Unified analysis of a class of blind feedforward symbol timing estimators employing second-order statistics

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***Abstract*— In this paper, all the previously proposed digital blind feedforward symbol timing estimators employing second-order statistics are casted into a unified framework. The finite sample mean-square error (MSE) expression for this class of estimators is established. Simulation results are also presented to corroborate the analytical results. It is found that the feedforward conditional maximum likelihood (CML) estimator and the square law nonlinearity (SLN) estimator with a properly designed pre-filter perform the best and their performances coincide with the asymptotic conditional Cramer-Rao bound (CCRB), which is the performance lower bound for the class of estimators under consideration.**

I. Introduction

The problem of digital blind feedforward symbol timing estimation assumes recovery of the timing delay of the received signal based on the oversampled and unsynchronized received samples. Many algorithms were proposed in the literature to solve this problem. They include the nonlinearity-based estimators (square law [1], logarithmic [6], absolute value and fourth order [7]), cyclostationary-based estimators [2], [3], [4], a two samples per symbol ad-hoc estimator [8] and its modified version [9], feedforward conditional maximum likelihood (CML) estimator [10] and the square law nonlinearity (SLN) estimator with pre-filter [5].

With so many estimators, designed using different philosophies and their performances analyzed independently under different assumptions, one would wonder whether we can have a general framework to analyze the performances of these estimators so that a fair and easy comparison can be made. This question was partially answered in [7], in which a technique for evaluating the jitter performance of symbol timing estimators employing a zero-memory, general type of nonlinearity was presented. In this paper, we provide a more thorough analysis by formulating all the blind feedforward symbol timing estimators employing second-order statistics (which include the estimators in [1], [2], [3], [4], [5], [8], [9] and [10]) into a single estimation framework, and then by deriving the finite sample mean-square error (MSE) expression for this class of estimators. The MSE expression for any individual estimator can be obtained from the general expression by setting suitable parameters. The analytical results are compared with the computer simulation results, and it is found that both sets of results match very well. Furthermore, it is found that within the class of estimators employing second-order statistics, the SLN estimator with a properly designed pre-filter [5] and the feedforward CML estimator [10] perform the best and their performances coincide with the conditional Cramer-Rao bound (CCRB) [11], which is the performance lower bound for the class of estimators under consideration.

II. Unified formulation for symbol timing estimators employing second-order statistics

For linear modulations transmitted through AWGN channels, the oversampled received signal can be written as

$$r(n) \triangleq e^{j\theta_o}\sqrt{E_s/T}\sum_i d_i g(nT_s - iT - \varepsilon_o T) + \eta(nT_s) \ , \quad (1)$$

where θ_o is the unknown phase offset; E_s is the symbol energy; d_i stands for the zero-mean unit variance, independently and identically distributed (i.i.d.) complex valued symbols being transmitted; $g(t) \triangleq g_t(t) \star g_r(t)$ is the combined response[1] of the unit energy transmit filter $g_t(t)$ and the receiving filter $g_r(t)$; T is the symbol period; $T_s \triangleq T/Q$ with Q being the oversampling ratio; $\varepsilon_o \in [0, 1)$ is the unknown symbol timing delay to be estimated and $\eta(nT_s)$ stands for the samples of filtered noise. It is assumed that the noise samples before receive filtering is complex-valued circularly distributed white Gaussian with power density N_o.

In this paper, we consider the class of estimators taking the following general form:

$$\hat{\varepsilon} = -\frac{1}{2\pi}\arg\left\{\sum_{k=0}^{K-1}\Lambda(k)e^{-j2\pi k/K}\right\}, \quad (2)$$

where $\Lambda(k) = \mathbf{r}^H\mathbf{B}_k\mathbf{r}$ with $\mathbf{r} \triangleq [r(0),\ r(1),\ ...,\ r(L_oQ-1)]^T$ is the observation vector of length L_o symbols and $\mathbf{B}_k$ is a fixed matrix of dimension $L_oQ \times L_oQ$. Let us now consider some special cases.

A. Cyclic correlation-based estimator

The cyclic correlation-based estimator [3] is given by

$$\hat{\varepsilon} = -\frac{1}{2\pi}\arg\left\{\sum_{n=0}^{L_oQ-\tau-1} r^*(n)r(n+\tau)e^{-j\pi\tau/Q}e^{-j2\pi n/Q}\right\}, \quad (3)$$

[1]Notation $\star$ stands for convolution.

for $Q \geq 3$ and some integer lag $\tau \geq 0$. Note that different values of τ result in different previously proposed estimators in the literature ($\tau = 0$ corresponds to the estimators proposed in [1] and [4], $\tau = Q$ corresponds to the estimator in [2]).

