INTEGRATION BASED FREQUENCY OFFSET ESTIMATE FOR THE MIMO-OFDM SYSTEM

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ABSTRACT

In this paper, we focus on the frequency offset estimation for a MIMO system with OFDM transmission technique. In our system, we directly approximate the *a posterior* distribution employing the Gauss-Hermite Integration in the preamble interval. Using this proposed approach, a better frequency offset estimation can be achieved in relatively large frequency offsets compared to the extended Kalman filter based approach over a quasi-static channel environment.

1. INTRODUCTION

To estimate unknown parameters through observations, we need to compute the *a posterior* distribution. For a linear dynamic model and linear observation, the Kalman filter gives an optimal solution to estimate them when the uncertainties is modeled as Gaussian. However, when a part of the parameters are coupled nonlinearly into observations, it is very difficult to compute the *a posterior* density in the multi-dimensional space. For this reason, the first-order nonlinear filter, called the extended Kalman filter (EKF) has been proposed and widely used [1]. However, since the accuracy of the EKF mainly depends on the stability of the Jacobian matrix, the EKF may diverge with a numerically unstable Jacobian matrix. For example, in a large frequency offset, the EKF divergence may occur more frequently. For this reason, more powerful nonlinear filters such as a Gaussian sum particle filter [2], a particle EKF (PEKF) [3],[1], and unscented Kalman filter (UKF) [4] have been proposed. The particle filter approximates the *a posterior* distribution as a set of particles, where each of particles consists of hypothesized state, and the corresponding weight. However, particle filter based structures still require the Jacobian matrix. The UKF performs in general worse than the EKF. To eliminate the need of the Jacobian matrix, we shall directly approximate the *a posterior* distribution using the Gauss-Hermite quadrature. With this quadrature, we can efficiently compute the required *a posterior* distribution [5]. The outline of this paper is as follows. The system and channel models are presented in Section 2, then the proposed frequency offset estimation algorithm is discussed in Section 3. Section 4 provides simulation parameters and results, and Section 5 contains conclusions.

2. SIGNAL AND CHANNEL MODELS FOR MIMO-OFDM SYSTEMS

In this paper, we consider a baseband model for a received MIMO OFDM signal over a multipath fading channel. The notation used for the MIMO-OFDM system includes the following:

- N_f, N_t, N_r : number of multipaths and antennas in transmitter and receiver.
- $K, N-1$: number of subcarriers and OFDM data symbols in one packet.
- $T_g, T_d \triangleq KT_s, T_s$: guard time interval, OFDM data symbol interval, and sampling time.
- T_p : preamble interval, $T_p = M_pT_s$.
- $\mathbf{A}$, $\mathbf{a}$, $(\mathbf{A})_{l,m}$, $(\mathbf{a})_k$: a matrix, a vector, the (l, m) element of the matrix $\mathbf{A}$, and the k-th element of the vector $\mathbf{a}$.

The symbols p, q, k, n are used as indices for the transmit antenna, the receiver antenna, the subcarrier, and the OFDM data symbol respectively. One packet is composed of $(N - 1)$ OFDM data symbols with one preamble symbol which is made up of M_p subcarriers. A guard time interval T_g is also included in each data symbol to eliminate ISI. Input symbols $\{d_k^p(n)\}$ drive the p-th modulator (a K-point IFFT) which modulates the symbols onto K subcarriers. The symbols $d_k(n)$ are chosen from a complex-valued finite alphabet. For convenience, the same signal constellation is employed for all subcarriers and antennas, although the method presented here can be extended to variable-rate constellations. The n-th output of the p-th modulator is

$$s^p(t) = s_D^p(t)p_D(t - T_p - T_d - T_g - T_d^g(n-1)),$$
$$s_D^p(t) = \frac{1}{\sqrt{K}} \sum_{k=0}^{K-1} d_k^p(n) e^{j2\pi k(t-T_p-T_d-T_g-T_d^g(n-1))/T_d}.$$

