THE ENERGY EFFICIENCY OF ON-OFF KEYING WITH PARTIAL CSI AND PEAK POWER CONSTRAINTS

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ABSTRACT

It is well known that On-Off Keying (OOK) suffers from a 3 dB SNR penalty relative to BPSK for coded transmission over a Rayleigh fading channel with perfect receiver channel state information (CSI). Previously, the cutoff rate has been used to show that this 3 dB penalty can be partially recovered if the OOK transmission probability can be varied, with full recovery in the limit of vanishing rates. We extend these results to the case of imperfect CSI. We show that the maximum penalty with partial CSI is 4 dB, and that there is a range of rates and CSI quality for which the penalty is instead a gain. By further varying the transmission probability of the '1' bit in OOK, we show that the 3 dB penalty can be recovered for small rates, but not at larger rates. Finally, we consider the energy penalty for using OOK under a peak power constraint.

1. INTRODUCTION

Energy-efficient signaling is a priority in most wireless communications systems. For a fixed communications objective and performance measure, it is desirable to transmit using as little energy as possible. For example, in a wireless military sensor network, transmitting large amounts of energy prematurely depletes sensor batteries, increases the probability of unwanted detection, increases node collisions due to the expanded communications range, and increases the complexity (and therefore, the size and weight) of the on-board power amplifier. Also due to energy constraints, many systems employ binary signalling, and further, are able to acquire only imperfect knowledge of the channel state at the receiver. Therefore, given binary signaling and imperfect receiver channel state information (CSI), we address the following: What is the energy-efficiency of equiprobable On-Off Keying (OOK) relative to antipodal signaling (BPSK)? Are significant gains realized if p, the probability of transmitting the '1' bit in OOK, is optimized? In the latter case, what is the penalty if a realistic peak power constraint is imposed (the peak to average power ratio is $1/p$)? We answer these questions over the entire (rate, CSI) region.

To study energy-efficiency, we will use the cutoff rate as a performance measure. The channel cutoff rate is a well-known information-theoretic metric [6] that has been frequently used to characterize the bit error rate, achievable information rates, and decoding complexity of coded transmission over wireless fading channels [2]. In this context, the cutoff rate has been studied under the assumptions of both no CSI and full CSI in [4] and [5]. Here, we seek to determine the energy required to attain a target cutoff rate R_o under partial CSI. We are motivated by [3], where it was shown that under perfect CSI, the well-known 3 dB penalty for using equiprobable OOK in place of BPSK [8], can be partially recovered if the OOK probability p can be varied, with full recovery in the limit of vanishing rates (this result also holds for the channel capacity, see, e.g., the discussion in [11]). However, when no CSI is available at the receiver, it is known that OOK is optimal ([1], [7]), which lends credence to a reexamination of this problem for partial CSI.

In Section 2 we describe the fading channel model and concisely provide our previous results that will be needed for the subsequent discussion. In Section 3 we derive the energy required to attain a target R_o for both OOK and BPSK. In Section 3.1, we consider equiprobable-OOK and show that the penalty is generally in the range $(-\infty, 4)$ dB with partial CSI (a negative penalty denotes a "gain"). Therefore, BPSK can be up to 4 dB more efficient than OOK under partial CSI, but on the other hand, there is also a range of (R_o, ω) over which OOK is more energy efficient. In Section 3.2, we asses the benefits of variable-probability OOK and show that, while the 3 dB penalty can be fully recovered at small rates, with an additional energy savings if the CSI quality exceeds a threshold, this penalty cannot be improved at large rates. Finally, we recognize that various regulations and hardware limitations will prevent the transmission of arbitrarily "peaky" inputs. Therefore, we consider performance under a peak power constraint in Section 4.

2. SYSTEM MODEL

2.1. Channel Model

We consider a single-user, time-varying fading channel with partial receiver CSI. The receiver's observation over a Rayleigh fading channel is given by

$$y_k = \sqrt{E}\, h_k s_k + n_k, \tag{1}$$

where k denotes discrete time, E is the average symbol energy, $h_k \sim \mathcal{CN}(0, \sigma_h^2)$ models i.i.d. fading[1], and $n_k \sim \mathcal{CN}(0, \sigma_N^2)$ models AWGN. The channel input $s_k \in \{A, -B\}$ is, without loss of generality, real valued, subject to the unit-energy constraint $p|A|^2 + (1-p)|B|^2 = 1$, where $0 \le p \le 1$ is the probability of transmitting A. Without loss of generality, we assume that $A \ge 1$ and $B \le 1$. We also assume that $\sigma_N^2 \ne 0$, $\sigma_h^2 \ne 0$.

