# KALMAN FILTER OF CHANNEL MODES IN TIME-VARYING WIRELESS SYSTEMS

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## ABSTRACT

In mobile communications the movement of the users makes the propagation channel to be time-varying. Algorithms that track channel variations have to trade between complexity and accuracy. Since the second-order statistic of time-varying channels is stationary, estimation of the channel can be reduced to track a set of r uncorrelated parameters. Based on this decomposition, in this paper we propose to simplify the optimum Kalman filter (KF) by tracking the r channel modes separately and by using the steady-state solution of the KF gain.

## 1. INTRODUCTION

In wireless communication systems the received signal in the discrete-time model is compactly described as the linear system

$$y_t = \mathbf{x}_t^H \mathbf{h}_t + n_t, \tag{1}$$

at time t vector  $\mathbf{h}_t = [h_t(0) \cdots h_t(W-1)]^T \in \mathbb{C}^{W \times 1}$  denotes the time-varying channel, the regressor  $\mathbf{x}_t = [x_t^* \ x_{t-1}^* \cdots x_{t-W+1}^*]^T \in \mathbb{C}^{W \times 1}$  contains transmitted symbols and  $n_t$  is the white Gaussian noise with  $E[|n_t|^2] = \sigma_n^2$ . The optimal estimate for the data sequence  $\{x_t\}$  from the received signals  $\{y_t\}$  is the Maximum Likelihood Sequence Estimate (MLSE) routinely implemented by the Viterbi Algorithm (VA). Since  $h_t$  is time-varying and performances are very demanding, it is required to exploit the benefits of the MLSE and optimal channel tracking. Joint MLSE and tracking is known to be infeasible in real systems as its computational complexity grows with the channel coherence time [1]. A practical solution consists in the Per Survivor Processing (PSP) [2], that embeds data aided channel estimation techniques into the VA. At each trellis step an independent channel tracking is accomplished for each survivor sequence. This permits to compute the metric and update the survivors. With respect to VA, the PSP adds the complexity due to the large number of channel estimators that have to be carried out simultaneously. The KF is known to have optimal channel tracking capabilities but it is infeasible to be implemented in PSP. Therefore common goal is to implement simplified algorithms still preserving the KF performance.

The dominating effort of the KF is the recursive update of a time-varying gain vector. A constant gain is obtained in [3] by introducing the assumption of uncorrelated channel taps and by forcing the error correlation matrix to admit a steady-state stationary solution. In [4] tracking is decomposed into a set of KFs on memoryless channel through an approximate prediction-feedback mechanism. The Least Mean Square (LMS) algorithm replaces the KF gain by the product between the regression term and a scalar gain  $\mu$ . In the (Simplified) Wiener LMS (WLMS) [5][6] the scalar

gain  $\mu$  is replaced by a linear filter optimized in the Minimum Mean Square Error (MMSE) sense.

In this paper we propose to simplify the KF by taking advantage of the modal structure of the channel vector  $\mathbf{h}_t$  as in [7]. We observe that in multipath propagation environments  $\mathbf{h}_t$  is the superposition of a large number of paths, that can be clustered into few (say  $r \leq W$ ) compound paths that are temporally resolvable. Each compound path is characterized by a stationary delay (at least for the time-scale of the transmission considered here) and an amplitude that accounts for the time-varving fading (here assumed to be Rayleigh distributed). In this case the channel vector is  $\mathbf{h}_t \sim \mathcal{CN}(0, \mathbf{R}_h)$  and it has rank-deficient covariance  $\mathbf{R}_h$  with  $r = \operatorname{rank}[\mathbf{R}_h] < W$ . Accordingly,  $\mathbf{h}_t$  can be decomposed into the stationary modes of  $\mathbf{R}_h$  and the corresponding uncorrelated time-varying modal amplitudes. Using this property, the channel estimation can be restricted to track this reduced set of uncorrelated modal amplitudes. The uncorrelation is not enough to simplify the KF, but it motivates us to design a novel ad hoc algorithm. A system model reparametrization, based on some reasonable hypothesis on the transmitted signal, permits to exploit the channel modes independence and decomposes the tracking to a set of parallel filters. The resulting algorithm, herein referred to as Simplified Kalman Filter (SKF), outperforms the other sub-optimal techniques since it applies the exact KF recursion. The complexity saving, in the order of LMS algorithm, is only due to the signal reparametrization and the statistical properties of the modal amplitudes within the tracking.