Here, $p_D(t)$ is a pulse with finite support on $[0, T_d)$, and $T_d^g = T_d + T_g$. The channel between the p-th transmit and q-th receive antenna, $\{f_l^{p,q}(n)\}$, is modeled by a tapped delay line (TDL) such that the n-th received signal at the q-th antenna is

$$y^q(t) = \sum_{p=1}^{N_t} \sum_{l=0}^{N_f-1} f_l^{p,q}(n) s_D^p(t - lT_s) + n^q(t).$$

It is assumed in the sequel that

- Multipath delay spread : $N_f T_s < T_g$.
- Channel: a set of channels $\{f_l^{p,q}(n)\}$ is assumed to be constant over one OFDM symbol duration but varies from symbol to symbol.
- Receiver: receiver is assumed to be matched to the transmitted pulse.

The additive noise $n^q(t)$ is circular white Gaussian with spectral density $2N_0$. Having eliminated the guard interval, the n-th OFDM data symbol vector in the time domain is now given by

$$\mathbf{y}^q(n) \triangleq \sum_{p=1}^{N_t} \tilde{\mathbf{D}}^p(n)\mathbf{f}^{p,q}(n) + \mathbf{z}^q(n), \qquad (1)$$

where

$$\mathbf{z}^q(n) \sim \mathcal{N}(\mathbf{z}^q(n); \mathbf{0}, 2N_0/T_s\mathbf{I}_K),$$
$$\mathbf{f}^{p,q}(n) \triangleq \left[f_0^{p,q}(n), f_1^{p,q}(n), \ldots, f_{N_f-1}^{p,q}(n)\right]^T \in \mathcal{C}^{N_f},$$
$$\tilde{\mathbf{D}}^p(n) \triangleq \begin{bmatrix} \tilde{d}_0^p(n) & \tilde{d}_{K-1}^p(n) & \ldots & \tilde{d}_{K-N_f+1}^p(n) \\ \tilde{d}_1^p(n) & \tilde{d}_0^p(n) & \ldots & \tilde{d}_{K-N_f+2}^p(n) \\ \vdots & \vdots & \ldots & \ldots \\ \tilde{d}_{K-1}^p(n) & \tilde{d}_{K-2}^p(n) & \ldots & \tilde{d}_{K-N_f}^p(n) \end{bmatrix},$$

where $\tilde{\mathbf{D}}^p(n)$ is a non-symmetric circulant matrix specified by $\text{cir}(\{\tilde{d}_k^p(n)\})$, and $\{\tilde{d}_k^p(n)\} = IFFT(\{d_k^p(n)\})$.

Here, $\mathcal{N}(\mathbf{x}; \mathbf{m}_x, \Sigma_x)$ denotes a circular Gaussian density with mean vector $\mathbf{m}_x$ and covariance matrix Σ_x. Also, $\mathbf{I}_i \triangleq \mathbf{I}_{i\times j}, i = j$, denotes an $i \times j$ identity matrix. With some discrepancies between the transmitter and receiver, we assume that there is a frequency offset, such that (1) is expressed as

$$\mathbf{y}^q(n) \triangleq \mathbf{\Delta}(\varepsilon(n)) \sum_{p=1}^{N_t} \tilde{\mathbf{D}}^p(n)\mathbf{f}^{p,q}(n) + \mathbf{z}^q(n). \qquad (2)$$

With the frequency offset $\Delta f(n)$, a normalized frequency offset is defined as $\varepsilon(n) \triangleq \Delta f(n) K T_s$, from which a $K \times K$ matrix $\mathbf{\Delta}(\varepsilon(n))$ is defined as

$$\mathbf{\Delta}(\varepsilon(n)) \triangleq \text{diag}\{1, e^{j2\pi\varepsilon(n)/K}, \ldots, e^{j2\pi(K-1)\varepsilon(n)/K}\}. \qquad (3)$$