If we decompose the summation term in (3) into Q polyphase components and define $n_u(k) \triangleq \lfloor (L_oQ - \tau - 1 - k)/Q \rfloor$, we can write the expression inside $\arg\{\ \}$ as

$$\sum_{k=0}^{Q-1} \underbrace{e^{-j\pi\tau/Q} \sum_{n=0}^{n_u(k)} r^*(nQ+k)r(nQ+k+\tau)}_{\Lambda_{CC}(k)} e^{-j2\pi k/Q}. \tag{4}$$

Therefore, the cyclic correlation-based estimator takes the form of (2) with $K = Q$. Expressing $\Lambda_{CC}(k)$ into matrix form, we have $\Lambda_{CC}(k) = \mathbf{r}^H \mathbf{B}_k^{CC} \mathbf{r}$, where $\mathbf{B}_k^{CC}$ is a $L_oQ \times L_oQ$ matrix with its $(nQ+k, nQ+k+\tau)^{th}$ element ($n = 0, 1, ..., n_u(k)$) equal to $e^{-j\pi\tau/Q}$ and other elements equal zero.

B. Lee's estimator and the modified estimator

A two samples per symbol estimator was proposed by Lee in [8]. Later, this estimator was modified to remove its asymptotic bias [9]. The modified version of Lee's estimator can be written as

$$\begin{aligned}\hat{\varepsilon} = -\frac{1}{2\pi} \arg\Big\{ \gamma \sum_{n=0}^{L_oQ-1} |r(n)|^2 e^{jn\pi} \\ + \sum_{n=0}^{L_oQ-2} \mathfrak{Re}[r^*(n)r(n+1)] e^{j(n-0.5)\pi} \Big\},\end{aligned} \tag{5}$$

with $Q = 2$ and γ is a constant depending on the pulse shape $g(t)$. If $g(t)$ is a raised cosine pulse with roll-off factor ρ, then $\gamma = 8\sin(\pi\rho/2)/(\rho\pi(4-\rho^2))$ [9]. The original Lee's estimator can be obtained by setting $\gamma = 1$. Now rewrite the expression inside $\arg\{\ \}$ of (5) as follows:

$$\begin{aligned}&\underbrace{\gamma \sum_{n=0}^{L_o-1} |r(nQ)|^2}_{\Lambda_{Lee}(0)} + \underbrace{\sum_{n=0}^{L_o-1} \mathfrak{Re}[r^*(nQ)r(nQ+1)]}_{\Lambda_{Lee}(1)} e^{-j\pi/2} \\ &+ \underbrace{\gamma \sum_{n=0}^{L_o-1} |r(nQ+1)|^2}_{\Lambda_{Lee}(2)} e^{-j\pi} \\ &+ \underbrace{\sum_{n=0}^{L_o-2} \mathfrak{Re}[r^*(nQ+1)r(nQ+2)]}_{\Lambda_{Lee}(3)} e^{-j3\pi/2}.\end{aligned} \tag{6}$$

Then the estimator in (5) can also be expressed in the form of (2) with $K = 4$. With the fact that $\mathfrak{Re}(x) = (x + x^*)/2$ and expressing $\Lambda_{Lee}(k)$ in matrix form, we have $\Lambda_{Lee}(k) = \mathbf{r}^H \mathbf{B}_k^{Lee} \mathbf{r}$, where[2] $\mathbf{B}_0^{Lee} = \gamma \mathbf{I}_{L_o} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $\mathbf{B}_1^{Lee} = \mathbf{I}_{L_o} \otimes \begin{bmatrix} 0 & 0.5 \\ 0.5 & 0 \end{bmatrix}$, $\mathbf{B}_2^{Lee} = \gamma \mathbf{I}_{L_o} \otimes \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, and

$$\mathbf{B}_3^{Lee} = \begin{bmatrix} 0 & \mathbf{0}_{1\times 2(L_o-1)} & 0 \\ \mathbf{0}_{2(L_o-1)\times 1} & \mathbf{I}_{L_o-1} \otimes \begin{bmatrix} 0 & 0.5 \\ 0.5 & 0 \end{bmatrix} & \mathbf{0}_{2(L_o-1)\times 1} \\ 0 & \mathbf{0}_{1\times 2(L_o-1)} & 0 \end{bmatrix}. \tag{7}$$

[2]Notation $\otimes$ denotes the Kronecker product.

