,1:L_h\}}\mathbf{F}^H_{\{:,1:L_h\}}$.

3.2. Correlation in channel squared spectrum

Since the DFT matrix $\mathbf{F}$ is full rank, the rank of $\mathbf{F}_{\{:,1:L_h\}}$ is L_h, and consequently $\text{rank}\left\{\mathbf{F}_{\{:,1:L_h\}}\mathbf{F}^H_{\{:,1:L_h\}}\right\} = L_h$ [10]. In the following, we denote by $\odot$ the element-wise Hadamard product [10]. Multiplication in frequency domain corresponds to convolution in time-domain. Therefore $\tilde{\mathbf{h}}(k) \odot \tilde{\mathbf{h}}(k)$ in frequency is related via IDFT to $\mathbf{h}(k) * \mathbf{h}(k)$ in time, where $*$ denotes the convolution product. Since there are $2L_h - 1$ degrees of freedom in $\mathbf{h}(k) * \mathbf{h}(k)$, the dimension of the subspace associated to squared frequency responses $\tilde{\mathbf{h}}(k) \odot \tilde{\mathbf{h}}(k)$ is $2L_h - 1$. Then, let us state the following theorem:

Theorem 1. *Let $\mathbf{x}$ be a $N \times 1$ vector such that $\mathbf{x} = \mathbf{A}\mathbf{x}$, where $\mathbf{A}$ is a $N \times N$ matrix of rank $L < N$. Then, the squared vector $\mathbf{x} \odot \mathbf{x}$ lies in the column space of $\mathbf{A} \odot \mathbf{A}$.*

Proof is given in the Appendix. From (4), $\tilde{\mathbf{h}}(k) = \mathbf{A}\tilde{\mathbf{h}}(k)$, and according to the above theorem, the squared spectrum $\tilde{\mathbf{h}}(k) \odot \tilde{\mathbf{h}}(k)$ lies in the column space of $\mathbf{A} \odot \mathbf{A}$. Also, since the space of squared channel spectrum is of dimension $2L_h - 1$, we conclude that $\text{rank}\{\mathbf{A} \odot \mathbf{A}\} = 2L_h - 1$.

Hence, because $2L_h - 1 < N$ in practice, low rank model arises from correlation in channel squared spectrum and subspace methods may be developed.

4. ONE-SHOT SUBSPACE BLIND CFO ESTIMATION

In the following, we introduce a blind CFO estimator which aims at restoring correlation in channel squared frequency response. Unlike the majority of blind CFO recovery techniques, the proposed estimator needs only a single OFDM block to operate with (i.e. no time averaging needs to be performed). Hence it allows finding estimate for each block independently. This is obviously significant advantage in case of time-selective channels. Next, a brief description of the proposed algorithm is provided in the case of real symbol modulations. From now on, we drop the time index k, for simplicity of the notation.

Let us consider the noise-free case in equations (1-3) and compute the element-wise Hadamard product $\mathbf{u}_\mu \odot \mathbf{u}_\mu$ as

$$\mathbf{u}_\mu \odot \mathbf{u}_\mu \quad = \quad (\mathbf{M}_{\mu-\epsilon}\mathbf{D}_{\tilde{\mathbf{h}}}\mathbf{a}) \odot (\mathbf{M}_{\mu-\epsilon}\mathbf{D}_{\tilde{\mathbf{h}}}\mathbf{a}), \qquad (5)$$

where we defined $\mathbf{M}_{\mu-\epsilon} = \mathbf{F}\mathbf{C}^*_\mu\mathbf{C}_\epsilon\mathbf{F}^H$. In case of perfect frequency synchronization, $\mu = \epsilon$ and $\mathbf{M}_{\mu-\epsilon} = \mathbf{I}_N$, where $\mathbf{I}_N$ is the $N \times N$ identity matrix. Then (5) becomes

$$\mathbf{u}_\epsilon \odot \mathbf{u}_\epsilon = (\mathbf{D}_{\tilde{\mathbf{h}}}\mathbf{a}) \odot (\mathbf{D}_{\tilde{\mathbf{h}}}\mathbf{a}) = \tilde{\mathbf{h}} \odot \tilde{\mathbf{h}}, \qquad (6)$$