Here, we assume that $\mathbf{\Delta}(\varepsilon(n))$ is independent of a receiver index. With this assumption, the received signal from all receiver antennas is expressed as

$$\mathbf{y}(n) \triangleq \tilde{\mathbf{\Delta}}(\varepsilon(n))\tilde{\mathbf{D}}(n)\mathbf{f}(n) + \mathbf{n}(n), \qquad (4)$$

where

$$\mathbf{y}(n) \triangleq [\mathbf{y}^1(n)^T, \ldots, \mathbf{y}^{N_r}(n)^T]^T \in \mathcal{C}^{N_r K},$$
$$\tilde{\mathbf{\Delta}}(\varepsilon(n)) \triangleq \mathbf{I}_{N_r} \otimes \mathbf{\Delta}(\varepsilon(n)), \tilde{\mathbf{D}}(n) \triangleq \mathbf{I}_{N_r \times N_t} \otimes \tilde{\mathbf{D}}^{1,N_t}(n),$$
$$\tilde{\mathbf{D}}^{1,N_t}(n) \triangleq \begin{bmatrix} \tilde{\mathbf{D}}^1(n) & \ldots & \tilde{\mathbf{D}}^{N_t}(n) \end{bmatrix} \in \mathcal{C}^{K \times N_f N_t},$$
$$\mathbf{f}(n) \triangleq [\mathbf{f}^{1,1}(n)^T, \ldots, \mathbf{f}^{N_t,1}(n)^T, \ldots, \mathbf{f}^{N_t,N_r}(n)^T]^T,$$
$$\mathbf{n}(n) \sim \mathcal{N}(\mathbf{n}(n); \mathbf{0}, 2N_0/T_s\mathbf{I}_{N_r K}). \qquad (5)$$

In (5), $\otimes$ represents the Kronecker product.

3. NONLINEAR PARAMETER ESTIMATION BASED ON THE GAUSS-HERMITE INTEGRATION

In this section, we firstly review a numerical approximate technique using the Gauss-Hermite quadrature [5]. After then we directly apply it to the computation of the *a posterior* distribution in the multi-dimensional space.

3.1. Gauss-Hermite Integration

It is well known that the univariate integrals of the following form can be approximated as [5]

$$\int_{-\infty}^{\infty} f(t)e^{-t^2}dt \approx \sum_{i=1}^{n} w_i f(t_i), \qquad (6)$$

where $w_i = \frac{2^{n-1} n! \sqrt{\pi}}{n^2[H_{n-1}(t_i)]^2}$, and t_i is the i-th zero of the Hermite polynomial $H_n(t)$. That is, a function $f(t)$ is approximated by a polynomial of degree of $2n-1$. Also, note that an approximation by the Gauss-Hermite integration (GH-I) works well when a function $f(t)$ is relatively smooth. The closeness of this approximation improves with n [5]. Through the GH-I, the integral is efficiently approximated by a set of finite points $\{t_i\}$ and the function $f(t_i)$ at these points being multiplied in the summation by the weights $\{w_i\}$. For a Gaussian random variable $\mathbf{x}(n) \in \mathcal{R}^{n_x}$ with a positive definite covariance matrix $\boldsymbol{\Sigma}$,$2\boldsymbol{\Sigma} = \mathbf{L}\mathbf{L}^T$, we have

$$\int_{\mathcal{R}^{n_x}} f(\mathbf{x}(n))\mathcal{N}(\mathbf{x}(n);\hat{\mathbf{x}}(n),\boldsymbol{\Sigma})d\mathbf{x}(n) \approx \sum_{i_1=1}^{M_1}\cdots\sum_{i_{n_x}=1}^{M_{n_x}} f(\mathbf{x}^{i_1,i_2,\ldots,i_{i_n}}(n))w^{i_1,i_2,\ldots,i_{n_x}(n)} \quad (7)$$