During each symbol interval, the receiver has an estimate of the channel, $\widehat{h}_k$, and (1) can be rewritten as

$$y_k = \sqrt{E}\, \widehat{h}_k s_k + \sqrt{E}\, \widetilde{h}_k s_k + n_k,$$

[1]The notation $x \sim \mathcal{CN}(\theta, \sigma^2)$ denotes a complex Gaussian random variable x with mean θ and with independent real and imaginary parts, each having variance $\sigma^2/2$.

where $\widetilde{h}_k \triangleq h_k - \widehat{h}_k$ is the residual error in the channel estimate. We assume that both the estimate and the residual error are zero-mean Gaussian and independent, i.e., $\widehat{h}_k \sim \mathcal{CN}(0, \widehat{\sigma}^2)$, $\widetilde{h}_k \sim \mathcal{CN}(0, \widetilde{\sigma}^2)$, and $\widehat{\sigma}^2 + \widetilde{\sigma}^2 = \sigma_h^2$. MMSE estimation schemes exist that satisfy this assumption, and in [7], we describe one such pilot symbol assisted modulation (PSAM) scheme. However, this information is not needed for the present discussion. We assume that codewords are decoded using the ML-detector which treats s_k as the channel input and the pair $(y_k, \widehat{h}_k)$ as the channel output. Finally, we define the normalized variance of the channel estimate,

$$\omega \triangleq \widehat{\sigma}^2 / \sigma_h^2,$$

as the *CSI quality* available at the receiver. Note that $\omega = 0$ denotes no CSI, and $\omega = 1$ denotes perfect CSI at the receiver.

2.2. Previous Results on Cutoff Rate

For a fixed symbol set (p, A, B), the cutoff rate for the system with outputs $(y_k, \widehat{h}_k)$ and input s_k is given by [6]

$$R_o = -\log_2 \sum_{s_1, s_2 = \{A, -B\}} Q(s_1) Q(s_2) \int_y \sqrt{P(y, \widehat{h}|s_1) P(y, \widehat{h}|s_2)} dy, \tag{2}$$

where $Q(A) = p$, $Q(-B) = 1 - p$, and $P(y, \widehat{h}|s)$ is the p.d.f. of the channel outputs, conditioned upon the channel input.

We have previously evaluated (2), and shown that BPSK ($p = 1/2, A = -B = 1$) is cutoff rate-optimal for full receiver CSI. For arbitrary ω, the BPSK cutoff rate is [7]

$$R_o = -\log_2 \left\{ 1 - \frac{\omega}{2} \frac{\mathcal{K}}{\mathcal{K} + 1} \right\}, \tag{3}$$

where $\mathcal{K} \triangleq \sigma_h^2 E / \sigma_N^2$ is the received SNR. Similarly, we have shown that OOK ($p, A = \sqrt{1/p}, B = 0$), where p is chosen to maximize (2), is cutoff rate optimal for no receiver CSI. For arbitrary ω and p, the OOK cutoff rate was shown to be [7]

$$R_o = -\log_2 \left\{ 1 + 2p(1-p) \left[\frac{\sqrt{1 + \mathcal{K}(1-\omega)\frac{1}{p}}}{1 + \mathcal{K}(2-\omega)\frac{1}{4p}} - 1 \right] \right\}. \tag{4}$$

3. MINIMUM ENERGY FOR TARGET CUTOFF RATE

We seek to minimize the energy required to attain a target rate R_o. Let us define

$$\lambda \triangleq 1 - 2^{1 - R_o}, \tau \triangleq 1 - 2^{-R_o}, \text{ and } \mu \triangleq 1 - \tau / (2p(1-p)) .$$

From (4), we find the energy required for OOK(p) to be [2] [3]

$$\mathcal{K}_{\mathrm{OOK}(p)} = \begin{cases} \frac{2(1-\omega) - \mu^2(2-\omega) + \sqrt{4(1-\omega)^2 + \mu^2(2-\omega)(3\omega - 2)}}{(1/(4p))\mu^2(2-\omega)^2}, \\ \qquad \text{for } R_o < -\log_2 [1 - 2p(1-p)] \\ \infty, \quad \text{otherwise} \end{cases}. \tag{5}$$

[2]OOK(p) denotes the On-Off Keying input where the probability of transmitting '1' is p; i.e., the symbol set ($p, A = \sqrt{1/p}, B = 0$).

