CHANNEL EQUALIZATION AND PHASE NOISE SUPPRESSION FOR OFDM SYSTEMS IN A TIME-VARYING FREQUENCY SELECTIVE CHANNEL USING PARTICLE FILTERING

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ABSTRACT

In this paper we address the problem of channel equalization and phase noise suppression in orthogonal frequency division multiplexing (OFDM) systems. For OFDM systems, random phase noise introduced by the local oscillator causes two effects: the common phase error (CPE), and the intercarrier interference (ICI). The performance of coherent OFDM systems greatly depends on the ability to accurately estimate the *effective* dynamic channel i.e., the combined effect of the CPE and the time-varying frequency selective channel. The proposed approach uses a pilot tone aided *particle filter* to track/estimate the effective dynamic channel in the time domain and equalizes in the frequency domain. The particle filter is efficiently implemented by combining sequential importance sampling, principles of Rao-Blackwellization, and strategies stemming from the auxiliary particle filter. Simulation results are provided to illustrate the effectiveness of the proposed algorithm.

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has been adopted in a number of applications. These include the digital audio broadcasting (DAB), digital video broadcasting (DVB) standards, and the wireless LAN standards, such as IEEE 802.11a, and HiperLAN2. OFDM is also considered as a natural candidate for 4G cellular systems and beyond, because of its efficient use of bandwidth, ability to combat impulsive noise, and robustness against multipath fading. However, OFDM systems suffer from some drawbacks as well, and one is the increased sensitivity to random phase noise (PN) that is introduced by the local oscillator.

PN in OFDM systems causes two effects. The first is a random phase rotation that is common to all subcarriers, that is appropriately referred to as the common phase error (CPE). The second is the introduction of intercarrier interference (ICI), resulting from the loss of orthogonality between each subcarrier. Indeed, many researchers [3] have studied the effects of PN in OFDM systems.

Several authors have also proposed various schemes for PN compensation. In [2, 5], the chosen approach was to counter rotate the received signal constellation, via an estimate of the CPE term. In this paper, we present a pilot tone aided algorithm that jointly equalizes the channel and compensates for the CPE in a time–varying frequency selective channel. The algorithm is based on the time domain tracking/estimation of the *effective* dynamic channel, i.e., the combined effect of the CPE, and the time-varying frequency selective channel, so that we may realize effective frequency domain equalization. However, for our estimates of interest, the optimal Bayesian estimators are analytically intractable. Hence, for online estimation of the *effective* dynamic channel, we resort to modern Bayesian methods known as particle filtering. The basic idea behind particle filters is to approximate the posterior distribution of interest with a set of weighted random samples (particles). In practice, these methods provide a means of approximating the optimal Bayesian estimators for nonlinear, possibly non–Gaussian dynamical systems. Indeed, as the number of particles become very large, the approximations approach the true optimal Bayesian estimators [1].

This paper is organized as follows. In Section 2, we introduce the baseband OFDM system. In Section 3, we introduce the dynamic state space model. Section 4 reviews fundamentals of particle filter, and Section 5, provides a discussion of our proposed algorithm. Section 6 presents some simulation results, and concludes this paper.

2. SYSTEM MODEL

The baseband OFDM system under consideration is shown in Figure 1.

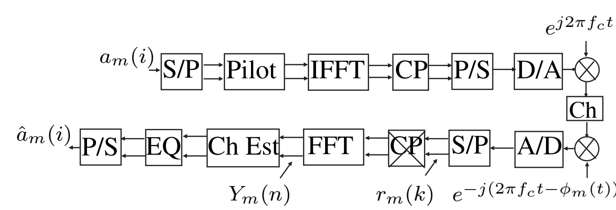

Fig. 1. Baseband OFDM System

From the information source, $\log_2(M)$ bits are encoded into M-QAM symbols $a_m(i)$, where $a_m(i)$ denotes the M-QAM symbol at the i-th subcarrier, of the m-th OFDM block, or symbol. Note that m is an index in time. Then, P pilot tones are inserted into $a_m(i)$, such that:

$$a_m(i) \quad = \quad \begin{cases} c_{pilot}(i) & i \in \Omega \\ \text{information data} & i \notin \Omega \end{cases}$$