where

$$\begin{aligned}
\mathbf{x}^{i_1,i_2,\ldots,l_{n_x}}(n) &\triangleq \hat{\mathbf{x}}(n) + \mathbf{L}\tau^{i_1,i_2,\ldots,i_{n_x}}(n),\\
w^{i_1,i_2,\ldots,i_{n_x}}(n) &\triangleq \frac{1}{\pi^{n_x/2}} w_{M_1}^{i_1}(n) w_{M_2}^{i_2}(n)\ldots w_{M_{n_x}}^{i_{n_x}}(n),\\
\tau^{i_1,i_2,\ldots,i_{n_x}}(n) &\triangleq [\tau_{M_1}^{i_1}(n),\tau_{M_2}^{i_2}(n),\ldots,\tau_{M_{n_x}}^{i_{n_x}}(n)]^T. \quad (8)
\end{aligned}$$

Using the Cholesky decomposition, we can express the multivariate integration as a Cartesian product of an univariate integration [6]. In (8), $\{\tau_j^{i_j}(n), w_j^{i_j}(n)\}$ is a set of abscissas and weights for an univariate GH-I in the j-th dimension [6].

3.2. Frequency Offset Estimate for the MIMO-OFDM System

To simplify our approach, we assume that the channel vector is known exactly at the receiver and we use only the preamble symbols to estimate the frequency offset. The system model and observation equations are of the form,

$$\begin{aligned}
\varepsilon(n) &= \alpha_\varepsilon \varepsilon(n-1) + w(n),\\
\mathbf{y}(n) &= \tilde{\boldsymbol{\Delta}}(\varepsilon(n))\tilde{\mathbf{D}}(n)\mathbf{f}(n) + \mathbf{n}(n),\\
w(n) &\sim \mathcal{N}(w(n);0,q_\varepsilon),\\
\mathbf{n}(n) &\sim \mathcal{N}(\mathbf{n}(n);\mathbf{0},2N_0/T_s\mathbf{I}_{N_rK}). \quad (9)
\end{aligned}$$

Note that $\mathbf{y}(n)$ is nonlinear in terms of $\varepsilon(n)$, which is evolving through time according to the system equation. The constant $\alpha_\varepsilon \in \mathcal{R}$ is assumed to be known exactly. In the filtering process, the best estimate $\varepsilon(n)$ given a cumulative observation $\tilde{\mathbf{y}}^n \triangleq \{\mathbf{y}(1),\ldots,\mathbf{y}(n)\}$ is $\hat{\varepsilon}(n|n) \triangleq E[\varepsilon(n)|\tilde{\mathbf{y}}^n]$. To find this we need to compute the density $p(\varepsilon(n)|\tilde{\mathbf{y}}^n)$, which can be obtained from

$$\begin{aligned}
p(\varepsilon(n)|\tilde{\mathbf{y}}^{n-1}) &= \int_{\mathcal{R}} p(\varepsilon(n)|\varepsilon(n-1))p(\varepsilon(n-1)|\tilde{\mathbf{y}}^{n-1})d\varepsilon(n-1),\\
p(\varepsilon(n)|\tilde{\mathbf{y}}^n) &= \frac{p(\mathbf{y}(n)|\varepsilon(n))p(\varepsilon(n)|\tilde{\mathbf{y}}^{n-1})}{\int_{\mathcal{R}} p(\mathbf{y}(n)|\varepsilon(n))p(\varepsilon(n)|\tilde{\mathbf{y}}^{n-1})d\varepsilon(n)} \quad (10)
\end{aligned}$$

When $p(\varepsilon(n-1)|\tilde{\mathbf{y}}^{n-1})$ is approximate as Gaussian

$$p(\varepsilon(n-1)|\tilde{\mathbf{y}}^{n-1}) \approx \mathcal{N}(\varepsilon(n-1);\hat{\varepsilon}(n-1|n-1),P_\varepsilon(n-1|n-1)),$$

then the following density is an immediate result

$$p(\varepsilon(n)|\tilde{\mathbf{y}}^{n-1}) = \mathcal{N}(\varepsilon(n);\hat{\varepsilon}(n|n-1),P_\varepsilon(n|n-1)), \quad (11)$$

where

$$\begin{aligned}
\hat{\varepsilon}(n|n-1) &\triangleq \alpha_\varepsilon \hat{\varepsilon}(n-1|n-1),\\
P_\varepsilon(n|n-1) &\triangleq \alpha_\varepsilon^2 P_\varepsilon(n-1|n-1) + q_\varepsilon.
\end{aligned}$$

