EFFICIENT OFDM SYMBOL ESTIMATION ALGORITHM OVER TIME-FREQUENCY-SELECTIVE FADING CHANNELS

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ABSTRACT

In orthogonal frequency division multiplexing (OFDM) systems, time-frequency-selective, equivalently time-variant multipath, fading destroys subcarrier orthogonality, resulting in intercarrier interference (ICI). OFDM systems with guard interval (GI) can prevent intersymbol interference (ISI) caused by frequency-selective fading, but do not combat ICI caused by time-selective fading. To mitigate the effects of ICI, several estimation methods have been proposed. These conventional algorithms are highly complex in general due to the large-sized matrix inversion. In this paper, we propose an efficient OFDM symbol estimation scheme, called iterative sequential neighbor search (ISNS) algorithm, which achieves an enhanced performance with a moderate complexity compared with the conventional methods. The validity of the proposed estimation scheme is demonstrated by computer simulations.

1. INTRODUCTION

OFDM is one of the most promising schemes due to its ability to combat the frequency-selectivity of the channel and to achieve a high spectral efficiency. In OFDM system, the transmission bandwidth is decomposed into many narrow orthogonal subchannels so that each subchannel can be regarded as a flat fading. A guard-time interval (GI) maintains the orthogonality between subcarriers against frequency-selective fading in case the maximum channel delay spread is shorter than the GI. However, in a time-frequency-selective fading channel, the orthogonality is lost, resulting in intercarrier interference (ICI) that significantly degrades the performance of the system. OFDM is being considered as a candidate for future generation mobile communication standards to be operated at high levels of mobility and at high transmit frequencies, which lead to the channel to be time-frequency selective. Therefore, it is necessary to mitigate the ICI caused by channel variations.

Several approaches have been developed to reduce the effect of ICI. In [1], ICI coefficients with small power are ignored to reduce the computational complexity of frequency-domain minimum mean squared error (MMSE) OFDM symbol estimation. In [2], a two-term Taylor series expansion based channel model is introduced to analyze the effect of ICI and an adaptive MMSE method is proposed. An ICI reduction method based on time-domain filtering is introduced in [3]. These schemes require the complexity more than $O(N^2)$ operations, where N is the size of FFT/IFFT in an OFDM system.

In this paper, we propose an efficient OFDM symbol estimation algorithm, iteratively searching neighboring symbols in the constellation, to improve the bit error rate (BER) in the presence of ICI. The algorithm first roughly estimates the OFDM symbol block, ignoring all or most ICI terms. This results in the reduction of complexity of total symbol estimation processing. Comparing to the previous methods which simply ignore small-valued ICI terms, we find our proposed algorithm yields enhanced performance with approximate $\mathrm{O}(N^2)$ complexity, which is associated with the number of iterations and OFDM symbol block error calculations.

$Notation$: A bold face letter denotes a vector or a matrix; $[\bullet]^*$ denotes complex conjugate; $[\bullet]^T$ denotes transpose; $[\bullet]^H$ denotes conjugate transpose; $[\bullet]^+$ denotes pseudo-inverse; $\mathbf{I}_N$ denotes $N \times N$ identity matrix; $D_p(\mathbf{A})$ is the diagonal matrix including up to p^{th} sub-diagonal and super-diagonal with the same terms as matrix $\mathbf{A}$; $[\mathbf{A}]_{m,n}$ denotes the element in the m^{th} row and n^{th} column of matrix $\mathbf{A}$; $|| \bullet ||$ denotes the Frobenius norm; Expectation is denoted by $E\{\bullet\}$; in general, a lower case letter stands for a time-domain signal while an upper case letter denotes a frequency-domain signal.