[3]Henceforth, we will indicate that a target cutoff rate is unattainable by stating that the required energy "equals" ∞.

Note that for $p < \frac{1}{2}$, some target cutoff rates cannot be achieved. Similarly, we find the energy required to achieve a target cutoff rate for BPSK from (3) to be

$$\mathcal{K}_{\mathrm{BPSK}} = \begin{cases} \frac{2\tau}{\omega - 2\tau}, & \text{for } R_o < -\log_2(1 - \omega/2) \\ \infty, & \text{for } R_o \geq -\log_2(1 - \omega/2) \end{cases}. \tag{6}$$

The CSI quality ω limits the rates attainable with BPSK.

3.1. BPSK and equiprobable OOK

When full receiver CSI is available, using OOK($1/2$) instead of BPSK incurs a 3 dB energy penalty for all R_o [8]. Here, we examine this penalty under partial CSI, and show that the maximum penalty is 4 dB. We also show that this penalty decreases for smaller values of ω and R_o, eventually becoming a "gain".

Define the energy penalty incurred for using BPSK in place of OOK($1/2$) to be

$$\gamma \triangleq \frac{\mathcal{K}_{\mathrm{BPSK}}}{\mathcal{K}_{\mathrm{OOK}(1/2)}}; \quad \gamma_{\mathrm{dB}} \triangleq 10 \log_{10} \gamma,$$

so that $\gamma_{\mathrm{dB}} < 0$ indicates a penalty for using OOK($1/2$) (therefore, in our notation, the well-known result states that $\gamma_{\mathrm{dB}} = -3$ for full receiver CSI, and for all R_o). Substitution yields

$$\gamma = \frac{(1 - \frac{\omega}{2})^2 (\omega - (1 + \lambda))^{-1} (1 + \lambda) \lambda^2}{-\lambda^2 (1 - \frac{\omega}{2}) + (1 - \omega) + \sqrt{\lambda^2 (-1 + \frac{\omega}{2})(1 - \frac{3}{2}\omega) + (1 - \omega)^2}},$$

for $R_o < -\log_2(1 - \omega/2)$, which we plot in Figure 1. The following remarks are in order:

R1. For small R_o, the 3 dB penalty for using OOK($1/2$) persists, even under partial CSI. It is easy to show that $\gamma_{\mathrm{dB}} \to -3$ as $R_o \to 0$, for *all* ω.

R2. For non-vanishing R_o however, the 3 dB penalty rule no longer holds. It is clear from the figure, that there exists a (R_o, ω) region where $\gamma_{\mathrm{dB}} \leq -3$. It can be shown that (proofs have been omitted due to space limitations):

(a) The maximum energy penalty occurs for some $\omega \in (0.8, 1)$, which is an increasing function of R_o,

(b) the penalty is greater than 3 dB if and only if

$$\omega \in \left(\frac{2}{3}, 1 \right) \text{ and } R_o \leq -\log_2 \frac{1 - \sqrt{-3 + 6\omega - 2\omega^2}}{2(1 - \omega)},$$

and

(c) the largest penalty occurs at ($R_o = 1^-$, $\omega = 1^-$), and has an infimum of $\gamma_{\mathrm{dB,inf}} = 10 \log_{10}(2/5) \approx -4$ dB. Therefore, *there is at most an additional* 1 *dB increase in the penalty due to imperfect receiver CSI.*

R3. Conversely, OOK($1/2$) may actually provide an energy "gain" for some values of R_o and ω. Setting $\gamma = 1$, we find the (R_o, ω) curve for which these two constellations are equally energy efficient to be given by the valid root of the third order polynomial,

$$\lambda^3 - 3\lambda^2 + 4 \left(\frac{1 - \omega}{\omega} \right)^2 \lambda + 4 \left(\frac{1 - \omega}{\omega} \right)^2 .$$

Based on the results in **R1**-**R3**, we can partition the (R_o, ω) plane as shown in Figure 2.