C. Feedforward CML estimator

The feedforward symbol timing estimator based on the conditional ML principle was proposed in [10]. Unfortunately, the results in [10] cannot be directly applied here since the original estimator was derived under the assumption that the noise samples are independent of each other, but in the signal model (1), the noise samples are correlated due to the receiver filtering. However, since the correlations between noise samples are related to the receiving filter (which is known), we can whiten the filtered noise samples by pre-multiplying the observation vector $\mathbf{r}$ with $(\boldsymbol{\varphi}^{-1/2})^H$, where $\boldsymbol{\varphi}$ is the correlation matrix of the noise vector, with its elements given by $[\boldsymbol{\varphi}]_{ij} = \int_{-\infty}^{\infty} g_r^*(t) g_r(t - (i-j)T/Q) dt$, and $\boldsymbol{\varphi}^{-1/2}$ denotes any square root of $\boldsymbol{\varphi}^{-1}$ (e.g., Cholesky decomposition) such that $\boldsymbol{\varphi}^{-1/2}(\boldsymbol{\varphi}^{-1/2})^H = \boldsymbol{\varphi}^{-1}$. Then the results of [10] can be applied readily to this transformed observation vector $(\boldsymbol{\varphi}^{-1/2})^H \mathbf{r}$. Following this direction, it can be shown that the feedforward CML symbol timing estimator is given by

$$\hat{\varepsilon} = -\frac{1}{2\pi} \arg\left\{ \sum_{k=0}^{K-1} \Lambda_{CML}(k) e^{-j2\pi k/K} \right\}, \tag{8}$$

where $K \geq 3$ and $\Lambda_{CML}(k) = \mathbf{r}^H \mathbf{B}_k^{CML} \mathbf{r}$ with

$$\mathbf{B}_k^{CML} \triangleq \boldsymbol{\varphi}^{-1} \mathbf{A}_\varepsilon (\mathbf{A}_\varepsilon^H \boldsymbol{\varphi}^{-1} \mathbf{A}_\varepsilon)^{-1} \mathbf{A}_\varepsilon^H \boldsymbol{\varphi}^{-1} \Big|_{\varepsilon = k/K}, \tag{9}$$

$$\mathbf{A}_\varepsilon \triangleq [\mathbf{a}_{-L_g}(\varepsilon),\ \mathbf{a}_{-L_g+1}(\varepsilon),\ ...,\ \mathbf{a}_{L_o+L_g-1}(\varepsilon)], \tag{10}$$

$$\begin{aligned}\mathbf{a}_i(\varepsilon) \triangleq [&g(-iT - \varepsilon T),\ g(T_s - iT - \varepsilon T),\ ..., \\ &g((L_oQ-1)T_s - iT - \varepsilon T)]^T,\end{aligned} \tag{11}$$

and L_g denotes the number of symbols affected by the inter-symbol interference (ISI) introduced by one side of $g(t)$.

D. Estimators with pre-filter

In [5], a properly designed pre-filter was applied to the SLN estimator and the modified Lee's estimator to improve their performances at medium and high SNRs. In general, the pre-filtering technique can be applied to the general estimator (2). In that case, the observation vector is composed of samples from the output of pre-filter. That is, $\Lambda_{PRE}(k) = \mathbf{x}^H \mathbf{B}_k \mathbf{x}$ with $\mathbf{x} \triangleq [x(0),\ x(1),\ ...,\ x(L_oQ-1)]^T$ and $x(n) \triangleq r(n) \star h(n)$ is the further filtered (apart from the receiver filtering) received signal samples through the pre-filter $h(n)$. If $h(n)$ is of finite length L_p, then $x(n) = \sum_{v=0}^{L_p-1} h(v) r(n-v)$ and

$$\mathbf{x} = \underbrace{\begin{bmatrix} h(L_p-1) & ... & h(0) & & \\ & h(L_p-1) & ... & h(0) & \\ & \ddots & ... & \ddots & \\ & & h(L_p-1) & ... & h(0) \end{bmatrix}}_{\mathbf{H}} \tilde{\mathbf{r}} \tag{12}$$

where $\tilde{\mathbf{r}} \triangleq [r(-L_p+1),\ r(-L_p+2),\ ...,\ r(L_oQ-1)]^T$. Therefore, the general estimator with pre-filter takes the form:

$$\hat{\varepsilon} = -\frac{1}{2\pi}\arg\left\{\sum_{k=0}^{K-1}\Lambda_{PRE}(k)e^{-j2\pi k/K}\right\}, \quad (13)$$

where $\Lambda_{PRE}(k) = \tilde{\mathbf{r}}^H\mathbf{H}^H\mathbf{B}_k\mathbf{H}\tilde{\mathbf{r}} \triangleq \tilde{\mathbf{r}}^H\mathbf{B}_k^{PRE}\tilde{\mathbf{r}}$. For example, for the cylic correlation-based estimator with pre-filter, we have $\mathbf{B}_k^{PRE} = \mathbf{H}^H\mathbf{B}_k^{CC}\mathbf{H}$.

Notice that, due to pre-filtering, although the observation vector **x** is of length L_oQ, the length of effective observation $\tilde{\mathbf{r}}$ (before pre-filtering) is L_oQ+L_p-1. Also, $\mathbf{B}_k^{PRE}$ is of dimension $(L_oQ+L_p-1)\times(L_oQ+L_p-1)$, rather than $(L_oQ-1)\times(L_oQ-1)$. Of course, if there is no pre-filter (i.e., $h(n)=\delta(n)$), all the equations in this subsection would reduce to that of the original estimator.

III. Performance analysis

A. Performance bound

In [11], the asymptotic CCRB was introduced for symbol timing estimation problem. The asymptotic CCRB is a lower bound tighter than the modified Cramer-Rao bound (MCRB), but still a valid lower bound on the variance of any consistent estimator that is quadratic with respect to the received signal (which is the class of estimators under consideration). However, the asymptotic CCRB in [11] was derived assuming white Gaussian noise samples, therefore, the whitening technique similar to that in Section II-C has to be applied in order to include the effect of the receiving filter. Applying the results of [11] to the whitened observation vector $(\boldsymbol{\varphi}^{-1/2})^H\mathbf{r}$, it can be shown that for fixed ε_o,

$$\text{CCRB}^{as}(\varepsilon_o) = \frac{1}{2\text{tr}(\tilde{\mathbf{D}}_{\varepsilon_o}^H\mathbf{\Psi}_{\varepsilon_o}\tilde{\mathbf{D}}_{\varepsilon_o})}\left(\frac{E_s}{N_o}\right)^{-1} \quad (14)$$

where $\tilde{\mathbf{D}}_\varepsilon \triangleq \frac{1}{\sqrt{Q}}d\mathbf{A}_\varepsilon/d\varepsilon$ and $\mathbf{\Psi}_\varepsilon \triangleq \boldsymbol{\varphi}^{-1} - \boldsymbol{\varphi}^{-1}\mathbf{A}_\varepsilon(\mathbf{A}_\varepsilon^H\boldsymbol{\varphi}^{-1}\mathbf{A}_\varepsilon)^{-1}\mathbf{A}_\varepsilon^H\boldsymbol{\varphi}^{-1}$. Since the symbol timing delay ε_o is assumed to be uniformly distributed in $[0,1)$, the average asymptotic CCRB can be calculated by numerical integration of (14).

B. MSE expression

In this section, we present the MSE expression for the general estimator (2). The derivation procedures follow closely to that in [10]. The only difference is that, the MSE expression in [10] was derived under the assumption of white noise, while in this paper, the correlation of noise has to be taken into consideration. This can be easily done by modifying just a few lines of the derivations in [10]. Due to space limitation, only the results are presented. Interested readers can refer to [10]. It can be shown that for a true timing delay ε_o, the MSE of the general estimator (2) is given by

$$\text{MSE}(\varepsilon_o) \triangleq \mathbf{E}[(\hat{\varepsilon}-\varepsilon_o)^2] = -\left(\frac{1}{2\pi}\right)^2\frac{\mathfrak{Re}(\phi_1)-\phi_2}{\mathfrak{Re}(\phi_1)+\phi_2}, \quad (15)$$

where

$$\phi_1 \triangleq e^{j4\pi\varepsilon_o}\sum_{k_1=0}^{K-1}\sum_{k_2=0}^{K-1}\mathbf{E}[\Lambda(k_1)\Lambda(k_2)]e^{-j2\pi k_1/K}e^{-j2\pi k_2/K}, \quad (16)$$

$$\phi_2 \triangleq \sum_{k_1=0}^{K-1}\sum_{k_2=0}^{K-1}\mathbf{E}[\Lambda(k_1)\Lambda^*(k_2)]e^{-j2\pi k_1/K}e^{j2\pi k_2/K}. \quad (17)$$