2. OFDM SYSTEM MODELING

The discrete-time baseband equivalent OFDM system model under consideration is illustrated in Fig. 1. In OFDM systems, input bits are mapped or coded to frequency-domain symbols (e.g. QPSK, 16-QAM etc.), and divided into blocks of length N after serial-to-parallel conversion, and modulated by means of the N-point IFFT transform. Each block of N modulated time-domain samples is extended with a cyclic prefix (CP), which is a copy of the last samples of the IFFT output. For the i^{th} OFDM symbol block $\mathbf{X}_F^i = [X_0^i, X_1^i, ..., X_{N-1}^i]^T$ with variance σ_S^2, the time-domain signal $\mathbf{X}_T^i = [x_0^i, x_1^i, ..., x_{N-1}^i]^T$, converted by the N-point IFFT, is represented as

$$\mathbf{X}_T^i = \mathbf{F}_N \mathbf{X}_F^i, \tag{1}$$

where the time-domain sample, which is the N-point IFFT output, is represented as

$$x_n^i = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} X_m^i e^{j2\pi nm/N}, \; 0 \le n \le N-1, \tag{2}$$

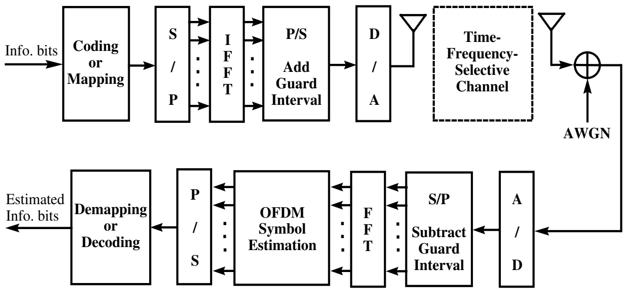

Fig. 1. Baseband equivalent block diagram for an OFDM system

and $\mathbf{F}_N$ is an N-point IFFT matrix. The i^{th} received block of time-domain OFDM samples after the removal of CP is

$$\mathbf{R}^i = [r_0^i, r_1^i, ..., r_{N-1}^i]^T = \mathbf{H}^i \mathbf{X}_T^i + \mathbf{Z}_T^i, \tag{3}$$

where $\mathbf{Z}_T^i = [z_0^i, z_1^i, ..., z_{N-1}^i]^T$ denotes the vector of time-domain circularly symmetric zero-mean white complex Gaussian noise (A WGN) with variance σ_Z^2. The complex matrix, known perfectly to the receiver, is given by

$$\mathbf{H}^i = \begin{bmatrix} h_{0,0}^i & h_{0,N-1}^i & \cdots & h_{0,1}^i \\ h_{1,1}^i & h_{1,0}^i & \cdots & h_{1,2}^i \\ \vdots & \vdots & \ddots & \vdots \\ h_{N-1,N-1}^i & h_{N-1,N-2}^i & \cdots & h_{N-1,0}^i \end{bmatrix}, \tag{4}$$

where $h_{n,l}^i, 0 \le n \le N-1, 0 \le l \le L-1$, is the channel impulse response (CIR) during the i^{th} OFDM symbol block interval at instant n and lag l, and the received sample of the i^{th} OFDM symbol block can be written as

$$r_n^i = \sum_{l=0}^{L-1} h_{n,l}^i x_{n-l}^i + z_n^i, \; 0 \le n \le N-1. \tag{5}$$

Since the CP as a GI is inserted, which is longer than the maximum delay spread L of channel, intersymbol interference (ISI) can be easily overcome. That is, the CP is a guard space between consecutive OFDM symbol blocks and removed before FFT processing at the receiver. The received time-domain samples are transformed into frequency-domain symbols by N-point FFT operation. The demodulated OFDM symbol block is expressed as

$$\mathbf{Y}^i = \left[Y_0^i, ..., Y_{N-1}^i\right]^T = \mathbf{F}_N^H \mathbf{H}^i \mathbf{F}_N \mathbf{X}^i + \mathbf{Z}_F^i = \mathbf{G}^i \mathbf{X}^i + \mathbf{Z}_F^i$$
$$= \begin{bmatrix} g_{0,0}^i & \cdots & g_{0,N-1}^i \\ \vdots & \ddots & \vdots \\ g_{N-1,0}^i & \cdots & g_{N-1,N-1}^i \end{bmatrix} \begin{bmatrix} X_0^i \\ \vdots \\ X_{N-1}^i \end{bmatrix} + \begin{bmatrix} Z_0^i \\ \vdots \\ Z_{N-1}^i \end{bmatrix} \tag{6}$$

where $\mathbf{F}_N^H$ is the N-point FFT matrix, $\mathbf{G}^i = \mathbf{F}_N^H \mathbf{H}^i \mathbf{F}_N$ is an $N \times N$ matrix with elements given by