since $\mathbf{D}_{\tilde{\mathbf{h}}}$ is diagonal and $\mathbf{a} \odot \mathbf{a} = \left[a_0^2, \ldots, a_{N-1}^2\right]^T = [1, \ldots, 1]^T$ under the assumption of unit energy real-valued modulations ($a_i \in \{-1, 1\},\ i = 0, \ldots, N-1$). Therefore $\mathbf{u}_\epsilon \odot \mathbf{u}_\epsilon$ is equal to the squared spectrum $\tilde{\mathbf{h}} \odot \tilde{\mathbf{h}}$. Consequently, it inherits the same correlation properties. Frequency mismatch ($\mu \neq \epsilon$) leads to intercarrier interference (ICI) and alters the components of $\mathbf{u}_\mu \odot \mathbf{u}_\mu$. Hence, their correlation structure becomes different from the one of $\tilde{\mathbf{h}} \odot \tilde{\mathbf{h}}$.

This leads to the idea of restoring the correlation induced by Fourier transforms in case of perfect synchronization. It is performed by maximizing the projection of $\mathbf{u}_\mu \odot \mathbf{u}_\mu$ in the subspace spanned by $\tilde{\mathbf{h}} \odot \tilde{\mathbf{h}}$, or equivalently, by minimizing the projection in the orthogonal subspace. Initial correlation is restored for $\mu = \epsilon$. Since $\tilde{\mathbf{h}} \odot \tilde{\mathbf{h}}$ lies in the column space of $\mathbf{A} \odot \mathbf{A}$ and the rank of the latter is $2L_h - 1 < N$, subspace technique may then be constructed. Note that virtual subcarriers (i.e. carriers carrying no data) are not needed to ensure low rank model, which naturally arises from correlation in channel spectrum.

Let us define the squared norm of the projection of $\mathbf{u}_\mu \odot \mathbf{u}_\mu$ to the orthogonal subspace of $\mathbf{A} \odot \mathbf{A}$ as cost function $\mathcal{C}$,

$$\mathcal{C}(\mu) = \left\| \mathbf{\Pi}^\perp_{\mathbf{A}\odot\mathbf{A}} (\mathbf{u}_\mu \odot \mathbf{u}_\mu) \right\|^2, \qquad (7)$$

where $\|\ \|^2$ is the squared Euclidean norm and $\mathbf{\Pi}^\perp_{\mathbf{A}\odot\mathbf{A}}$ denotes the projection matrix to subspace orthogonal to the columns of $\mathbf{A} \odot \mathbf{A}$. As the matrix $\mathbf{A}$ depends only on the DFT size N and the channel length L_h, it may be computed offline, as well as the projection matrix $\mathbf{\Pi}^\perp_{\mathbf{A}\odot\mathbf{A}}$. The cost function $\mathcal{C}(\mu)$ is periodic with period 1, because replacing μ by $\mu + 1$ only produces a shift by one of the OFDM subcarriers, and therefore the correlation features remain unchanged.

An estimate $\hat{\epsilon}$ of the CFO is found by minimizing the norm of vector $\mathbf{u}_\mu \odot \mathbf{u}_\mu$ in orthogonal subspace of $\mathbf{A} \odot \mathbf{A}$ as

$$\hat{\epsilon} = \arg \min_{\mu \in [0,1)} \mathcal{C}(\mu). \qquad (8)$$

The spanned subspaces as well as the cost function to be minimized are depicted in Figures 1 and 2, respectively. Numerical solution to (8) may be found e.g. using a gradient descent method. Computational cost is not prohibitive due to a one dimensional search space with unique minimum (see Fig. 2). Above results may be extended to complex constant modulus constellations. Due to lack of space, this will be presented in a forthcoming paper.

5. SIMULATIONS

In this section simulation results are reported. The OFDM system parameters are chosen as follows: the carrier frequency is $f_0 = 2.4$ GHz, the number of subcarriers is set to $N = 64$ and the available

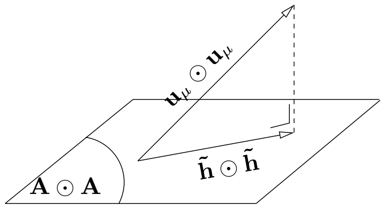

Fig. 1. Subspaces used in proposed blind CFO estimation.