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where Ω denotes the set of pilot tone locations. In particular, we choose Ω to satisfy [4]:

$$\Omega = \{i | i = kS, \qquad \text{with } k = 0, \ldots, P-1\}$$

where N is the number of subcarriers, and $S = \frac{N}{P}$ is the spacing between each pilot tone. After pilot tone insertion, $\{a_m(i)\}_{i=0}^{N-1}$ are sent to the IDFT, and a cyclic prefix (CP) is introduced to remove inter-symbol interference (ISI).

The time varying frequency selective channel $h(t, \tau)$ is assumed to be *quasi-static* over one OFDM symbol, with L independent propagation paths. The additive white noise is denoted by $n(t)$.

At the receiver, the *free-running* local oscillator introduces PN $\phi_m(t)$. Because we assume perfect frequency and timing synchronization, we can write the k-th received sample of the m-th OFDM symbol as:

$$r_m(k) = \sum_{l=0}^{L-1} h_{m,l} s_m(k-l) e^{j\phi_m(k)} + n_m(k)$$

where $n_m(k)$ is assumed to be zero mean complex Gaussian whose variance is σ_n^2, and $\phi_m(k)$ is a sample of the PN process at the output of the free-running local oscillator. Since we make the assumption that the entire channel impulse response lies within the CP, i.e., $N_{cp} \geq L-1$. The discarding of the CP followed by the DFT of $\{r_m(k)\}_{k=0}^{N-1}$ yields:

$$\begin{aligned} Y_m(n) \quad &= \quad a_m(n) H_m(n) \underbrace{I_m(0)}_{CPE} \\ &+ \underbrace{\sum_{\substack{i=0 \\ i \neq n}}^{N-1} a_m(i) H_m(i) I_m(n-i)}_{ICI} + W_m(n) \end{aligned} \tag{1}$$

where

$$\begin{aligned} H_m(n) &= \sum_{l=0}^{L-1} h_{m,l} e^{-j\frac{2\pi n l}{N}} \\ I_m(n) &= \frac{1}{N} \sum_{k=0}^{N-1} e^{j\phi_m(k)} e^{-j\frac{2\pi n k}{N}} \end{aligned}$$

and $W_m(n) = DFT\{n_m(k)\}$. From (1), we recognize that PN introduces two problems. The first problem is the additional phase variation of the desired sample $a_m(n)H_m(n)$ by the CPE $arg\{I_m(0)\}$, and the second is the ICI, which results from the loss of orthogonality between each sub-carrier. Moreover, if we define $H_m^{eff}(n) = H_m(n)I_m(0)$ as the *effective* channel response, then one can see that it is crucial to obtain accurate estimates of $H_m^{eff}(n)$, so that we may reliably recover $a_m(n)$. Zero-forcing (ZF) equalization follows after channel estimation, and the transmitted symbol can be estimated by $\hat{a}_m(n) = \frac{Y_m(n)}{\hat{H}_m^{eff}(n)}$ for $n = 0, \ldots, N-1$ where $\hat{H}_m^{eff}(n)$ is the estimate of the effective channel response, $H_m^{eff}(n)$.

3. STATE SPACE MODEL

Particle filters require a process equation, and a observation equation. The aim of this section is to develop the required dynamic state space (DSS) model, through the known statistics of the channel, and PN process.