$$g_{n,k}^i = \frac{1}{\sqrt{N}} \sum_{l=0}^{L-1} H_{l,n-k}^i e^{-j2\pi kl/N}, \; 0 \le n, k \le N-1, \tag{7}$$

with the output of the FFT for the CIR $h_{n,l}^i$ as

$$H_{l,n-k}^i = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} h_{m,l}^i e^{-j2\pi(n-k)m/N}, 0 \le n, k \le N-1, \tag{8}$$

the demodulated frequency-domain symbols via the FFT can be expressed as follows

$$Y_n^i = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \sum_{l=0}^{L-1} X_k^i H_{l,n-k}^i e^{-j2\pi kl/N} + Z_n^i, \tag{9}$$

and frequency-domain AWGN vector $\mathbf{Z}_F^i$ is the output of N-point FFT for time-domain AWGN vector $\mathbf{Z}_T^i$.

3. OFDM SYMBOL ESTIMATION SCHEMES

In this section, we describe the OFDM symbol estimation schemes in the presence of ICI. If the channel is assumed to be time-invariant during the OFDM symbol block and an adequate number of subcarriers with a CP of adequate length are used, then subcarrier orthogonality is maintained and the received signal can be prevented from being damaged by ISI. In this case, the OFDM signal can be easily compensated for by a one-tap frequency-domain equalizer. However, time-selective fading channels cause a loss of subcarrier orthogonality, resulting in ICI. The ICI leads to an error floor in BER, increasing with the relative Doppler frequency Δf_D, which is the product of the Doppler frequency f_D and OFDM symbol block duration T_S, respectively.

The demodulated frequency-domain OFDM symbol block in (10) can be rewritten as follows

$$\begin{aligned} \mathbf{Y}^i &= \mathbf{G}^i \mathbf{X}^i + \mathbf{Z}_F^i \\ &= D_0(\mathbf{G}^i)\mathbf{X}^i + \left[\mathbf{G}^i - D_0(\mathbf{G}^i)\right] \mathbf{X}^i + \mathbf{Z}_F^i \\ &= \mathbf{A}^i \mathbf{X}^i + \mathbf{A}_\triangle^i \mathbf{X}^i + \mathbf{Z}_F^i, \end{aligned} \tag{10}$$

where $\mathbf{A}^i$ is a $N \times N$ diagonal matrix with the same terms as $\mathbf{G}^i$, the main diagonal elements of $\mathbf{A}_\triangle^i$ are zeros, of which the other elements are equal to $\mathbf{G}^i$. It can be observed that the first and second terms (i.e., $\mathbf{A}^i$ and $\mathbf{A}_\triangle^i \mathbf{X}^i$) on the right side of (10) show the multiplicative distortion at the desired subchannels and the ICI, respectively, that is,

$$\left[\mathbf{A}^i\right]_{n+1,k+1} = \begin{cases} \frac{1}{\sqrt{N}} \sum\limits_{l=0}^{L-1} H_{l,n-k}^i e^{-j2\pi kl/N} & n = k \\ 0 & n \ne k \end{cases}, \tag{11}$$

$$\left[\mathbf{A}_\triangle^i\right]_{n+1,k+1} = \begin{cases} \frac{1}{\sqrt{N}} \sum\limits_{l=0}^{L-1} H_{l,n-k}^i e^{-j2\pi kl/N} & n \ne k \\ 0 & n = k \end{cases}. \tag{12}$$

Hence, $\mathbf{A}^i \mathbf{X}^i$ denotes the desired term without ICI, which represents the contribution from the same symbol, and $\mathbf{A}_\triangle^i \mathbf{X}^i$ denotes ICI terms, which represents the contribution from the other frequency-domain symbols within the OFDM symbol block. Therefore, the demodulated frequency-domain symbols is

$$\begin{aligned} Y_n^i &= \tfrac{1}{\sqrt{N}} \sum_{l=0}^{L-1} X_n^i H_{l,0}^i e^{-j2\pi kl/N} \\ &+ \tfrac{1}{\sqrt{N}} \sum_{k \ne n}^{N-1} \sum_{l=0}^{L-1} X_k^i H_{l,n-k}^i e^{-j2\pi kl/N} + Z_n^i, \; 0 \le n \le N-1. \end{aligned} \tag{13}$$