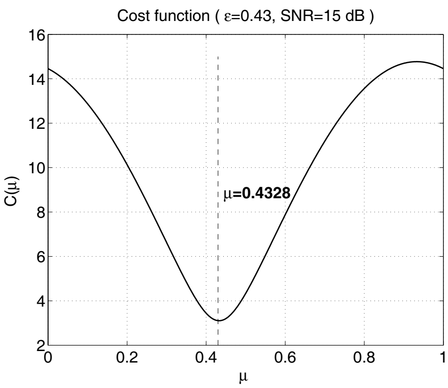

Fig. 2. Cost function, real case.

bandwidth is $B = 0.5$ MHz. The length L of the cyclic prefix is 4. Hence, the dimension of the subspace spanned by squared channel spectrum is 7, i.e. rank $\{\mathbf{A} \odot \mathbf{A}\} = 7$. BPSK modulation is used. Results are given first in the case of time-invariant frequency selective channel and constant carrier offset. Comparison is made with the Cramér-Rao bound (CRB). Finally, simulations are reported for both time-varying channel and offset.

5.1. Time-invariant frequency selective channels, constant CFO

The normalized frequency offset is set as $\epsilon = 0.43$. The wireless channel is considered to be deterministic but unknown to the receiver. That not only affects the transmission and the proposed algorithm, but also the (CRB). The channel impulse response chosen for our simulations has four transmission paths and is the following 4×1 vector:

$$\mathbf{h} = \begin{bmatrix} 0.0731 - 0.8702j \\ 0.3613 - 0.4503j \\ -0.1098 + 0.4476j \\ -0.0270 - 0.0942j \end{bmatrix}.$$

Numerical gradient descent method was used to solve the minimization problem in (8). Squared spectra before and after CFO correction are plotted in Figure 3 against the true one (i.e. with the true channel frequency response and perfect synchronization). The proposed algorithm restores correlation among subcarriers with high fidelity, in both amplitude and phase, without any knowledge of the frequency selective channel, under noise and severe frequency mismatch conditions (SNR = 15 dB and $\epsilon = 0.43$ in Fig. 3).

The Mean Square Error (MSE) is chosen as an error criterion for carrier offset estimation:

$$\mathrm{MSE} = \mathrm{E}\,|\hat{\epsilon} - \epsilon|^2. \tag{9}$$

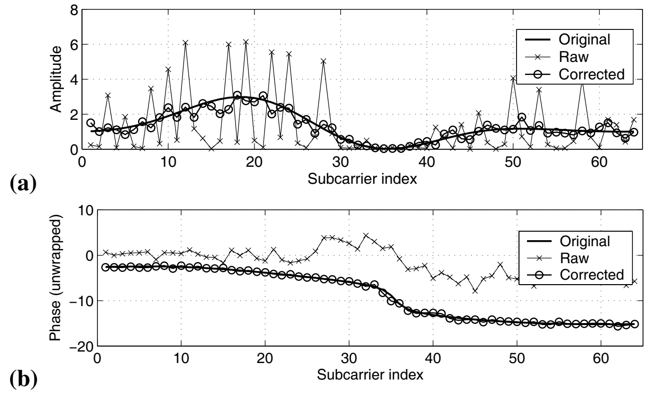

Fig. 3. Squared spectrum, BPSK modulation case, SNR=15 dB and $\epsilon = 0.43$. Both (a) amplitude and (b) phase of squared spectrum are restored with high fidelity, under noise and severe frequency mismatch.

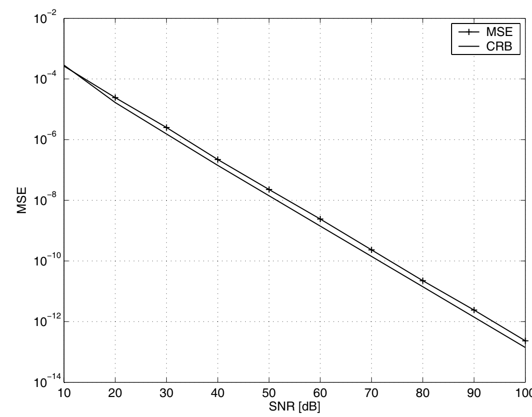

Fig. 4. MSE and CRB vs. SNR, BPSK case, time-invariant frequency selective channel, $\epsilon = 0.43$ and single OFDM block. MSE is 2.25 dB above from the CRB, hence close to optimal performance is achieved with only one block.

Plot of the MSE versus SNR is depicted in Figure 4. Performance is compared to the CRB derived in [1] for the blind CFO estimation problem in OFDM. The MSE of the proposed CFO estimator lies only 2.25 dB above from the CRB, which defines the smallest achievable variance among the class of unbiased estimators. Hence CRB is almost achieved, using only a single OFDM block. Another remarkable property is that neither pilot nor virtual subcarriers are required [2], nor any time averaging over the received blocks. These properties make the proposed method attractive for estimating time-varying CFO in time and frequency selective channels. This case is studied next.

