# Iterative Interference Suppression for Pseudo Random Postfix OFDM based Channel Estimation

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***Abstract*— This contribution[1] proposes an iterative channel impulse response (CIR) estimation scheme in the context of the Pseudo-Random-Postfix OFDM (PRP-OFDM) modulation. While conventional techniques reduce noise and interference of OFDM data symbols on the CIR estimates by simple mean value calculation, the new proposal uses soft decoder outputs in order to perform iterative OFDM data symbol interference cancellation. In a typical example for 64QAM constellations, the mean-square-error (MSE) of the CIR estimates is improved by approx. 12dB after three iterations. Based on PRP-OFDM postfixes only, it is thus possible to perform channel estimation for higher order constellations (64QAM and higher) with an initial CIR estimation over a small observation window (for 64QAM typically 30 to 40 OFDM symbols). Any loss in throughput due to additional redundancy for CIR estimation purposes, e.g. pilot tones, learning symbols, etc. is avoided at the cost of an increase in decoding complexity.**

## I. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) seems the preferred modulation for modern broadband communication systems. Indeed, the OFDM inherent robustness to multi-path propagation and its appealing low complexity equalization scheme makes it suitable either for high speed modems over twisted pair (digital subscriber lines xDSL), terrestrial digital broadcasting (Digital Audio and Video Broadcasting: DAB, DVB) and 5GHz Wireless Local Area Networks (WLAN) IEEE802.11a [1], [2].

All these systems rely on the insertion of a Cyclic Prefix (CP-OFDM) to turn the linear convolution into a set of parallel attenuations in the discrete frequency domain. We assume in this paper a coherent modulation scheme which requires to estimate the Channel Impulse Response (CIR) for performing the decoding at the receiver. An issue with CP-OFDM is that this estimation step is usually relying on the insertion of learning symbols and pilot tones resulting in a loss in useful throughput especially for high Doppler environments. In order to address this issue and enhance WLAN mobility, a new modulation scheme has recently been proposed: the Pseudo Random Postfix OFDM (PRP-OFDM) modulation [3], [4] replacing the cyclic prefix extension by a known postfix weighted by a pseudo random scalar sequence changing at the OFDM block rate [5]. This way, unlike for classical OFDM modulators, the receiver can exploit an additional information: the prior knowledge of a part of the transmitted block and track channel variations. [3] details several equalization and decoding schemes compatible with PRP-OFDM.

With PRP-OFDM, an estimate of the CIR can be derived by a simple averaging of the summation of the postfix with the head of the time domain OFDM received block [3]. The averaging is required in order to cancel the interference of the samples carrying useful information on the pseudo-random postfix. A practical trade-off needs to be established between the length of the averaging window and the resulting amount of residual interference impacting the channel estimation accuracy. For a large averaging window [6] shows that PRP-OFDM CIR estimation outperforms schemes relying on rotating pilot patterns interpolation in terms of mean-square-error (MSE) up to an SNR of approx. 15dB. When targeting higher SNRs (e.g. using high order constellations, 64QAM) the requirement in CIR MSE is such that the constraints put on the averaging window length lead to solutions which cannot be considered for implementation.

In order to solve this issue, this paper proposes an iterative CIR estimation scheme refining the first rough CIR estimates based on exploiting the outputs of a soft output decoder which makes PRP-OFDM suitable for high throughput systems. The added complexity introduced by this iterative data interference cancellation on the channel estimation can be mitigated when the system considered already implements an advance coding scheme such as turbo code which requires already the presence of a forward backward decoder in the receiver.

The paper is organized as follows. Section II settles the notations and defines the PRP-OFDM modulator. The new iterative channel estimation technique is compared to classical one and discussed in section III. Finally section IV provides simulation results.

## II. Notations and PRP-OFDM modulator

This section settles the baseband discrete-time block equivalent model of a $N$ carrier PRP-OFDM system. The $i$th $N \times 1$ input digital vector [2] $\tilde{\mathbf{s}}_N(i)$ is first modulated by the IFFT matrix $\mathbf{F}_N^H = \frac{1}{\sqrt{N}}\left(W_N^{ij}\right)^H, 0 \le i < N, 0 \le j < N$ and $W_N = e^{-j\frac{2\pi}{N}}$. Then, a deterministic postfix vector

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[2]Lower (upper) boldface symbols will be used for column vectors (matrices) sometimes with subscripts $N$, $D$ or $P$ emphasizing their sizes (for square matrices only); tilde will denote frequency domain quantities; argument $i$ will be used to index blocks of symbols; ${}^H$ (${}^T$) will denote Hermitian (Transpose).