3.1. Channel Model

The frequency selective Rayleigh fading channel is characterized by a tapped-delay model. Channel taps $\{h_{m,l}\}_{l=0}^{L-1}$ are assumed to be mutually uncorrelated, zero mean complex Gaussian coefficients, with Jake's Doppler spectrum [7]. Accordingly, the autocorrelation of the l-th channel tap $h_{m,l}$ for $l = 0, \ldots, L-1$ is given by:

$$r_l(k) = E[h_{m,l} h_{m+k,l}^*] = r_l(0) J_0(2\pi f_d k T) \tag{2}$$

where $r_l(0)$ denotes the power of the l-th channel tap, f_d denotes the maximum Doppler frequency, T denotes the duration of one OFDM symbol, and $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind. Exact modelling of (2) via an autoregressive moving-average (ARMA) model is impossible, because the autocorrelation function is nonrational. However, several authors have approximated Jakes model with an autoregressive (AR) model. We follow a similar approach, and model the temporal dynamics of $h_{m,l}$, though a AR(2) model [6]:

$$h_{m,l} = -a_1 h_{m-1,l} - a_2 h_{m-2,l} + v_{m,l}, \qquad l = 0, \ldots, L\text{-}1 \tag{3}$$

where $v_{m,l}$ is a zero mean complex Gaussian random variable with variance σ_l^2. Model coefficients a_1,a_2, and σ_l^2 are chosen so that the autocorrelation of (3) closely matches (2).

3.2. Phase Noise Model

Discrete time Wiener PN $\phi(n)$ corresponds to samples of $\phi(t)$ at time $t = nT_s$, where $T_s = T/(N + N_{cp})$ is the sampling period of the receiver A/D converter. It can be shown that:

$$\phi(n) = \phi(n-1) + w(n) \tag{4}$$

where $w(n)$ is a zero mean Gaussian random variable with variance $\sigma_w^2 = 2\pi B T_s$. In the sequel, we will refer to BT as the phase noise rate. Furthermore, for small $\phi(n)$ the m-th CPE θ_m can be approximated by the average of the PN that was sampled during the useful (i.e. data) portion of the m-th OFDM symbol. This fact, together with (4), yields the desired CPE process equation [2]:

$$\theta_m = \theta_{m-1} + \tilde{w}_m \tag{5}$$

where $\tilde{w}_n$ is a zero mean Gaussian random variable with variance $\sigma_{cpe}^2 = \left(\frac{2N^2+1}{3N} + N_{cp}\right)\sigma_w^2$.

3.3. Observation Model

Our starting point is (1). At known pilot tone locations, the least squares (LS) estimate of the effective channel response $H_m^{eff}(n)$ is:

$$\hat{H}_m^{eff}(n) \quad = \quad \frac{Y_m(n)}{c_{pilot}(n)}, \qquad n \in \Omega$$

We proceed by stacking $\{\hat{H}_m^{eff}(n)\}_{n \in \Omega}$ into a $P \times 1$ vector $\boldsymbol{H}_{LS}^{eff}$, so that we may write:

$$\boldsymbol{H}_{LS}^{eff} \quad = \quad \boldsymbol{V} I_m(0) \boldsymbol{h}_m + \boldsymbol{I}_m + \boldsymbol{Z}_m$$

where $\boldsymbol{h}_m = [h_{m,0}, \ldots, h_{m,L-1}]^T$ is a vector of channel taps, $\boldsymbol{I}_m$ is a vector of ICI quantities, $\boldsymbol{Z}_m$ is an AWGN vector, and $\boldsymbol{V}$ is the following Vandermonde DFT Matrix:

$$\boldsymbol{V} \quad = \quad \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_N^S & \ldots & W_N^{S(L-1)} \\ \ldots & \ldots & \ldots & \ldots \\ 1 & W_N^{(P-1)S} & \ldots & W_N^{(P-1)S(L-1)} \end{bmatrix}$$

with $W_N = e^{-j2\pi/N}$. Assuming $P = L$, the Vandermonde matrix is non-singular and hence invertible. Therefore, a noisy estimate of $I_m(0)\boldsymbol{h}_m$ is given by:

$$\boldsymbol{Y}_m = \boldsymbol{V}^{\boldsymbol{-1}}\boldsymbol{H}_{LS}^{\mathit{eff}} = I_m(0)\boldsymbol{h}_m + \tilde{\boldsymbol{I}}_m + \tilde{\boldsymbol{Z}}_m \tag{6}$$

Equation (6) provides a coarse estimate of $I_m(0)\boldsymbol{h}_m$, and can be seen as an observation equation in our DSS model.