To show the relative significance of ICI terms, the average ICI power statistic is defined as follows,

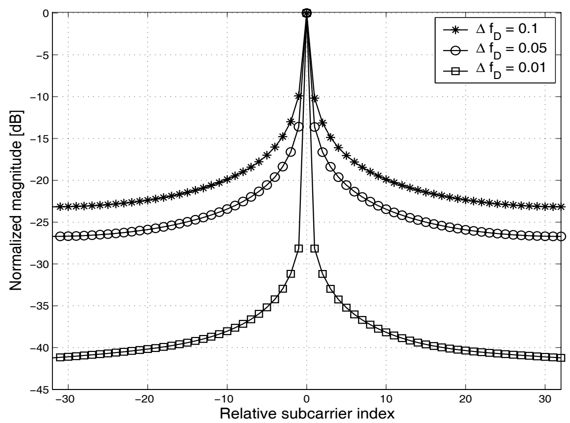

Fig. 2. Normalized average ICI leakage through relative subcarrier indices with 64-point FFT/IFFT

$$\Xi_p = \frac{1}{N}\sum_{k=0}^{N-1} E\left\{\left|\frac{1}{\sqrt{N}}\sum_{l=0}^{L-1} H_{l,p}^{i} e^{-j2\pi kl/N}\right|^2\right\}, \qquad (14)$$
$$-(N-1) \le p \le N-1,$$

where p is a relative subcarrier index (e.g. p=0 for the desired subchannel). Fig. 2 shows the normalized average ICI power statistic Ξ_P/Ξ_0 for three different relative Doppler frequencies, where we can see that the most power of ICI is concentrated in the neighborhood of the desired subchannel and gradually decreases apart from the desired subchannel.

3.1. Conventional methods

The zero-forcing (ZF) and linear minimum mean squared error (MMSE) symbol estimation methods are given by [4]

$$\hat{\mathbf{X}}_{ZF}^{i} = \left[\mathbf{G}^{i}\right]^{+}\mathbf{Y}^{i}, \qquad (15)$$

$$\hat{\mathbf{X}}_{MMSE}^{i} = \left[\mathbf{G}^{i\,H}\mathbf{G}^{i} + \frac{\sigma_Z^2}{\sigma_S^2}\mathbf{I}_N\right]^{-1}\mathbf{G}^{i\,H}\mathbf{Y}^{i}. \qquad (16)$$

In general, the conventional OFDM symbol estimation methods exhibit relatively good performance at low Δf_D. That is, with a time-invariant channel, $\mathbf{G}^i$ is a diagonal matrix. Hence, ZF and MMSE symbol estimations can be implemented in $O(N)$ operations. However, with a time-frequency-selective channel, an implementation of (15) and (16) usually requires non-trivial inversion of the $N \times N$ matrix with $O(N^3)$ operations, which may be prohibitively complex for practical applications.

From Fig. 3, we can observe that since most power is concentrated in the neighborhood of the diagonal line in (10), the ICI terms that do not significantly affect $\mathbf{Y}^i$ can be neglected and is assumed as [1, 4]

$$g_{n,k}^{i} = 0, \ |n-k| > p, \qquad (17)$$

where $2p$ denotes the number of dominant ICI terms against n^{th} frequency-domain symbol. Hence, the complexity of conventional ZF and MMSE symbol estimation methods can be reduced by using $D_p(\mathbf{G}^i)$ instead of $\mathbf{G}^i$ in (15) and (16) as follows

$$\hat{\mathbf{X}}_{ZF}^{i} = \left[D_p(\mathbf{G}^{i})\right]^{+}\mathbf{Y}^{i}, \qquad (18)$$

$$\hat{\mathbf{X}}_{MMSE}^{i} = \left[D_p(\mathbf{G}^{i})^H D_p(\mathbf{G}^{i}) + \frac{\sigma_Z^2}{\sigma_S^2}\mathbf{I}_N\right]^{-1} D_p(\mathbf{G}^{i})^H\mathbf{Y}^{i}. \qquad (19)$$

3.2. Proposed symbol estimation algorithm

Before proposing our Iterative Sequential Neighbor Search (ISNS) algorithm, we define the following OFDM symbol error metrics

$$\xi_n^i = \left|Y_n^i - \sum_{k=0}^{N-1} g_{n,k}^{i}\tilde{X}_k^i\right|^2. \qquad (20)$$

We now propose and develop an efficient OFDM symbol estimation algorithm by minimizing the above OFDM symbol error metric ξ_n^i with respect to received OFDM symbol block.