In the above equations,

$$\begin{aligned}\mathbf{E}[\Lambda(k_1)\Lambda(k_2)] &= tr[\mathbf{B}_{k_1}^T\mathbf{R}_{\varepsilon_o}]tr[\mathbf{B}_{k_2}^T\mathbf{R}_{\varepsilon_o}] \\ &\quad + tr[\mathbf{B}_{k_1}^T\mathbf{R}_{\varepsilon_o}\mathbf{B}_{k_2}^T\mathbf{R}_{\varepsilon_o}] + c(k_1,k_2), \quad (18)\\ \mathbf{E}[\Lambda(k_1)\Lambda^*(k_2)] &= tr[\mathbf{B}_{k_1}^T\mathbf{R}_{\varepsilon_o}]tr[\mathbf{B}_{k_2}\mathbf{R}_{\varepsilon_o}] \\ &\quad + tr[\mathbf{B}_{k_1}^T\mathbf{R}_{\varepsilon_o}\mathbf{B}_{k_2}\mathbf{R}_{\varepsilon_o}] + c(k_1,k_2), \quad (19)\end{aligned}$$

where $tr[.]$ denotes the trace of a matrix,

$$\mathbf{R}_\varepsilon \triangleq \frac{E_s}{T}\mathbf{G}_\varepsilon + \frac{N_oQ}{T}\boldsymbol{\varphi}, \quad (20)$$

$$[\mathbf{G}_\varepsilon]_{ij} \triangleq \sum_{n=-\infty}^{\infty}g^*(iT/Q-nT-\varepsilon T)g(jT/Q-nT-\varepsilon T), \quad (21)$$

$$\begin{aligned}c(k_1,k_2) &\triangleq \frac{E_s^2}{T^2}(m_4-2) \\ &\times\sum_{n=-\infty}^{\infty}[\mathbf{a}_n(\varepsilon_o)^H\mathbf{B}_{k_1}\mathbf{a}_n(\varepsilon_o)][\mathbf{a}_n(\varepsilon_o)^H\mathbf{B}_{k_2}\mathbf{a}_n(\varepsilon_o)],\end{aligned} \quad (22)$$

with $\mathbf{a}_n(\varepsilon_o)$ defined in (11), and $m_4 = \mathbf{E}[|d_i|^4]$ is the fourth order moment of the transmitted symbols. As the symbol timing delay ε_o is assumed to be uniformly distributed in $[0,1)$, the average MSE is calculated by numerical integration of (15).

IV. Numerical examples and discussions

In this section, the general analytical MSE expression presented in the last section will be plotted as a function of E_s/N_o for different estimators. The analytic results are compared with the corresponding simulation results and the asymptotic CCRB. All the results are generated assuming i.i.d. QPSK data, $L_o=100$, both $g_t(t)$ and $g_r(t)$ are square root raised cosine pulses with $\rho=0.3$, $L_g=3$, and ε_o is uniformly distributed in the range $[0,1)$. The carrier phase θ_o is generated as a uniformly distributed random variable in the range $[-\pi,\pi)$, and assumed constant during each estimation. Each simulation point is obtained by averaging 10^4 simulation runs. The asymptotic CCRB is computed assuming $Q=2$. In this paper, the results of the following representative estimators are presented: 1) Modified Lee's estimator [9], 2) Feedforward CML estimator [10], 3) SLN estimator [1] and 4) SLN estimator with pre-filter [5] where the pre-filter being used is $h(n)=g(t)\cos(2\pi t/T)|_{t=nT/Q}$ for $n=-5Q,...,5Q$ (i.e., $L_p=10Q+1$) [5]. Notice that the first two estimators assume an oversampling ratio $Q=2$, while the last two estimators assume an oversampling ratio $Q=4$.