3.4. Dynamic State Space Model

Use of (3), (5), and (6), leads to the considered DSS model:

$$\begin{aligned}\boldsymbol{x}_m &= \boldsymbol{F}\boldsymbol{x}_{m-1} + \boldsymbol{V}_m\\ \theta_m &= \theta_{m-1} + \tilde{w}_m\\ \boldsymbol{Y}_m &= \boldsymbol{G}(\theta_m)\boldsymbol{x}_m + \tilde{\boldsymbol{I}}_m + \tilde{\boldsymbol{Z}}_m\end{aligned}$$

where $\boldsymbol{x}_m = [\boldsymbol{h}_m^T\, \boldsymbol{h}_{m-1}^T]^T$. The matrices $\boldsymbol{F}$ and $\boldsymbol{G}(\theta_m)$ have the form of:

$$\boldsymbol{F} = \begin{bmatrix} -a_1 I_L & -a_2 I_L \\ I_L & 0_L \end{bmatrix}, \qquad \boldsymbol{G}(\theta_m) = \begin{bmatrix} \hat{I}_m(0)\cdot I_L & 0_L \end{bmatrix}$$

where in place of $I_m(0)$, we have substituted the convenient approximation $I_m(0) \approx \hat{I}_m(0) = e^{j\theta_m}$ [2], with I_L and 0_L denoting the $L \times L$ identity and all zero matrix respectively. The notation $\boldsymbol{V}_m$ denotes a vector of the form $\boldsymbol{V}_m = [\boldsymbol{v}_m^T, 0_{L\times 1}^T]^T$ where $0_{L\times 1}$ is a $L \times 1$ vector of zeroes, and $\boldsymbol{v}_m$ is a $L \times 1$ vector of white Gaussian noise with covariance matrix $E[\boldsymbol{v}_m\boldsymbol{v}_m^H] = diag(\sigma_0^2, \ldots, \sigma_{L-1}^2)$. $\tilde{\boldsymbol{I}}_m$ and $\tilde{\boldsymbol{Z}}_m$ are vectors of transformed ICI and noise components respectively. Moreover, we make the assumptions in [5], so that for large N the elements of $\tilde{\boldsymbol{I}}_m$ approximately follow a zero mean complex Gaussian distribution with variance $\sigma_I^2 = 2\pi BT/3P$. In addition, the elements of $\tilde{\boldsymbol{Z}}_m$ follow a complex Gaussian distribution with zero mean and variance $\sigma_Z^2 = \sigma_n^2/P$.

The main objective is to obtain the minimum mean square error (MMSE) estimates of $\{\hat{I}_m(0)h_{m,l}\}_{l=0}^{L-1}$:

$$E[\hat{I}_m(0)\boldsymbol{h}_m|\boldsymbol{Y}_{1:m}] = \iint \hat{I}_m(0)\boldsymbol{h}_m p(\theta_m, \boldsymbol{h}_m|\boldsymbol{Y}_{1:m})d\theta_m d\boldsymbol{h}_m$$

where the density $p(\theta_m, \boldsymbol{h}_m|\boldsymbol{Y}_{1:m})$ denotes the filtering posterior probability density function (PDF), and the notation $(\cdot)_{1:m}$, indicates all the elements from time 1 to time m. Unfortunately, the posterior PDF $p(\theta_m, \boldsymbol{h}_m|\boldsymbol{Y}_{1:m})$ is analytically intractable. Thus, we propose to numerically approximate $p(\theta_m, \boldsymbol{h}_m|\boldsymbol{Y}_{1:m})$ via particle filtering, so that we may ultimately estimate the posterior expectation $E[\hat{I}_m(0)\boldsymbol{h}_m|\boldsymbol{Y}_{1:m}]$.