The outline of the paper is as follows. In Sect. 2 the modal structure of the channel is explained. In Sect. 3 and 4 we review the KF and derive the SKF. The simulation results are shown in Sect. 5. Finally Sect. 6 provides our conclusions.

## 2. MODAL STRUCTURE FOR THE MULTIPATH CHANNEL VECTOR

According to the WSSUS multipath model the channel variability is modelled as  $\mathbf{R}_h(k) = \mathrm{E}[\mathbf{h}_t \mathbf{h}_{t+k}^H] = \mathbf{R}_h \rho(k)$  (see [7] for details). The temporal autocorrelation  $\rho(k)$  of each fading amplitude depends on the Doppler frequency  $(f_D)$  normalized to the system bandwidth  $(f_c)$  according to the Jakes function  $\rho(k) = J_o(2\pi k f_D/f_c)$  (Clarke model [8]).

Under the rank-*r* assumption for the matrix  $\mathbf{R}_h$ , the latter can be equivalently rewritten as  $\mathbf{R}_h = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H$ , where the  $W \times r$ matrix  $\mathbf{U}$  and the  $r \times r$  diagonal matrix  $\mathbf{\Lambda}$  collect, respectively, the eigenvectors and eigenvalues of  $\mathbf{R}_h$ . The channel can be reparametrized accordingly in term of channel modes  $\mathbf{U}$  as

$$\mathbf{h}_t = \mathbf{U}\mathbf{b}_t,\tag{2}$$

where the modal amplitudes are  $\mathbf{b}_t \sim \mathcal{CN}(0, \mathbf{\Lambda})$ . From (2) it follows that the *r* modal amplitudes  $\mathbf{b}_t$  have autocorrelation matrix  $\mathbf{R}_b(k) = E[\mathbf{b}_t \mathbf{b}_{t+k}^H] = \mathbf{\Lambda}\rho(k)$ , they are therefore uncorrelated and they preserve the same temporal correlation function  $(\rho(k))$ of the *W* channel parameters  $\mathbf{h}_t$ . Notice that since the delays are constant over a large time scale, the basis **U** can be consistently estimated from a sample covariance matrix  $\hat{\mathbf{R}}_h$  calculated from the channel estimates over several training sequences, see [7] for details. In this paper we assume that the time-scale is large enough to consider that **U** and *r* can be handled as known. In signal model (1) the basis **U** can be grouped into the known regressor  $\mathbf{c}_t^H = \mathbf{x}_t^H \mathbf{U}$  so that

$$y_t = \mathbf{c}_t^H \mathbf{b}_t + n_t. \tag{3}$$

The transmitted symbols are assumed to be independent and identically distributed with  $E[|x_t|^2] = \sigma_x^2$ , the entries of  $\mathbf{c}_t^H$  are thus uncorrelated and  $\mathbf{R}_c = E[\mathbf{c}_t \mathbf{c}_t^H] = \sigma_x^2 \mathbf{I}_r$ . We further assume that the channel vector is normalized so that  $E[|\mathbf{h}_t|^2] = tr[\mathbf{R}_h] = 1$ , the signal-to-noise ratio is defined as  $SNR = \sigma_x^2/\sigma_n^2$ .

#### 3. KALMAN FILTER (KF)

When variations of the received signal can be modelled by a dynamic state model, the KF is known to be the linear MMSE estimator that approaches the lower bound on channel estimation if the Gaussian assumptions hold true. In the following we introduce some assumptions on the temporal variations of the channel parameters and we briefly recall the KF algorithm for the channel estimation from (3). A simplified version of the KF will be then discussed in the following section.

For Kalman filtering the Clarke model of fading can be conveniently approximated as autoregressive (AR) models of finite order [5]. Let the time-varying channel be modelled as first-order AR (extension to higher orders is straightforward)

$$\mathbf{b}_t = \alpha \mathbf{b}_{t-1} + \mathbf{w}_t,\tag{4}$$

 $\alpha \in R$  and the driving noise  $\mathbf{w}_t \sim \mathcal{CN}(0, \mathbf{R}_w)$  is white. The covariance  $\mathbf{R}_w$  is diagonal and collects a scaled amplitudes power profile ( $\mathbf{\Lambda}$ ). In KF the a-priori ( $\hat{\mathbf{b}}_{t|t-1}$ ) and a-posteriori ( $\hat{\mathbf{b}}_{t|t}$ ) estimates of the modal amplitudes are obtained as:

$$\widehat{\mathbf{b}}_{t|t-1} = \alpha \widehat{\mathbf{b}}_{t-1|t-1}, \tag{5}$$

$$\widehat{\mathbf{b}}_{t|t} = \widehat{\mathbf{b}}_{t|t-1} + \mathbf{k}_t (y_t - \mathbf{c}_t^H \widehat{\mathbf{b}}_{t|t-1}), \qquad (6)$$

where the time-varying KF gain vector is

$$\mathbf{k}_{t} = \frac{\mathbf{P}_{t|t-1}\mathbf{c}_{t}}{\sigma_{n}^{2} + \mathbf{c}_{t}^{H}\mathbf{P}_{t|t-1}\mathbf{c}_{t}},$$
(7)

for  $\mathbf{P}_{t-1|t-1} = \alpha^2 (\mathbf{I} - \mathbf{k}_{t-1} \mathbf{c}_{t-1}^H) \mathbf{P}_{t-1|t-2} + \mathbf{R}_w$ . Once defined the error for the a-priori estimate  $\mathbf{\tilde{b}}_{t|t-1} = \mathbf{b}_t - \mathbf{\hat{b}}_{t|t-1}$ , it is  $\mathbf{P}_{t|t-1} = \mathbf{E}[\mathbf{\tilde{b}}_{t|t-1}\mathbf{\tilde{b}}_{t|t-1}^H]$ . Basically the KF updates the channel estimate (6) on the basis of the prediction error  $\varepsilon_{t|t-1} = y_t - \mathbf{c}_t^H \mathbf{\hat{b}}_{t|t-1}$  and the gain  $\mathbf{K}_t$ .

## 4. SIMPLIFIED KALMAN FILTER (SKF)

Although the dynamic state model (4) is decoupled, the KF estimate errors are still correlated (i.e.,  $\mathbf{P}_{t|t-1}$  is not diagonal) making the gain updating (7) computationally expensive. We propose here

to reduce the tracking (6) to a set of parallel equations and avoid the gain computation (7) by making  $\mathbf{K}_t$  time-unvarying.

Let the received signal  $y_t$  be transformed by the regressor  $\mathbf{c}_t$ as  $\mathbf{c}_t y_t$ , since each realization  $\mathbf{c}_t \mathbf{c}_t^H$  can be decomposed in the covariance matrix  $\mathbf{R}_c$  and the zero mean deviation  $\mathbf{Z}_t = \mathbf{c}_t \mathbf{c}_t^H - \mathbf{R}_c$ , the observation (3) can be equivalently written as

$$\mathbf{c}_t y_t = \mathbf{R}_c \mathbf{b}_t + \mathbf{Z}_t \mathbf{b}_t + \mathbf{c}_t n_t.$$
(8)

By adding and subtracting  $\mathbf{Z}_t \widehat{\mathbf{b}}_{t|t-1}$ , it is

$$\mathbf{g}_t = \mathbf{c}_t y_t - \mathbf{Z}_t \widehat{\mathbf{b}}_{t|t-1} = \mathbf{R}_c \mathbf{b}_t + \mathbf{Z}_t (\mathbf{b}_t - \widehat{\mathbf{b}}_{t|t-1}) + \mathbf{c}_t n_t.$$
(9)

It is worth noticing that the a-priori estimate  $\hat{\mathbf{b}}_{t|t-1}$  can be regarded as deterministic at *t*-th step and it does not affect the signal statistic. The term  $\mathbf{g}_t$  in (9) consists in the scaled modal amplitudes ( $\mathbf{R}_c \mathbf{b}_t = \sigma_x^2 \mathbf{b}_t$ ) and a noise term  $\boldsymbol{\eta}_t = \mathbf{Z}_t(\tilde{\mathbf{b}}_{t|t-1}) + \mathbf{c}_t n_t$  (also referred as gradient noise in [5]).  $\boldsymbol{\eta}_t$  is composed of  $\mathbf{c}_t n_t$  due to the AWGN and the term  $\mathbf{Z}_t \tilde{\mathbf{b}}_{t|t-1}$  (or feedback noise [6]) that depends on the a-priori error  $\tilde{\mathbf{b}}_{t|t-1}$ . When comparing (9) with (3), it is quite clear that the regressor is made stationary ( $\mathbf{R}_c$  instead of  $\mathbf{c}_t$ ) to the detriment of the noise ( $\boldsymbol{\eta}_t$  instead of  $n_t$ ).