3.2. OOK with variable probability

From [3], we know that when full receiver CSI is available, the 3 dB penalty for using OOK($1/2$) (in place of BPSK) is partially recovered by using OOK(p^*) instead, where p^* is the energy-efficient transmission probability. It was shown that a full recovery is possible as $R_o \to 0$. However, in the last section, we showed that OOK($1/2$) may provide either penalty or a gain relative to BPSK, when only imperfect CSI is available. Therefore, we will not discuss how much we are able to "recover" by using OOK(p^*) in place of BPSK. Rather, we will discuss how much we are able to gain by using OOK(p^*) in place of OOK($1/2$).

The transmission probability p^* that minimizes the energy required to attain R_o, for a given ω, is given by

$$p^* = \arg \min_{0 \le p \le 1/2} \mathcal{K}_{\mathrm{OOK}(p)} \tag{7}$$

and, clearly, the resulting required energy is given by $\mathcal{K}_{\mathrm{OOK}(p^*)}$. In general, (7) does not yield a closed form expression (CFE) for p^*. However, it is easy to verify that as $R_o \to 0, p^* \to 0$ and that as $R_o \to 1, p^* \to 1/2$. Additionally, when full CSI is available ($\omega = 1$), (7) yields the CFE

$$p^* = \sqrt{\frac{1 - 2^{-R_o}}{2}}. \tag{8}$$

When no CSI is available, and R_o is small ($R_o \to 0$), (7) yields

$$p^* = \alpha(1-2^{-R_o}), \alpha \triangleq \left[\frac{7+\sqrt[3]{199-3\sqrt{33}}+\sqrt[3]{199+3\sqrt{33}}}{6}\right],$$

which implies that the transmission probability grows logarithmically in R_o.

Next, we define χ to be the energy penalty for using OOK($1/2$) in place of OOK(p^*),

$$\chi \triangleq \frac{\mathcal{K}_{\mathrm{OOK}(1/2)}}{\mathcal{K}_{\mathrm{OOK}(p^*)}}, \quad \chi_{\mathrm{dB}} \triangleq 10\log_{10}\chi\ .$$

Note that $\chi_{\mathrm{dB}} \ge 0$ since OOK(p^*) will always be at least as energy efficient as OOK($1/2$). We plot χ_{dB} in Figure 3 for small R_o, and make the following remarks:

R4. *For small rates ($R_o \to 0$).* It was seen in the previous section that the 3 dB OOK($1/2$) penalty persists for all values of ω. Note that with OOK(p^*) there is a 3 dB gain for (approximately) $\omega \in (0.4, 1)$. Therefore, the well-known 3 dB penalty is recovered with OOK(p^*), not just for full CSI as shown in [3], but also for moderate to large values of CSI. In addition, for $\omega \in (0.0, 0.4)$, OOK(p^*) "more than recovers" the 3 dB penalty suffered by OOK($1/2$). That is, OOK(p^*) is *more energy-efficient than BPSK*. As $\omega \to 0$, the energy savings grows arbitrarily large.

R5. *For large rates ($R_o \to 1$).* As $R_o \to 1$, the gain of OOK(p^*) relative to OOK($1/2$) approaches 0 dB for all ω. This is because as $R_o \to 1$, $p^* \to 1/2$ (see the discussion after Equation (7)), making the two inputs equivalent. In particular, the maximum penalty for using OOK($1/2$) in place of BPSK was seen to be 4 dB, which occurs as ($R_o \to 1, \omega \to 1$). Therefore, *this 4 dB penalty persists, even when OOK(p^*) is used in place of OOK($1/2$).*