4. PARTICLE FILTER

The particle filter utilizes a weighted set of samples, to approximate the filtering posterior PDF $p(\boldsymbol{x}_m, \theta_m|\boldsymbol{Y}_{1:m})$. Thus at time m, if we draw N_p samples $\{\boldsymbol{x}_m^{(i)}, \theta_m^{(i)}\}_{i=1}^{N_p}$ from a *importance function* $\pi(\boldsymbol{x}_m, \theta_m\, |\boldsymbol{x}_{1:m-1}, \theta_{1:m-1}, \boldsymbol{Y}_{1:m})$, and recursively update the importance weights $\{w_m^{(i)}\}_{i=1}^{N_p}$ as:

$$w_m^{(i)} \propto w_{m-1}^{(i)} \frac{p(\boldsymbol{Y}_m|\boldsymbol{x}_m^{(i)}, \theta_m^{(i)})p(\boldsymbol{x}_m^{(i)}, \theta_m^{(i)}|\boldsymbol{x}_{m-1}^{(i)}, \theta_{m-1}^{(i)})}{\pi(\boldsymbol{x}_m^{(i)}, \theta_m^{(i)}|\boldsymbol{x}_{1:m-1}^{(i)}, \theta_{1:m-1}^{(i)}, \boldsymbol{Y}_{1:m})} \tag{7}$$

We have for the empirical approximation of $p(\boldsymbol{x}_m, \theta_m|\boldsymbol{Y}_{1:m})$:

$$\hat{p}(\boldsymbol{x}_m, \theta_m|\boldsymbol{Y}_{1:m}) = \sum_{i=1}^{N_p} \tilde{w}_m^{(i)}\delta((\boldsymbol{x}_m, \theta_m) - (\boldsymbol{x}_m, \theta_m)^{(i)})$$

where $\tilde{w}_m^{(i)} = [\sum_{j=1}^{N_p} w_m^{(j)}]^{-1}w_m^{(i)}$ is the *normalized* importance weight, and $\delta(\cdot)$ is the Dirac delta function.

In practice, however, particle filtering suffer from *the Degeneracy problem*. That is, after a few iterations, all but a few particles possess insignificant weights. The result is an inefficient particle filter. Typically, the prescribed solution is to *resample* the particles, and the basic idea is to discard particles with weak importance weights and to multiply ones with sizable importance weights [1].

5. AUXILIARY RAO-BLACWELLIZED PARTICLE FILTER

For our DSS model, it is possible to design a better algorithm that yields estimates with lower variances. The idea is to exploit the inherent linear sub-structure of our given DSS model. Consider the joint posterior distribution $p(\boldsymbol{x}_m, \theta_{1:m}|\boldsymbol{Y}_{1:m})$ written as:

$$p(\boldsymbol{x}_m, \theta_{1:m}|\boldsymbol{Y}_{1:m}) = p(\boldsymbol{x}_m|\theta_{1:m}, \boldsymbol{Y}_{1:m})p(\theta_{1:m}|\boldsymbol{Y}_{1:m}) \tag{8}$$

It is clear that we can obtain the Gaussian PDF $p(\boldsymbol{x}_m|\theta_{1:m}, \boldsymbol{Y}_{1:m})$, via a Kalman filter, and that we can approximate the marginal posterior distribution $p(\theta_{1:m}|\boldsymbol{Y}_{1:m})$ with a particle filter. This approach is commonly known as the Rao-Blackwellized particle filter (RBPF) [1]. Now, at time m, assume for an estimate of $p(\theta_{1:m}|\boldsymbol{Y}_{1:m})$ we have:

$$\hat{p}(\theta_{1:m}|\boldsymbol{Y}_{1:m}) = \sum_{i=1}^{N_p} \tilde{w}_m^{(i)}\delta(\theta_{1:m} - \theta_{1:m}^{(i)}) \tag{9}$$