Similarly to methods that derive analytical results of the tracking performance [5][9], here the gradient noise  $\eta_t$  can be easily analyzed when assuming as uncorrelated all the consecutive regression vectors:  $E[\mathbf{c}_t \mathbf{c}_{t+k}^H] = 0 \ \forall k \neq 0$ . Accordingly, entries of matrix  $\mathbf{Z}_t$  depend only on transmitted signals at time t whereas the a-priori estimate error  $\mathbf{\tilde{b}}_{t|t-1}$  is related only to the data up to time t-1. Therefore  $\mathbf{Z}_t$  is uncorrelated to  $\mathbf{\tilde{b}}_{t|t-1}$  so that  $\eta_t$  can be proved to be zero mean, white and uncorrelated with the parameters  $\mathbf{b}_{\tau} \ \forall \tau$ . The covariance matrix  $\mathbf{R}_{\eta} = E[\eta_t \eta_t^H]$  is the sum of the AWGN and the feedback noise contributions:

$$\mathbf{R}_{\eta} = \sigma_x^2 \sigma_n^2 \mathbf{I}_r + \mathbf{E}[\mathbf{Z}_t \widetilde{\mathbf{b}}_{t|t-1} \widetilde{\mathbf{b}}_{t|t-1}^H \mathbf{Z}_t^H].$$
(10)

When evaluating the expectations over  $\mathbf{\tilde{b}}_{t|t-1}$  and  $\mathbf{Z}_t$  separately, the second term in (10) is  $\mathbf{E}[\mathbf{Z}_t\mathbf{P}_{t|t-1}\mathbf{Z}_t^H] = \mathbf{E}[\mathbf{c}_t\mathbf{c}_t^H\mathbf{P}_{t|t-1}\mathbf{c}_t\mathbf{c}_t^H] - \mathbf{R}_c\mathbf{P}_{t|t-1}\mathbf{R}_c$  and involves fourth order moments of the regressors. For independent and circular Gaussian distributed entries of  $\mathbf{c}_t$ , it is [5]:

$$\mathbb{E}[\mathbf{c}_t \mathbf{c}_t^H \mathbf{P}_{t|t-1} \mathbf{c}_t \mathbf{c}_t^H] = \sigma_x^4 [tr(\mathbf{P}_{t|t-1}) \mathbf{I}_r + \mathbf{P}_{t|t-1}].$$
(11)

The Gaussian assumption is here a practical solution to avoid the exact computation of the symbol probability density function, that depends on the basis U (a negligible improvement in performance can be achieved when using the exact probability density function). Covariance (10) reduces to

$$\mathbf{R}_{\eta} = (\sigma_x^2 \sigma_n^2 + tr(\mathbf{P}_{t|t-1}) \sigma_x^4) \mathbf{I}_r.$$
 (12)

The basic idea of the proposed simplified KF is to apply the KF recursion to the signal  $\mathbf{g}_t$  in (9) so that the modal amplitudes estimate is updated by the prediction error here obtained by comparing the modified observation  $\mathbf{c}_t y_t - \mathbf{Z}_t \hat{\mathbf{b}}_{t|t-1}$  with the desired response  $\mathbf{R}_c \hat{\mathbf{b}}_{t|t-1}$ , or equivalently as

$$\widehat{\mathbf{b}}_{t|t} = \widehat{\mathbf{b}}_{t|t-1} + \mathbf{K}_t \mathbf{c}_t (y_t - \mathbf{c}_t^H \widehat{\mathbf{b}}_{t|t-1}),$$

Algorithm	AR-1	AR-2
LMS	2r + 1	2r + 1
WLMS	3r + 1	5r + 1
SKF/KFL	4r	6r
KF	$\mathcal{O}(r^3)$	$\mathcal{O}(r^3)$

 
 Table 1. Complexity comparison in terms of complex multiplications for each step.