4. PEAK-POWER CONSIDERATIONS

In the previous sections, we have seen that the flexibility of the OOK(p) symbol set results in significant energy savings at low target rates R_o and for smaller values of the CSI quality ω. However, using a small value of p increases the peak transmission power and lowers the system duty cycle, which adversely affects the performance of practical radios [9]. Therefore, we now investigate the SNR penalty incurred when a peak-to-average power ratio (PAPR) constraint is imposed. The PAPR for OOK(p) is $1/p$, and the resulting PAPR constraint $1/p \le \delta$ is therefore in the range

$$2 \le \delta < \infty.$$

The most strict constraint of $\delta = 2$ forces OOK($1/2$), while the most relaxed condition of $\delta \to \infty$ corresponds to "no" peak power constraint, allowing for use of OOK(p^*). For a general δ, the energy required to attain a target R_o using the peak power-constrained optimal transmission probability p^*_δ is $\mathcal{K}_{\mathrm{OOK}(p^*_\delta)}$, where

$$p^*_\delta = \arg \min_{\frac{1}{\delta} \le p \le \frac{1}{2}} \mathcal{K}_{\mathrm{OOK}(p)} = \max\left(\frac{1}{\delta}, p^*\right). \tag{9}$$

The last equality in (9) requires that $\mathcal{K}_{\mathrm{OOK}(p)}$ be a monotonic increasing function of the argument $|p - p^*|$, i.e., it should have only one critical point, a minima, for $p \in (0, 1/2)$. We have shown this for the ($\omega = 1$, any R_o) and ($\omega = 0, R_o$ small) cases, and will *assume* this form in general for numerical evaluation. The analytic results below *do not* require this assumption.

Define ψ_δ to be the (non-negative) energy penalty for using OOK(p^*_δ) in place of OOK(p^*),

$$\psi_\delta = \frac{\mathcal{K}_{\mathrm{OOK}(p^*_\delta)}}{\mathcal{K}_{\mathrm{OOK}(p^*)}}, \quad \psi_{\delta,\,\mathrm{dB}} \triangleq 10\log_{10}\psi_\delta, \tag{10}$$

The following remarks are in order:

R6. *For fixed ω.* The penalty $\psi_{\delta,\mathrm{dB}}$ depends on δ, R_o and ω. It is generally decreasing in δ and R_o, and vanishes ($\psi_{\delta,\mathrm{dB}} \to 0$) as $R_o \to 1$ or as $\delta \to \infty$.

R7. *For no CSI ($\omega = 0$).* In Figure 4, we plot $\psi_{\delta,\,\mathrm{dB}}$, for several values of δ. It can be shown that $\psi_{\delta,\,\mathrm{dB}}$ becomes unbounded as $R_o \to 0$.

R8. *For full CSI ($\omega = 1$).* The penalty function is found by substituting (8) and (9) into (10), yielding

$$\psi_\delta = \begin{cases} 2\frac{\sqrt{2-2^{1-R_o}}-1}{(\delta-2)+2/\delta-\delta 2^{-R_o}}, \\ \qquad\qquad \text{for } R_o < -\log_2\left(1-2/\delta^2\right) \\ 1, \text{ for } R_o \ge -\log_2\left(1-2/\delta^2\right) \end{cases}.$$

The maximum energy penalty is bounded by 3 dB, with equality when both $\delta = 2$ and $R_o \to 0$ are true.

C1. For a fixed peak power constraint $\delta = \delta_0$, the penalty is upper bounded by the $R_o \to 0$ case yielding

$$\psi_{\delta_o} \le \lim_{R_o \to 0} \psi_{\delta_o} = \frac{\delta_0}{\delta_0 - 1}.$$

C2. Similarly, for a fixed R_o, the penalty is upper bounded by the $\delta = 2$ case yielding:

$$\psi_\delta(R_o) \le \psi_2(R_o) = \frac{\sqrt{2-2^{1-R_o}}-1}{1/2 - 2^{-R_o}},$$

where $\psi_\delta(R_o)$ indicates the dependency of ψ_δ on R_o.

5. DISCUSSION

We have examined the well-known 3 dB penalty that exists when using equiprobable On-Off Keying in place of BPSK for communications when only imperfect receiver CSI is available. We have shown that this penalty now occupies a range $(-\infty, 4)$ dB depending on the target rate, and the available CSI quality. By varying the probability of the '1' bit in OOK, we find that full energy recovery is possible for small rates, and that additional energy savings are obtained if the CSI quality exceeds a threshold, but that no improvement is possible at large target rates. We have also quantified the energy penalty imposed by a peak power constraint.