$\mathbf{c}_D = (c_0, \ldots, c_{D-1})^T$ weighted by a pseudo random value $\alpha(i) \in \mathbb{C}, |\alpha(i)| = 1$ is appended to the IFFT outputs $\mathbf{s}_N(i)$. A pseudo random $\alpha(i)$ prevents the postfix time domain signal from being deterministic and avoids thus spectral peaks [3]. With $P = N + D$, the corresponding $P \times 1$ transmitted vector is $\mathbf{s}_P(i) = \mathbf{F}_{\text{ZP}}^H \tilde{\mathbf{s}}_N(i) + \alpha(i)\mathbf{c}_P$, where

$$\mathbf{F}_{\text{ZP}}^H = \left[\frac{\mathbf{I}_N}{\mathbf{0}_{D,N}}\right]_{P\times N} \mathbf{F}_N^H \quad \text{and} \quad \mathbf{c}_P = \left(\mathbf{0}_{1,N}\ \mathbf{c}_D^T\right)^T$$

The samples of $\mathbf{s}_P(i)$ are then sent sequentially through the channel modeled here as a $L$th-order FIR $H(z) = \sum_{n=0}^{L-1} h_n z^{-n}$ of impulse response $\mathbf{h}_P = (h_0, \cdots, h_{L-1})$. The OFDM system is designed such that the postfix duration exceeds the channel memory $L \le D$.

Let $\mathbf{H}_{\text{ISI}}(P)$ and $\mathbf{H}_{\text{IBI}}(P)$ be respectively the Toeplitz inferior and superior triangular matrices of first column: $[h_0, h_1, \cdots, h_{L-1}, 0, \rightarrow, 0]^T$ and first row $[0, \rightarrow, 0, h_{L-1}, \cdots, h_1]$. As already explained in [7], the channel convolution can be modeled by $\mathbf{r}_P(i) = \mathbf{H}_{\text{ISI}}\mathbf{s}_P(i) + \mathbf{H}_{\text{IBI}}\mathbf{s}_P(i-1) + \mathbf{n}_P(i)$. $\mathbf{H}_{\text{ISI}}(P)$ and $\mathbf{H}_{\text{IBI}}(P)$ represent respectively the intra and inter block interference. Since $\mathbf{s}_P(i) = \mathbf{F}_{\text{ZP}}^H \tilde{\mathbf{s}}_N(i) + \alpha(i)\mathbf{c}_P$, we have:

$$\mathbf{r}_P(i) = (\mathbf{H}_{\text{ISI}} + \beta_i \mathbf{H}_{\text{IBI}})\mathbf{s}_P(i) + \mathbf{n}_P(i)$$

where $\beta_i = \frac{\alpha(i-1)}{\alpha(i)}$ and $\mathbf{n}_P(i)$ is the $i$th AWGN vector of element variance $\sigma_n^2$. Note that $\mathbf{H}_{\beta_i} = (\mathbf{H}_{\text{ISI}} + \beta_i \mathbf{H}_{\text{IBI}})$ is pseudo circulant: i.e. a circulant matrix whose $(D-1)\times(D-1)$ upper triangular part is weighted by $\beta_i$.

The expression of the received block is thus:

$$\begin{aligned} \mathbf{r}_P(i) &= \mathbf{H}_{\beta_{\text{i}}}\left(\mathbf{F}_{\text{ZP}}^H \tilde{\mathbf{s}}_N(i) + \alpha(i)\mathbf{c}_P\right) + \mathbf{n}_P(i) \qquad (1) \\ &= \mathbf{H}_{\beta_{\text{i}}} \begin{pmatrix} \mathbf{F}_N^H \tilde{\mathbf{s}}_N(i) \\ \alpha(i)\mathbf{c}_D \end{pmatrix} + \mathbf{n}_P(i) \end{aligned}$$

Please note that equation (1) is quite generic and captures also the CP and ZP (Zero Padding) modulation schemes. Indeed ZP-OFDM corresponds to $\alpha(i) = 0$ and CP-OFDM is achieved for $\alpha(i) = 0$, $\beta_i = 1 \forall i$ and $\mathbf{F}_{\text{ZP}}^H$ is replaced by $\mathbf{F}_{\text{CP}}^H$, where

$$\mathbf{F}_{\text{CP}}^H = \left[\frac{\mathbf{0}_{D,N-D} \mid \mathbf{I}_D}{\mathbf{I}_N}\right]_{P\times N} \mathbf{F}_N^H.$$

With these notations, CIR estimation is discussed in the following.