where the gain  $\mathbf{K}_t$  is here an  $r \times r$  matrix. The remainder equations are easily obtained by applying the KF based on model (4) and (9):

$$\widehat{\mathbf{b}}_{t|t-1} = \alpha \widehat{\mathbf{b}}_{t-1|t-1}, \tag{13}$$

$$\mathbf{K}_t = \mathbf{P}_{t|t-1} \mathbf{R}_c (\mathbf{R}_{\eta} + \mathbf{R}_c \mathbf{P}_{t|t-1} \mathbf{R}_c)^{-1}, \quad (14)$$

$$\mathbf{P}_{t|t-1} = \alpha^2 (\mathbf{I} - \mathbf{K}_{t-1} \mathbf{R}_c) \mathbf{P}_{t-1|t-2} + \mathbf{R}_w, \quad (15)$$

Recalling that  $\mathbf{R}_c = \sigma_x^2 \mathbf{I}_r$  and  $\mathbf{R}_w$  is diagonal, the matrices  $\mathbf{K}_t$ and  $\mathbf{P}_{t|t-1}$  in (14) and (15) preserve a diagonal structure during the iterations to yield to a set of *r* decoupled KFs except for the gain computation (14). Nevertheless it can be shown that the timeinvariance of the regression term  $\mathbf{R}_c$  lets  $\mathbf{K}_t$  converge to a stationary steady-state solution

$$\lim \mathbf{K}_t = \overline{\mathbf{K}}$$
.

This can be exploited to fully decouple the KFs and reduce the complexity by pre-evaluating  $\overline{\mathbf{K}}$  from (14) and (15). This yields to the following constant gain algorithm here referred to as SKF

$$\widehat{\mathbf{b}}_{t|t-1} = \alpha \widehat{\mathbf{b}}_{t-1|t-1}, \tag{16}$$

$$\widehat{\mathbf{b}}_{t|t} = \widehat{\mathbf{b}}_{t|t-1} + \overline{\mathbf{K}} \cdot \mathbf{c}_t \varepsilon_{t|t-1}.$$
(17)

The SKF structure recalls the method presented in [3] (here referred as KFL), where a constant gain  $\overline{\mathbf{K}}$  is obtained by forcing the KF recursion to admit a stationary solution and by isolating AWGN and coupling terms of the gain computation into a factor left as a free parameter. A practical solution to get decoupled equations accounts only for the AWGN. Thus, when compared with SKF, it follows that the KFL gain neglects the feedback noise term in  $\mathbf{R}_{\eta}$ . Such approximation is acceptable only for low SNR  $(tr(\mathbf{P}_{t|t-1}) \ll \sigma_n^2/\sigma_x^2)$ .

The WLMS algorithm [5] proposes a scalar gain  $\mu/\sigma_x^2$  instead of  $\overline{\mathbf{K}}$  in (17). Although both the techniques are optimized in MMSE sense, SKF outperforms WLMS since the former adapts the gain to the amplitudes power profile while the latter is constrained to use the same scalar gain  $(\mu/\sigma_x^2)$  for all the parameters.

#### 5. PERFORMANCE EVALUATION

In this section the complexity and the performance of the SKF are evaluated and compared with the other methods. The LMS is here designed with  $\mu = \frac{1}{r}$ , the other tracking techniques are optimized by approximating the Clarke fading by AR models [5], where the parameters are selected according to the normalized Doppler frequency  $f_D/f_c$  (here assumed as known). Table 1 shows the computational cost (complex multiplication for step) of the LMS, WLMS, SKF, KFL and KF for first (AR-1) and second (AR-2) order AR channel models. Cost of the KF is prohibitive for practical systems, thus motivating this research. Sub-optimal techniques reduces the complexity to O(r), the SKF has the same cost of KFL



Fig. 1. Steady-state MSE versus SNR. Simulation for LMS, KFL, WLMS, SKF and KF tracking over Clarke model channel (W = 8, P = 5,  $\tau = [1 \ 2.1 \ 2.4 \ 4.1 \ 4.5]T$ ,  $\sigma_i^2 = \sigma^2 2^{-i}$ ).

since they share the same structure. In the following simulation we have considered AR-2 channel modelling to better approximate the Clarke model  $\rho(k)$ .