It is also of interest to study energy efficiency from other perspectives, e.g., subject to a bandwidth utilization criterion (spectral efficiency), or in terms of the attainable information rate per unit of energy expended. These themes have been covered in [10] and [11] from the perspective of the channel capacity, and it would be of interest to see if additional insights can be drawn from the cutoff rate metric when the CSI quality is variable.

6. REFERENCES

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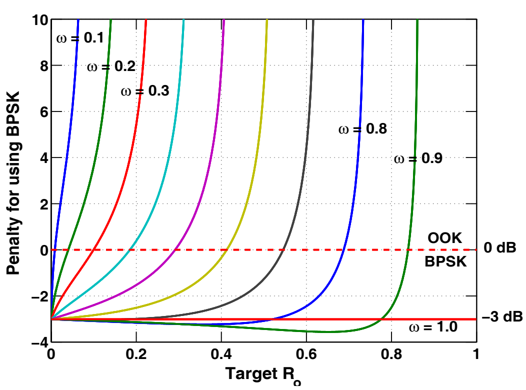

Fig. 1. Energy penalty γ_{dB} vs. target cutoff rate R_o for several values of CSI quality ω. A negative value of γ_{dB} indicates that BPSK is more energy efficient than OOK$(1/2)$.

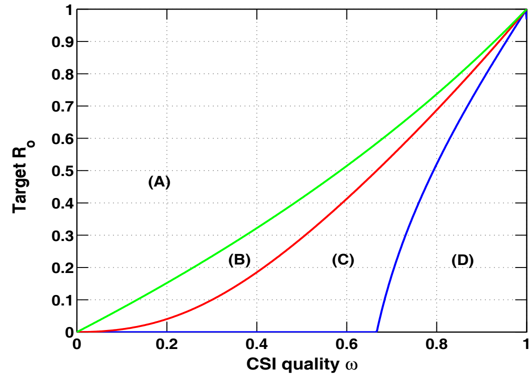

Fig. 2. A partitioning of the (R_o, ω) plane into regions where the energy penalty for using BPSK in place of OOK$(1/2)$ is: (A) "∞" dB, (B) between 0 dB and 3 dB, (C) between -3 dB and 0 dB, and (D) less than -3 dB. Note that the maximum penalty is ≈ 4 dB, and occurs for $R_o = 1^-$ and $\omega = 1^-$.

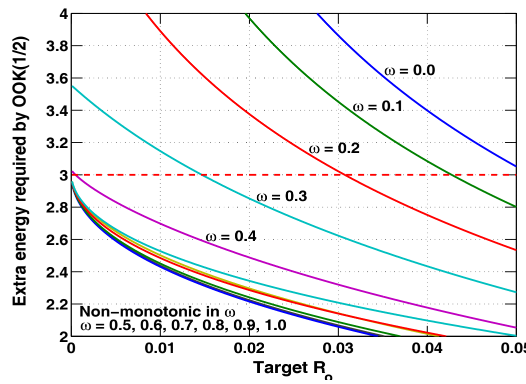

Fig. 3. A plot of χ_{dB}, the energy-penalty for using OOK$(1/2)$ in place of OOK(p^*), for various values of the target cutoff rate R_o (R_o small) and CSI quality ω. For larger values of ω, χ_{dB} is seen to be non-monotonic in ω.

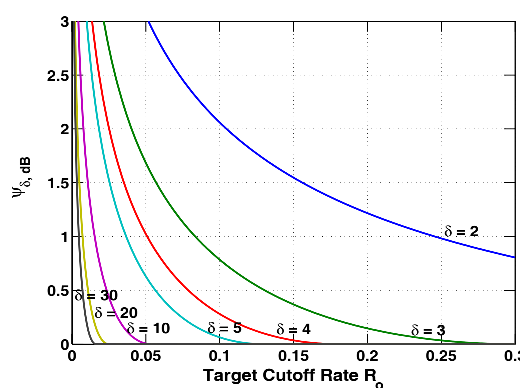

Fig. 4. A plot of $\psi_{\delta,\,\text{dB}}$, the energy-penalty incurred for PAPR constraints of $\delta = \{2, 3, 4, 5, 10, 20, 30\}$, and for no CSI.