However, in the formulation of (10), the received signal vector $\mathbf{y}(n)$ is a nonlinear function of $\varepsilon(n)$, such that it is hard to compute $p(\varepsilon(n)|\tilde{\mathbf{y}}^n)$. Thus, we propose a new numerical approximation technique to the *a posterior* distribution using the GH-I. Now using the GH-I, $\hat{\varepsilon}(n|n)$ and $P_\varepsilon(n|n)$ of $p(\varepsilon(n)|\tilde{\mathbf{y}}^n)$ are respectively computed as follows

$$\begin{aligned}
\hat{\varepsilon}(n|n) &= \frac{\int_{\mathcal{R}} \varepsilon(n)p(\mathbf{y}(n)|\varepsilon(n))p(\varepsilon(n)|\tilde{\mathbf{y}}^{n-1})d\varepsilon(n)}{\int_{\mathcal{R}} p(\mathbf{y}(n)|\varepsilon(n))p(\varepsilon(n)|\tilde{\mathbf{y}}^{n-1})d\varepsilon(n)},\\
P_\varepsilon(n|n) &= \frac{\int_{\mathcal{R}} \varepsilon(n)^2 p(\mathbf{y}(n)|\varepsilon(n))p(\varepsilon(n)|\tilde{\mathbf{y}}^{n-1})d\varepsilon(n)}{\int_{\mathcal{R}} p(\mathbf{y}(n)|\varepsilon(n))p(\varepsilon(n)|\tilde{\mathbf{y}}^{n-1})d\varepsilon(n)} - \hat{\varepsilon}(n|n)^2. \quad (12)
\end{aligned}$$

Denoting $\varepsilon_i(n)$ and w_i as the i-th zero of the Hermitian polynomial and its corresponding weight, we can readily approximate (12) according to

$$\begin{aligned}
\hat{\varepsilon}(n|n) &= \hat{\varepsilon}(n|n-1) + \sqrt{2P_\varepsilon(n|n-1)}(a),\\
P_\varepsilon(n|n) &= 2P_\varepsilon(n|n-1) \times (b - a^2),\\
a &\triangleq \frac{\sum_{i=1}^N w_i \varepsilon_i(n) f_1(\tilde{\varepsilon}_i(n))}{\sum_{j=1}^N w_j f_1(\tilde{\varepsilon}_j(n))},\\
b &\triangleq \frac{\sum_{i=1}^N w_i \varepsilon_i(n)^2 f_1(\tilde{\varepsilon}_i(n))}{\sum_{j=1}^N w_j f_1(\tilde{\varepsilon}_j(n))}. \quad (13)
\end{aligned}$$

In (13), a function $f_1(\varepsilon(n))$ is defined as

$$f_1(\varepsilon(n)) \triangleq \frac{1}{(2\pi N_0/T_s)^{KN_r/2}} e^{-\frac{||\mathbf{y}(n)-\tilde{\boldsymbol{\Delta}}(\varepsilon(n))\tilde{\mathbf{D}}(n)\mathbf{f}(n)||^2}{2N_0/T_s}}$$

and $\tilde{\varepsilon}_i(n) \triangleq \hat{\varepsilon}(n|n-1) + \sqrt{2P_\varepsilon(n|n-1)}\varepsilon_i(n)$. With (13), we can finally approximate $p(\varepsilon(n)|\tilde{\mathbf{y}}^n)$ as

$$p(\varepsilon(n)|\tilde{\mathbf{y}}^n) \approx \mathcal{N}(\varepsilon(n);\hat{\varepsilon}(n|n),P_\varepsilon(n|n)). \quad (14)$$

4. SIMULATION RESULTS

The following parameters are used in the simulations.