We consider a channel length W = 8 as composed by P = 5resolved Rayleigh-fading paths (i.e., r = 5) having delays  $\tau = [1$  $2.1 \; 2.4 \; 4.1 \; 4.5]T \; (T \text{ is the symbol time)}$  and exponential power delay profile  $\sigma_i^2 = \sigma^2 2^{-i}$  ( $\sigma^2$  is scaled to have  $E[|\mathbf{h}_t|^2] = 1$ ). The transmitted pulse is a raised cosine with roll-off 0.4. The maximum Doppler frequency is  $f_D = 500 Hz$  and the system bandwidth  $f_c = 200 \ KHz$ . The performance, in terms of the steadystate  $MSE = \lim_{t\to\infty} E[|\widetilde{\mathbf{b}}_{t|t}|^2]$ , is plotted versus the SNR in Fig. 1. It can be easily noticed that the SKF performance attains closely that of the KF over the whole SNR range. Compared to the LMS, the SKF provides a large performance improvement. At low SNR the LMS estimate is embedded in noise, whereas at high SNR the MSE slope presents a floor due to the lack of a-priori information on the dynamic model. The performance degradation is particularly evident for large number of paths and large Doppler frequency, thus making the LMS inadequate in tracking multipath fast-varving channels. Despite of the more efficient structure, even the KFL experiences an MSE floor. In this case the approximation done by neglecting the feedback noise covariance in the gain computation is acceptable at low SNR, but affects considerably the performance for  $SNR \ge 20 dB$ . The SKF yields also a constant gain of approx.  $2 \div 3dB$  in SNR as compared with the WLMS. As explained in Sect 4, the disadvantage of the WLMS is due to the constraint to update all the modal amplitudes by the same scalar gain regardless of their power profile. As a consequence, the difference between the SKF and the WLMS grows in a dense multipath environment (r large), when the fading amplitudes present different orders of magnitude, and vanishes for single path channel (r = 1).

In the simulation displayed in Fig. 2 the channel is the superposition of P = 3 paths with W = 5,  $\tau = [1 \ 1.8 \ 2.1]$ , exponential power delay profile,  $f_D = 700 \ Hz$  and  $f_c = 200 \ KHz$ . The SKF



Fig. 2. Steady-state MSE versus SNR. Simulations for LMS, KFL, WLMS, SKF and KF tracking over Clarke model channel (W = 5, P = 3,  $\tau = [1 \ 1.8 \ 2.1]T$ ,  $\sigma_i^2 = \sigma^2 2^{-i}$ ).

is still tight to the KF. The lower number of resolved paths (r = 3) reduces the performance loss of the WLMS (as the power profile is less dispersive) and of the KFL (as the contribution of the feedback noise decreases).

Finally we analyze the data detection performance for channel tracking used in conjuction with a PSP receiver. Here the VA metric at the *t*-th step is evaluated by using the a-priori amplitude estimate  $\mathbf{b}_{t|t-1}$  provided by the algorithm at the previous step [3]. The transmitted packets are composed by 150 symbols (300 bits over 4-PSK modulation) followed by W - 1 known tail symbols. The channel is the same as in the previous example and the CIR is assumed known at the beginning of each packet (optimal training). The SKF and the KFL precompute the optimum gain  $\overline{\mathbf{K}}$ , the KF initializes the error covariance matrix  $\mathbf{P}_{t|t-1}$  with the final value of the previous packet in order to avoid the initial transient. Fig. 3 shows the bit error rate (BER) versus the SNR for the LMS, KFL, WLMS, SKF, KF tracking and using known channel in VA. The simulation results reveal that the SKF is well suited to be embedded in the PSP receiver and provides performance comparable to the KF. On the contrary the LMS is degraded by the absence of the prediction  $\hat{\mathbf{b}}_{t|t-1}$  so it has to approximate the metric by using the outdated estimate  $\hat{\mathbf{b}}_{t-1|t-1}$ . This leads to a performance degradation in particular for fast-varying channel. At SNR = 30dB, the SKF PSP provides a gain in SNR of approx.  $1 \div 2dB$  with respect to KFL PSP, 4dB to WLMS PSP and  $7 \div 8dB$  to LMS PSP.

#### 6. CONCLUSIONS

The proposed SKF exploits a modal channel decomposition and a proper signal reparametrization to obtain an efficient KF implementation. The method permits to decompose the channel estimation problem into a set of parallel filters with constant gains, these can be precomputed once given the dynamic models (and the SNR). Analysis of the MSE over fast-varying channel shows that the SKF introduce negligible degradation with respect to the



**Fig. 3**. BER versus SNR for PSP receiver. Simulation for LMS, KFL, WLMS, SKF, KF tracking and known channel. Channel as in Fig. 2.

optimum KF and outperforms the other reduced-complexity techniques. The estimation efficiency and the complexity reduction make the SKF suited to be routinely used within the PSP receiver.

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