For the computation of $\mathbf{B}_k^{CML}$ and $\text{CCRB}^{as}(\varepsilon_o)$, there is a need to calculate $\boldsymbol{\varphi}^{-1}$. Unfortunately, numerical calculations show that, for the $g_r(t)$ under consideration, $\boldsymbol{\varphi}$ is not full rank (at least to the accuracy of Matlab). A main reason for rank deficiency is that, due to the nature of $g_r(t)$, when $|i-j|$ is large, the values of $[\boldsymbol{\varphi}]_{ij}$ are very very small but not zero. A way to get around this is to replace $\boldsymbol{\varphi}^{-1}$ by $\bar{\boldsymbol{\varphi}}^{-1}$, where

$$[\bar{\boldsymbol{\varphi}}]_{ij} = \begin{cases} [\boldsymbol{\varphi}]_{ij} & \text{if } |i-j| < L_{\varphi}Q \\ 0 & \text{otherwise.} \end{cases} \tag{23}$$

In this way, the matrix $\bar{\boldsymbol{\varphi}}$ can be made full rank, but at the same time, significant part of the correlation between noise samples can still be represented accurately. Since most of the correlation induced by $g_r(t)$ is confined to a duration of a few symbols, $L_{\varphi} = 4$ is used for the rest of the paper. Notice that the matrix $\boldsymbol{\varphi}$ in (20) need not to be replaced by $\bar{\boldsymbol{\varphi}}$ since no inversion is required.

Fig. 1 shows the results for the modified Lee's estimator and the feedforward CML estimator. It can be seen that the analytical and simulation results match very well. Furthermore, the feedforward CML estimator performs much better than the modified Lee's estimator at high E_s/N_o and its performance coincides with the asymptotic CCRB, meaning that the feedforward CML estimator is the best (in terms of MSE performance) within the class of symbol timing estimators employing second-order statistics. Fig. 2 shows the results for the SLN estimator with and without pre-filter. This figure also shows that the simulation results match the analytical results very well. Moreover, the figure shows that the application of a properly designed pre-filter removes the estimation error floor at high E_s/N_o and makes the performance of the resultant estimator reaches the asymptotic CCRB.

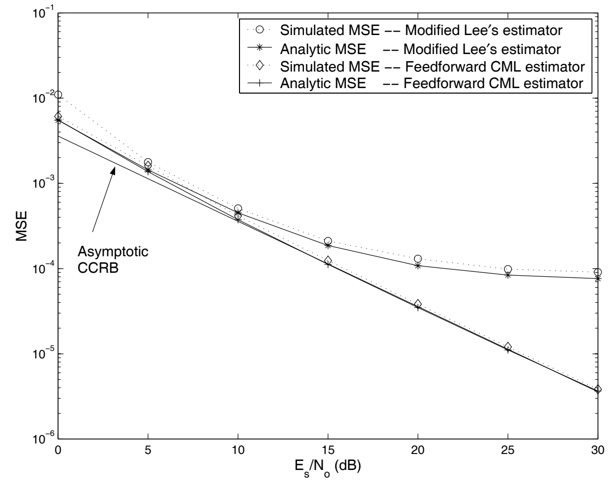

Fig. 1. Analytic and simulated MSEs for modified Lee's estimator and feedforward CML estimator.

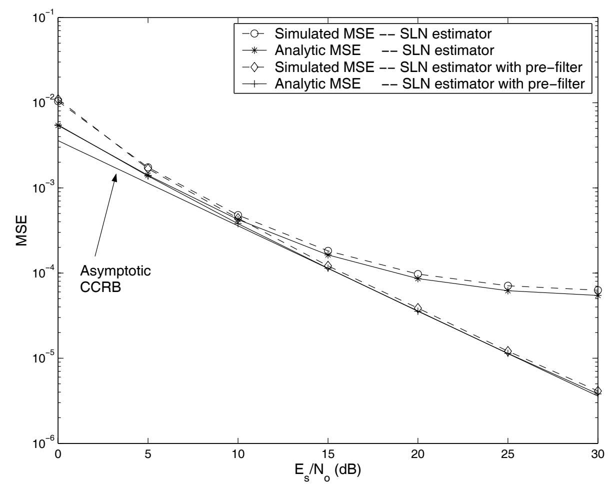

Fig. 2. Analytic and simulated MSEs for SLN estimator with and without pre-filter.

V. Conclusions

In this paper, all the previously proposed feedforward symbol timing estimators employing second-order statistics were formulated into a unified framework. The finite sample mean square error (MSE) expression and the asymptotic conditional Cramer-Rao bound (CCRB) for this class of estimators were established. It was found that the analytical and simulation results match very well. Furthermore, it was found that the feedforward CML estimator [10] and the SLN estimator with a properly designed pre-filter [5] perform the best and their performances coincide with the asymptotic CCRB.

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