Thus, by substituting (9) into (8) and marginalizing over $\theta_{1:m-1}$, we obtain an estimate of $p(\boldsymbol{x}_m, \theta_m|\boldsymbol{Y}_{1:m})$ that is given by:

$$\hat{p}(\boldsymbol{x}_m, \theta_m|\boldsymbol{Y}_{1:m}) = \sum_{i=1}^{N_p} \tilde{w}_m^{(i)} p(\boldsymbol{x}_m|\theta_{1:m}^{(i)}, \boldsymbol{Y}_{1:m})\delta(\theta_m - \theta_m^{(i)}) \tag{10}$$

where $p(\boldsymbol{x}_m|\theta_{1:m}^{(i)}, \boldsymbol{Y}_{1:m})$ is a complex Gaussian PDF with mean $\boldsymbol{x}_{m|m}^{(i)} = E[\boldsymbol{x}_m|\theta_{1:m}^{(i)}, \boldsymbol{Y}_{1:m}]$, and covariance $\boldsymbol{P}_{m|m}^{(i)} = cov[\boldsymbol{x}_m|\theta_{1:m}^{(i)}, \boldsymbol{Y}_{1:m}]$. However, unlike (7), the importance weights for the RBPF satisfy:

$$w_m^{(i)} \propto w_{m-1}^{(i)} \frac{p(\boldsymbol{Y}_m|\theta_{1:m}^{(i)}, \boldsymbol{Y}_{1:m-1})p(\theta_m^{(i)}|\theta_{m-1}^{(i)})}{\pi(\theta_m^{(i)}|\theta_{1:m-1}^{(i)}, \boldsymbol{Y}_{1:m})} \tag{11}$$

where $\pi(\theta_m|\theta_{1:m-1}, \boldsymbol{Y}_{1:m})$ denotes the importance function for θ_m. Moreover, we adopt the prior $p(\theta_m|\theta_{m-1})$ as our importance function, in which case the importance weights (11) simplify to:

$$w_m^{(i)} \propto w_{m-1}^{(i)} p(\boldsymbol{Y}_m|\theta_{1:m}^{(i)}, \boldsymbol{Y}_{1:m-1}) \tag{12}$$

where

$$p(\boldsymbol{Y}_m|\theta_{1:m}^{(i)}, \boldsymbol{Y}_{1:m-1}) = NC(\boldsymbol{Y}_m; \boldsymbol{Y}_{m|m-1}^{(i)}, \boldsymbol{S}_m) \tag{13}$$

is a complex Gaussian PDF with mean $\boldsymbol{Y}_{m|m-1}^{(i)} = E[\boldsymbol{Y}_m|\theta_{1:m}^{(i)}, \boldsymbol{Y}_{1:m-1}]$, and covariance $\boldsymbol{S}_m^{(i)} = cov[\boldsymbol{Y}_m|\theta_{1:m}^{(i)}, \boldsymbol{Y}_{1:m-1}]$. Notice that the Kalman filter efficiently computes the PDF's $p(\boldsymbol{x}_m|\theta_{1:m}^{(i)}, \boldsymbol{Y}_{1:m})$, and $p(\boldsymbol{Y}_m|\theta_{1:m}^{(i)}, \boldsymbol{Y}_{1:m-1})$ for (10) and (12) respectively. Therefore, it is apparent that a Kalman filter is associated with each particle, and that the RBPF utilizes a bank of Kalman filters to approximate the true filtering posterior distribution.