STEP 1. Obtain an initial OFDM symbol candidate by using

$$\tilde{\mathbf{X}}_{(1)}^{i} = \begin{cases} \left[D_p(\mathbf{G}^{i})\right]^{+}\mathbf{Y}^{i} \\ \left[D_p(\mathbf{G}^{i})^H D_p(\mathbf{G}^{i}) + \frac{\sigma_Z^2}{\sigma_S^2}\mathbf{I}_N\right]^{-1} D_p(\mathbf{G}^{i})^H\mathbf{Y}^{i} \end{cases}$$

with p as the number of dominant ICIs.

STEP 2. Calculate ξ_n^i, $0 \le n \le N-1$ using $\tilde{\mathbf{X}}_{(r=1)}^{i}$ with an iteration number r.

STEP 3. Start to search the neighboring symbols in the constellation and update as follows

For $n = 0 : N-1$

- Search the neighbor symbols $\{\hat{X}_{n(r)}^{i(s)}\}, 1 \le s \le S$, of $\tilde{X}_{n(r)}^{i}$ in the constellation, where S is the number of candidate symbols $\{\hat{X}_{n(r)}^{i(s)}\}$.
- Calculate OFDM symbol error $\hat{\xi}_n^{i(s)}$ using the searched candidate symbols $\{\hat{X}_{n(r)}^{i(s)}\}$.
- If there exist $\hat{\xi}_n^{i(s)}$ such as $\min\limits_{1\le s\le S}\hat{\xi}_n^{i(s)} < \xi_n^i$, update $\xi_n^i = \min\limits_{1\le s\le S}\hat{\xi}_n^{i(s)}$ and $\tilde{X}_{n(r)}^{i} = \hat{X}_{n(r)}^{i(s)}$. Otherwise, remain the previous $\xi_n^{i(s)}$ and $\tilde{X}_{n(r)}^{i}$.

End.

STEP 4. Repeat STEP 3 with $r \leftarrow r+1$ to get more updated ξ_n^i and $\tilde{X}_{n(r)}^{i}$.

STEP 5. The end of the algorithm.

As can be seen from the above algorithm, the initially estimated i^{th} OFDM symbol block is obtained through the simple inversion of $D_0(\mathbf{G}^i)$ or $D_p(\mathbf{G}^i)$, $p \ll N$ with a low complexity. In STEP 3, the estimator sequentially searches the neighboring symbol with a minimum OFDM symbol error in the constellation and update $\tilde{X}_{n(r)}^{i}$ and ξ_n^i with the best neighboring symbol, if it exists. We can observe that this procedure is sequentially done within the i^{th} OFDM symbol block. The above procedure is repeated until the iteration number r reaches the predetermined value. For example, the area of neighbor search in the 16-QAM constellation is shown in Fig. 3. Note that the computational complexity increases as the region of the search area in the constellation increases. In our simulations, we choose the first neighbor area.

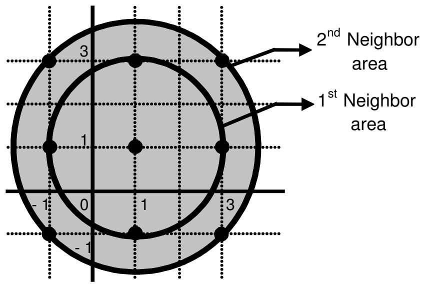

Fig. 3. Neighbor areas including neighbor symbols of 16-QAM symbol $(1 + i)$

4. SIMULATION RESULTS

Computer simulations are carried out to evaluate the performance of the proposed OFDM symbol estimation algorithm. It is assumed that the receiver knows the exact CIR with perfect carrier and symbol synchronization. With a square 16-QAM constellation, a 500 kHz bandwidth is divided into 64 subchannels, meaning the FFT/IFFT size N=64, and the OFDM symbol block duration $T_s = NT_0$=128 μs . The maximum channel delay is 4μs which is shorter than GI duration 6μs. Jake's model in [6] is used for the 2-ray Rayleigh fading channel, as the relative delays are 0 and 4μs with equal power of 0dB, respectively. Considered relative Doppler frequencies are 0.1 and 0.05 for the relatively fast and moderately time-varying channel, respectively. The performance criterion is the BER versus signal-to-noise ratio (SNR) for Δf_D = 0.1 and 0.05.