## III. Channel Estimation

Below the standard low-complexity PRP-OFDM CIR estimation technique [3] based on interference suppression by mean value calculation is briefly recalled. Then a description of the proposed iterative scheme improving the CIR estimate MSE follows. All derivations are detailed in the static context, extension to mobility environment is possible applying the techniques presented in [4], [6].

### A. Standard channel estimation

Define $\mathbf{H}_{\text{CIR}}(D) := \mathbf{H}_{\text{ISI}}(D) + \mathbf{H}_{\text{IBI}}(D)$ as the $D\times D$ circulant channel matrix of first row $row_0(\mathbf{H}_{\text{CIR}}(D)) = [h_0, 0, \rightarrow, 0, h_{L-1}, \cdots, h_1]$. Note that $\mathbf{H}_{\text{ISI}}(D)$ and $\mathbf{H}_{\text{IBI}}(D)$ contain respectively the lower and upper triangular parts of $\mathbf{H}_{\text{CIR}}(D)$.

Denoting by $\mathbf{s}_N(i) := [s_0(i), \cdots, s_{N-1}(i)]^T$, splitting this vector in 2 parts: $\mathbf{s}_{N,0}(i) := [s_0(i), \cdots, s_{D-1}(i)]^T$, $\mathbf{s}_{N,1}(i) := [s_{N-D}(i), \cdots, s_{N-1}(i)]^T$, and performing the same operations for the noise vector: $\mathbf{n}_P(i) := [n_0(i), \cdots, n_{P-1}(i)]^T$, $\mathbf{n}_{D,0}(i) := [n_0(i), \cdots, n_{D-1}(i)]^T$, $\mathbf{n}_{D,1}(i) := [n_{P-D}(i), \cdots, n_{P-1}(i)]^T$, the received vector $\mathbf{r}_P(i)$ can be expressed as:

$$\mathbf{r}_P(i) = \begin{pmatrix} \mathbf{H}_{\text{ISI}}(D)\mathbf{s}_{N,0}(i) + \alpha(i-1)\mathbf{H}_{\text{IBI}}(D)\mathbf{c}_D + \mathbf{n}_{D,0} \\ \vdots \\ \mathbf{H}_{\text{IBI}}(D)\mathbf{s}_{N,1}(i) + \alpha(i)\mathbf{H}_{\text{ISI}}(D)\mathbf{c}_D + \mathbf{n}_{D,1} \end{pmatrix}. \qquad (2)$$

As usual the transmitted time domain signal $\mathbf{s}_N(i)$ is assumed zero-mean. Thus the first $D$ samples $\mathbf{r}_{P,0}(i)$ of $\mathbf{r}_P(i)$ and its last $D$ samples $\mathbf{r}_{P,1}(i)$ can be exploited very easily to retreive the channel matrices relying on the deterministic nature of the postfix as follows:

$$\begin{aligned} \hat{\mathbf{r}}_{c,0} &:= \mathrm{E}\left[\frac{\mathbf{r}_{P,0}(i)}{\alpha(i-1)}\right] = \mathbf{H}_{\text{IBI}}(D)\mathbf{c}_D, \\ \hat{\mathbf{r}}_{c,1} &:= \mathrm{E}\left[\frac{\mathbf{r}_{P,1}(i)}{\alpha(i)}\right] = \mathbf{H}_{\text{ISI}}(D)\mathbf{c}_D. \end{aligned} \qquad (3)$$

Since $\mathbf{H}_{\text{ISI}}(D) + \mathbf{H}_{\text{IBI}}(D) = \mathbf{H}_{\text{CIRC}}(D)$ is circulant and diagonalizable in the frequency domain $\mathbf{F}_D$ combining equations (3) and using the commutativity of the convolution yields:

$$\begin{aligned} \hat{\mathbf{r}}_c &:= \hat{\mathbf{r}}_{c,0} + \hat{\mathbf{r}}_{c,1} = \mathbf{H}_{\text{CIRC}}(D)\mathbf{c}_D \\ &= \mathbf{C}_D \mathbf{h}_D = \mathbf{F}_D^H \tilde{\mathbf{C}}_D \mathbf{F}_D \mathbf{h}_D, \end{aligned} \qquad (4)$$

where $\mathbf{C}_D$ is a $D \times D$ circulant matrix with first row $row_0(\mathbf{C}_D) := [c_0, c_{D-1}, c_{D-2}, \cdots, c_1]$ and $\tilde{\mathbf{C}}_D := \text{diag}\{\mathbf{F}_D \mathbf{c}_D\}$.