We observe, however, that the prior $p(\theta_m|\theta_{m-1})$ is inefficient; it proposes samples $\{\theta_m^{(i)}\}_{i=1}^{N_p}$ without any knowledge of the current observation $\boldsymbol{Y}_m$. Thus to incorporate the recent observation into our proposals, we follow a strategy that is inspired by the Auxiliary particle filter (APF) [1]. The idea at time m is, to preselect (resample) the particles $\{\theta_{1:m-1}^{(i)}\}_{i=1}^{N_p}$ that have large predictive likelihoods $p(\boldsymbol{Y}_m|\theta_{1:m-1}^{(i)}, \boldsymbol{Y}_{1:m-1})$. Indeed, if we rewrite (12) as:

$$w_m^{(i)} \propto w_{m-1}^{(i)} p(\boldsymbol{Y}_m|\theta_{1:m-1}^{(i)}, \boldsymbol{Y}_{1:m-1}) \frac{p(\boldsymbol{Y}_m|\theta_{1:m}^{(i)}, \boldsymbol{Y}_{1:m-1})}{p(\boldsymbol{Y}_m|\theta_{1:m-1}^{(i)}, \boldsymbol{Y}_{1:m-1})} \tag{14}$$

then (14) suggests that we may instead preselect (resample) the particles $\{\theta_{1:m-1}^{(i)}\}_{i=1}^{N_p}$ according to the importance weights $\lambda_m^{(i)} \propto w_{m-1}^{(i)} p(\boldsymbol{Y}_m|\theta_{1:m-1}^{(i)}, \boldsymbol{Y}_{1:m-1})$, and that after resampling, set the weights to the so-called second stage importance weights $w_m^{(i)} \propto \frac{p(\boldsymbol{Y}_m|\theta_{1:m}^{(i)}, \boldsymbol{Y}_{1:m-1})}{p(\boldsymbol{Y}_m|\theta_{1:m-1}^{(i)}, \boldsymbol{Y}_{1:m-1})}$. This procedure is advantageous since it uses information $\boldsymbol{Y}_m$ at time m to select the most promising particles at time $m-1$. Unfortunately, it is difficult to evaluate $p(\boldsymbol{Y}_m|\theta_{1:m-1}, \boldsymbol{Y}_{1:m-1})$:

$$p(\boldsymbol{Y}_m|\theta_{1:m-1}, \boldsymbol{Y}_{1:m-1}) = \int p(\boldsymbol{Y}_m|\theta_{1:m}, \boldsymbol{Y}_{1:m-1}) p(\theta_m|\theta_{m-1}) d\theta_m$$

since $p(\boldsymbol{Y}_m|\theta_{1:m}, \boldsymbol{Y}_{1:m-1})$ as given by (13) depends on θ_m via a nonlinear measurement function $\boldsymbol{G}(\theta_m)$. However, for small process noise σ_{cpe}^2 such as the case we are considering, $p(\theta_m|\theta_{m-1})$ is well characterized by a sample $\mu_m \sim p(\theta_m|\theta_{m-1})$. Hence, if we make the approximation that $p(\theta_m|\theta_{m-1}) \approx \delta(\theta_m - \mu_m)$, the predictive likelihood can be approximated by:

$$\hat{p}(\boldsymbol{Y}_m|\theta_{1:m-1}, \boldsymbol{Y}_{1:m-1}) = p(\boldsymbol{Y}_m|\theta_m = \mu_m, \theta_{1:m-1}, \boldsymbol{Y}_{1:m-1}) \tag{15}$$

The resulting algorithm follows. At time m:

1. For $i = 1, \ldots, N_p$, set $\tilde{\boldsymbol{x}}_{m|m-1}^{(i)} = \boldsymbol{x}_{m|m-1}^{(i)}$, $\tilde{\boldsymbol{P}}_{m|m-1}^{(i)} = \boldsymbol{P}_{m|m-1}^{(i)}$, $\tilde{\theta}_{m-1}^{(i)} = \theta_{m-1}^{(i)}$ and sample $\mu_m^{(i)} \sim p(\theta_m|\tilde{\theta}_{m-1}^{(i)})$
2. For $i = 1, \ldots, N_p$, calculate first stage importance weights $\lambda_m^{(i)} \propto w_{m-1}^{(i)} \hat{p}(\boldsymbol{Y}_m|\tilde{\theta}_{1:m-1}^{(i)}, \boldsymbol{Y}_{1:m-1})$ using (15), and set $\sum_{i=1}^{N_p} \lambda_m^{(i)} = 1$.
3. Resample $\{\tilde{\boldsymbol{x}}_{m|m-1}^{(i)}, \tilde{\boldsymbol{P}}_{m|m-1}^{(i)}, \tilde{\theta}_{m-1}^{(i)}\}_{i=1}^{N_p}$ w.r.t importance weights to obtain $\{\boldsymbol{x}_{m|m-1}^{(i)}, \boldsymbol{P}_{m|m-1}^{(i)}, \theta_{m-1}^{(i)}\}_{i=1}^{N_p}$.
4. For $i = 1, \ldots, N_p$, draw new samples $\theta_m^{(i)} \sim p(\theta_m|\theta_{m-1}^{(i)})$.
5. For $i = 1, \ldots, N_p$, compute $\boldsymbol{x}_{m|m}^{(i)}$, and $\boldsymbol{P}_{m|m}^{(i)}$ using Kalman filter update equations.
6. For $i = 1, \ldots, N_p$, calculate the second stage importance weights $w_m^{(i)} \propto \frac{p(\boldsymbol{Y}_m|\theta_{1:m}^{(i)}, \boldsymbol{Y}_{1:m-1})}{\hat{p}(\boldsymbol{Y}_m|\theta_{1:m-1}^{(i)}, \boldsymbol{Y}_{1:m-1})}$, and set $\sum_{i=1}^{N_p} w_m^{(i)} = 1$.
7. For $i = 1, \ldots, N_p$, compute predicted state $\boldsymbol{x}_{m+1|m}^{(i)}$, and predicted covariance $\boldsymbol{P}_{m+1|m}^{(i)}$ using Kalman filter prediction equations. Set $m \rightarrow m+1$, and go back to step 1.

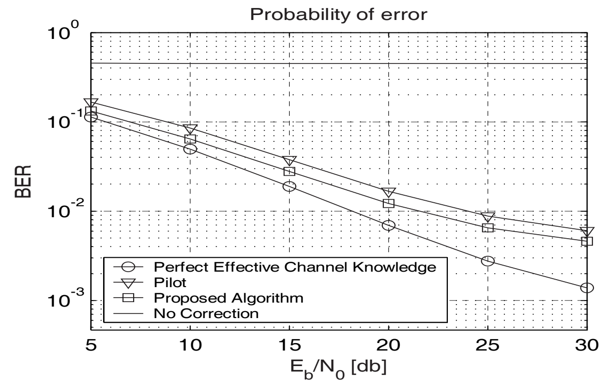

Fig. 2. BER for 16-QAM

6. SIMULATIONS AND CONCLUSIONS

We considered a 16-QAM OFDM system with system parameters $N = 128$, $P = 4$, and $N_{cp} = 8$. The total channel bandwidth was chosen to be $B_w = 1MHz$, and a four path i.e., $L = 4$ frequency selective channel was generated from Jakes fading model, with time-Doppler fading rate $f_dT = 0.04$. The adopted power delay profile, and time delay profile was [0,-13,-22,-28] (dB), and $[0, 1, 2, 3]$ μs respectively. The phase noise rate BT was set to 0.01, and the proposed algorithm was implemented with $N_p = 50$ particles. The BER was evaluated at each SNR for 8000 OFDM symbols, and figure 2 illustrates the results. Our algorithm results in 2-3 dB improvement over an approach based solely on pilot tones. For example, at 12 dB our algorithm is approximately 2 dB away from the ideal curve, while the latter is almost 4 dB away. In conclusion, we have proposed a new Rao-Blackwellization and Auxiliary particle filtering technique for channel equalization and phase noise suppression in OFDM systems. Results show about 2-3 dB improvement over a naive scheme based solely on an LS estimate of the effective channel using the available pilot tones.

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