- $K = 64$, $N_t = N_r = 4$, $N_p = 320$,
- $\mathbf{f}^{p,q}(n) = \{0.749, 0.502, 0.3365, 0.2256, 0.1512\}, \forall\, p, q$,

For a quasi-static channel, we assume that $\alpha_\varepsilon \approx 1.0$, $q_\varepsilon \approx 0.0$. For the QPSK subcarrier modulation, Figure 1 shows the absolute frequency offset estimation error at different frequency offsets over a quasi-static channel. We assume that the channel is exactly known at the receiver and apply the Gauss-Hermite filter only in the preamble interval [7]. At a reasonable high frequency offset error, the proposed scheme can work better than the EKF at SNRs in the range $(0, 16)$. However, the proposed scheme does not appear to have a performance advantage over the EKF based approach in relatively small frequency offset errors. For the OFDM data symbol interval, $N = 10$, we use the MIMO-OFDM data detector proposed by the authors [8], called the QRD-M with $M = 16$. Figure 2 is the corresponding BER curve with a different value of $\epsilon(n)$. It shows that with the Gauss-Hermite filter, we can have 4 [dB] gain at a 2×10^{-3} BER with $\epsilon(n) = 0.3$. However, at $\epsilon(n) = 0.1$ two approaches have almost the same performance. This is the expectation from Figure 1.

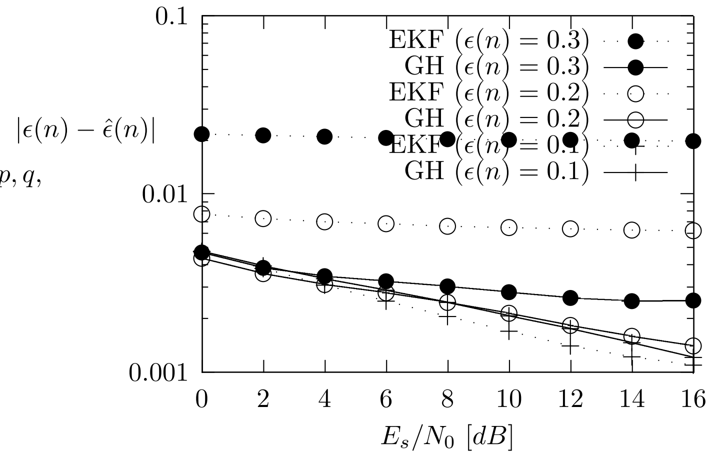

Fig. 1. Absolute frequency offset estimation error.

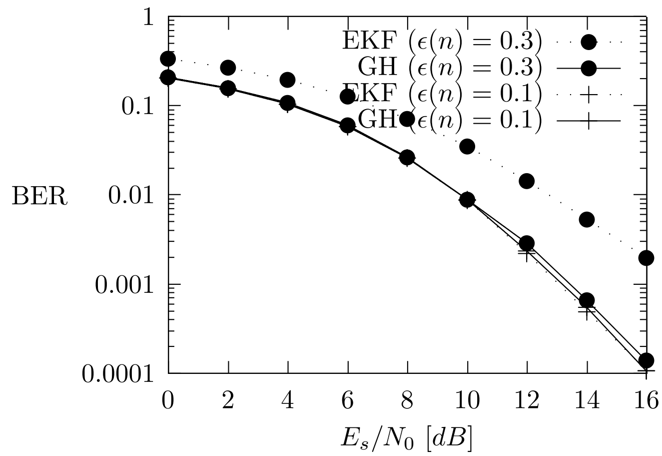

Fig. 2. BER performance of the MIMO-OFDM.

5. CONCLUSIONS

We propose a new frequency offset estimator for MIMO-OFDM over an exactly known quasi-static channel. As a nonlinear parameter estimator, we employ the Gauss-Hermite filter. Simulation results show significant performance improvements compared with the EKF based approach in a relatively large frequency offset for the uncoded MIMO-OFDM system.

6. REFERENCES

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