As a benchmark, we first show the performance comparisons of conventional ZF methods with the inversions of matrix $\mathbf{G}^i$, matrix $D_1(\mathbf{G}^i)$, and diagonal matrix $D_0(\mathbf{G}^i)$ to compare the performance of our proposed algorithm. Fig. 4 shows that the ISNS algorithm deploying ZF with $D_0(\mathbf{G}^i)$ and $D_1(\mathbf{G}^i)$, for an initial OFDM symbol candidate decision offers a great improvement in BER, even after 1-3 iterations, while the degree of performance improvement decreases as the iterations increase. As can be seen in Fig. 4(a), with Δf_D=0.1, about 4-5dB gain of the ISNS algorithm with $D_1(\mathbf{G}^i)$ is achieved at a BER of 2×10^{-2}, compared to the conventional ZF with $D_1(\mathbf{G}^i)$. On the other hand, the performance improvement of our algorithm is obtained at a cost of a little more complexity due to the number of iterations and calculation of the OFDM symbol errors. However, the resultant complexity is reasonable, since it is still $O(N^2)$, which is related to a function of the small values of iteration number r and considered dominant neighboring subcarrier number p, and the OFDM symbol error is sequentially calculated and updated within a OFDM symbol block at each iteration. This is to be compared with the conventional approaches mentioned in Sec. 3.1, where the complexities are more than $O(N^3)$ associated with the inversion of a large-sized matrix [5]. Moreover, the calculation of initial inversion of $D_p(\mathbf{G}^i)$, $p \geq 1$ can be further reduced by matrix decomposition with reference to [1].

5. CONCLUSION

Operation of OFDM system over a time-frequency-selective fading channel results in ICI and severe performance degradation. To mitigate the ICI within an OFDM symbol block, we have proposed a novel OFDM symbol estimation scheme, the ISNS algorithm. Compared with the conventional OFDM symbol estimation methods, our proposed algorithm exhibits better performance with a moderate complexity. Simulation results demonstrated that our proposed algorithm can enhance OFDM system performance in the presence of severe ICI, making it a powerful candidate for the next generation mobile communication systems.

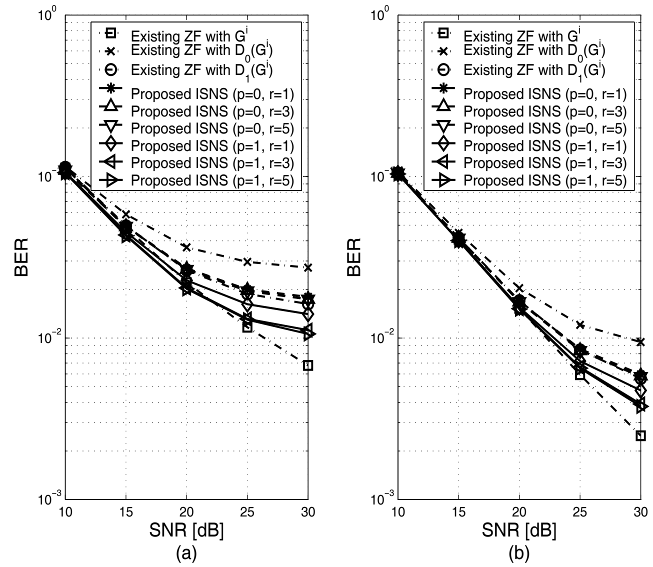

Fig. 4. BER performance vs. SNR of frequency-domain ZF methods and proposed ISNS algorithm using $D_0(\mathbf{G}^i)$ and $D_1(\mathbf{G}^i)$ for (a) Δf_D = 0.1 and (b) Δf_D = 0.05 with 64-point FFT/IFFT

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