5.2. Time-frequency selective channels, time-varying CFO

Block fading is now assumed, i.e. the channel stays constant within one OFDM block and varies from block to block. The wireless channel is considered to have four independent paths ($L_h = 4$) with unit variance Rayleigh i.i.d. distributed coefficients. The normalized frequency offset is assumed to be uniformly distributed in $[0, 1]$, and to vary block-wise.

Plot of the MSE versus SNR is depicted in Figure 5. Results are ensemble averaged on 10000 different channel and CFO realizations. Highly accurate tracking of time-varying CFO is achieved in time-frequency selective channels in a fully blind manner, for broad-range of SNRs (0 ot 100 dB).

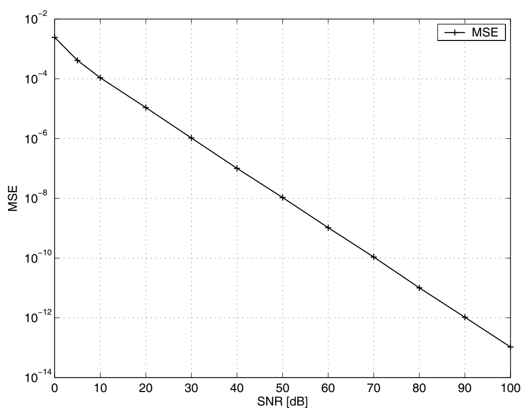

Fig. 5. MSE of normalized offset vs. SNR, BPSK case, i.i.d. Rayleigh block fading channel, uniformly distributed CFO in $[0, 1]$ and varying block-wise. Ensemble average (1000 blocks).

Performance is superior to existing blind [6, 8] or semi-blind techniques [5]. Numerous blind frequency synchronization methods have been proposed in the literature related to OFDM, but most of them need extensive time-averaging in order to get rid of the influence of both noise and data symbols. Hence they cannot perform well in estimating accurately CFO with a single OFDM block [6, 8]. Results are also comparable to pilot-aided CFO estimation techniques such as in [9]. A high-resolution subspace technique based on ESPRIT algorithm was proposed in [2], but requires virtual subcarriers to employ low rank signal model. Virtual subcarriers provide help especially for time and frequency synchronization problems, but at the expense of bandwidth efficiency, since those carriers do not carry any data. However, due to DFT and IDFT operations, low rank model is feasible in OFDM transmission provided that the channel length L_h is such that $2L_h - 1 < N$. This is generally the case for a well designed OFDM system.

6. CONCLUSIONS

In this paper, we propose a novel subspace based algorithm for blind carrier frequency offset estimation in OFDM. Exploiting correlation in squared spectrum of the channel is is the key idea of the method. Low rank signal model is obtained without virtual subcarriers. The proposed estimator performs frequency synchronization with only one OFDM block. Hence, no extensive time averaging is needed. Cramér-Rao bound is almost reached with a single OFDM block. Close to optimal estimation of time-varying carrier offset in time-frequency selective channels is achieved. Finally, the method extends also to complex constant modulus symbol modulations.

7. APPENDIX

Proof of Theorem 1. Let $\mathbf{x} = [x_1, \ldots, x_N]^T$ and let $\mathbf{a}_1, \ldots, \mathbf{a}_N$ be the $N \times 1$ column vectors of $\mathbf{A}$. Then, we exploit the relationship $\mathbf{x} = \mathbf{A}\mathbf{x}$ and express the squared vector $\mathbf{x} \odot \mathbf{x}$ as

$$\mathbf{x} \odot \mathbf{x} = (\mathbf{A}\mathbf{x}) \odot (\mathbf{A}\mathbf{x}) \tag{10}$$

$$= \left(\sum_{k_1=1}^{N} x_{k_1} \mathbf{a}_{k_2} \right) \odot \left(\sum_{k_2=1}^{N} x_{k_2} \mathbf{a}_{k_2} \right) \tag{11}$$

$$= \sum_{k_1=1}^{N} \sum_{k_2=1}^{N} x_{k_1} x_{k_2} \left(\mathbf{a}_{k_1} \odot \mathbf{a}_{k_2} \right) \tag{12}$$

where (11) follows from (10) due to the distributivity of the Hadamard product. Since rank $\{\mathbf{A}\} = L$, there exist a basis $\mathcal{B} = \{\mathbf{b}_1, \ldots, \mathbf{b}_L\}$ of vectors such that $\mathbf{a}_i = \sum_{l=1}^{L} \alpha_{il} \mathbf{b}_l$, $\alpha_{il} \in \mathbb{C}$, $i = 1, \ldots, N$.