Since in practice the expectation $\mathrm{E}[\cdot]$ in (3) is approximated by a mean value calculation over a limited number $Z$ of symbols, we can model the estimation error as noise $\tilde{\mathbf{n}}_D$.

Assuming both the received OFDM time domain data samples and $\mathbf{n}_P$ to be Gaussian of respective covariances $\sigma_s^2 \mathbf{I}_N$ and $\sigma_n^2 \mathbf{I}_P$, the covariance of $\tilde{\mathbf{n}}_D$ is $\mathbf{R}_{\tilde{\mathbf{n}}_D} = \mathrm{E}\left[\tilde{\mathbf{n}}_D \tilde{\mathbf{n}}_D^H\right] = \frac{\sigma_s^2 + \sigma_n^2}{Z}\mathbf{I}_D$. Thus, an estimate of the CIR $\hat{\mathbf{h}}_D$ can be retrieved by either a ZF or MMSE approach [8].

### B. The new iterative channel estimation

The iterative estimation scheme presented here requires an initial CIR estimate which is for example obtained by the technique presented above.

The iterative CIR estimation is performed in several steps:

1) Initial CIR estimation: at iteration $k = 0$, perform an initial CIR estimation $\hat{\mathbf{h}}^0(i)$, for example as proposed in section III-A.
2) Increment iteration index: $k \leftarrow k + 1$
3) Perform FEC decoding based on latest CIR estimates $\hat{\mathbf{h}}^{k-1}(i)$: buffer the outputs of the soft-output decoder

which indicate the bit-probabilities of the $l$th encoded bit of the constellation on carrier $n$ of OFDM symbol $i$: $p_l^k(x_n(i))$ with $n \in [0, \cdots, N-1]$ and $l \in [0, \cdots, log_2(Q)-1]$; $Q$ is the constellation order.

4) Interference estimation: as detailed in appendix I, the interference estimation $\mathbf{u}_P^k(i)$ from OFDM data symbol $i$ is generated based on the bit-probabilities $p_l^k(x_n(i))$ and the latest CIR estimates $\hat{\mathbf{h}}^{k-1}(i)$ as given by theorem 1.1.
5) Interference suppression: subtract estimated interference from received vector $\mathbf{r}_P(i)$ and form a new observation vector: $\bar{\mathbf{r}}_P^k(i) = \mathbf{r}_P(i) - \mathbf{u}_P^k(i)$.
6) CIR estimation: derive a new CIR estimate $\hat{\mathbf{h}}^k(i)$ from $\bar{\mathbf{r}}_P^k(i)$ e.g. as proposed in section III-A. The resulting $\hat{\mathbf{h}}^k(i)$ yields to a more accurate estimate since interference of the OFDM data symbols on the postfix convolved by the channel has been reduced.
7) Iterate: until a given performance criterion is met go to step 2.

The iterative CIR estimation is compatible to any FEC decoder which delivers at its output bit-probabilities of encoded information bits among which are the SOVA (Soft-Output-Viterbi-Algorithm) decoders and forward backward algorithm. If such a decoder is applied for the sake of CIR estimation only, the complexity increase is considerable. However, if the proposed technique is used in a system where iterative decoding is used anyhow (e.g. in the context of Turbo Codes, etc.), the additional complexity can be considered for implementation.

## IV. Simulation Results

In order to illustrate the performances of our approach, simulations have been performed in the IEEE802.11a [1] WLAN context: a $N = 64$ carrier 20MHz bandwidth broadband wireless system operating in the 5.2GHz band using a 16 sample postfix. The CP-OFDM modulator is replaced by a PRP-OFDM modulator. A rate $R = 1/2$, constraint length $K = 7$ Convolutional Code (CC) (o171/o133) is used before bit interleaving followed by 64QAM constellation mapping.