We now prove that the vector $\mathbf{x} \odot \mathbf{x}$ lies in the column space of $\mathbf{A} \odot \mathbf{A}$, that is each vector term $\mathbf{a}_{k_1} \odot \mathbf{a}_{k_2}$ in (12) lies in span $\{\mathbf{A} \odot \mathbf{A}\}$. First, for $k_1 = k_2 = k$, the term $\mathbf{a}_k \odot \mathbf{a}_k$ is by definition the kth column of $\mathbf{A} \odot \mathbf{A}$, and obviously lies in its column space. Then, the Hadamard product $\mathbf{a}_{k_1} \odot \mathbf{a}_{k_2}$ may be written using the basis $\mathcal{B}$ as

$$\mathbf{a}_{k_1} \odot \mathbf{a}_{k_2} = \sum_{l_1=1}^{L} \sum_{l_2=1}^{L} \alpha_{k_1 l_1} \alpha_{k_2 l_2} \left(\mathbf{b}_{l_1} \odot \mathbf{b}_{l_2} \right). \tag{13}$$

Finally for $k_1 \neq k_2$, $\mathbf{a}_{k_1} \odot \mathbf{a}_{k_2}$ is a linear combination of vectors $\mathbf{b}_{l_1} \odot \mathbf{b}_{l_2}$, as it is also for each column $\mathbf{a}_k \odot \mathbf{a}_k$ of $\mathbf{A} \odot \mathbf{A}$ (see (13)). Hence $\mathbf{a}_{k_1} \odot \mathbf{a}_{k_2}$ lies in the same vector space.

□

8. REFERENCES

[1] Roman T., Visuri S. and Koivunen V., "Performance bound for blind CFO estimation in OFDM with real-valued constellations", to appear IEEE Semiannual Vehicular Technology Fall Conference, Los-Angeles, USA, Sept. 26-29, 2004. Available at http://wooster.hut.fi/∼troman/publications

[2] Tureli U., Liu H. and Zoltowski M.D., "OFDM blind carrier offset estimation: ESPRIT", IEEE Transactions on Communications, Vol. 48, Issue: 9, 2000, pp. 1459-1461.

[3] Bolcskei H., "Blind estimation of symbol timing and carrier frequency offset in wireless OFDM systems", IEEE Transactions on Communications, Vol. 49, Issue: 6, 2001, pp. 988-999.

[4] Schmidl T.M. and Cox D.C., "Robust frequency and timing synchronization for OFDM", IEEE Transactions on Communications, Vol. 45, Issue: 12, 1997, pp. 1613-1621.

[5] Moose P.H., "A technique for orthogonal frequency division multiplexing frequency offset correction", IEEE Transactions on Communications, Vol. 42, Issue: 10, 1994, pp. 2908-2914.

[6] Ciblat P. and Serpedin E., "A fine blind frequency offset estimator for OFDM/OQAM systems", IEEE Transactions on Signal Processing, Vol. 52, Issue: 1, 2004, pp. 291-296.

[7] Ciblat, P., Vandendorpe, L., "Blind carrier frequency offset estimation for noncircular constellation-based transmissions", IEEE Transactions on Signal Processing, Vol. 51, Issue: 5, May 2003, pp. 1378-1389.

[8] Xiaoli M., Tepedelenlioglu, C., Giannakis, G.B., Barbarossa, S., "Non-data-aided carrier offset estimators for OFDM with null subcarriers: identifiability, algorithms, and performance", IEEE Journal on Selected Areas in Communications, Vol. 19, Issue: 12, Dec. 2001, pp. 2504-2515.

[9] Zhang Z., Zhao M., Zhou H., Liu Y. and Gao J., "Frequency offset estimation with fast acquisition in OFDM system", IEEE Communications Letters, Vol. 8, Issue: 3, March 2004, pp. 171-173.

[10] Horn R.A. and Johnson C.R., " Topics in Matrix Analysis", Cambridge University Press, 1991.