Monte Carlo simulations are run and averaged over 2500 realizations of a normalized BRAN-A [9] frequency selective channel without Doppler in order to obtain BER curves.

Based on a SOVA decoder, figure 1 illustrates for a fixed carrier-over-interference (C/I) ratio of $C/I = 24dB$ that the MSE of the CIR is decreased by approx. 12dB after three iterations using the new algorithm proposed in section III-B compared to the initial estimates obtained by the algorithm proposed in section III-A. This gain varies only slightly with the size of the observation window for the mean-value calculation *postfix convolved by CIR plus noise*. The BER results (of decoded bits) for a mean-value calculation window size of 30 and 40 symbols are given by Figure 2 and Figure 3 respectively. For a mean-value calculation over 30 symbols, the BER performance starts to saturate for SNRs above approx. 20dB between a BER of $10^{-4}$ and $10^{-5}$. This effect is no longer observed for a 40 symbols window size within the considered BER range above $10^{-5}$. Compared to an IEEE802.11a CP-OFDM based implementation with channel estimation over two learning symbols, an average gain below 0.5dB is obtained for PRP-OFDM with mean-value calculation over 30 symbols and approx. a 1dB gain for PRP-OFDM with mean-value calculation over 40 symbols when iterating.

Note that the short size of the mean-value calculation window also makes the proposed scheme applicable in high Doppler scenarios and to packet based transmission schemes where a packet contains a small number of OFDM symbols.

## V. Conclusion

A new iterative interference cancellation scheme for PRP-OFDM based systems has been proposed. In a typical example the MSE of the resulting CIR estimated is improved by approx. 12dB over three iterations. This makes PRP-OFDM modulators applicable to higher order constellations, e.g. 64QAM, etc. For the reasons given in section IV, the proposed scheme can be applied in high mobility scenarios without losing throughput nor spectral efficiency compared to CP-OFDM systems designed for a static environment, since no additional redundancy in terms of pilot tones, learning symbols, etc. is necessary.

## Appendix I
## Interference Estimation

Let describe in this section the details of the proposed interference estimation at iteration $k$.

*Theorem 1.1:* Define the last CIR estimated $\hat{\mathbf{h}}^{k-1}(i)$ represented by matrix $\mathbf{H}_{\beta_\mathrm{i}}^{k-1}(i)$ multiplication. Denote by $\tilde{\mathbf{y}}_N^{k-1}(i) = \tilde{\mathbf{s}}_N(i) + \tilde{\mathbf{w}}_N^{k-1}$ the frequency domain equalized vector $\mathbf{r}_P(i)$ ($\tilde{\mathbf{w}}_N^{k-1}$ representing the residual error) preformed with the CIR estimate $\hat{\mathbf{h}}^{k-1}(i)$ of the previous step. The optimum time domain interference estimate in the minimum MSE sense is thus given by

$$\mathbf{u}_P^k(i) = \sum_{\mathbf{a}_N \in [a_0, \cdots, a_{Q-1}]^N} p(\tilde{\mathbf{s}}_N(i) = \mathbf{a}_N | \tilde{\mathbf{y}}_N^{k-1}(i)) \mathbf{H}_{\beta_\mathrm{i}}^{k-1} \mathbf{F}_{\mathrm{ZP}}^H \mathbf{a}_N \tag{5}$$

$\{a_n \in \mathbb{C}, n \in [0, \cdots, Q-1]\}$ is the set of constellation symbols (alphabet) and $Q$ the constellation order.□

**Practical aspects:**

The expression $p(\tilde{\mathbf{s}}_N(i) = \mathbf{a}_N | \tilde{\mathbf{y}}_N^{k-1}(i))$ is calculated using Bayes' rule:

$$p(\tilde{\mathbf{s}}_N(i) = \mathbf{a}_N | \tilde{\mathbf{y}}_N^{k-1}(i)) = \frac{p(\tilde{\mathbf{y}}_N^{k-1}(i) | \tilde{\mathbf{s}}_N(i) = \mathbf{a}_N) p(\tilde{\mathbf{s}}_N(i) = \mathbf{a}_N)}{p(\tilde{\mathbf{y}}_N^{k-1}(i))} \tag{6}$$

$p(\tilde{\mathbf{s}}_N(i) = \mathbf{a}_N) = \prod_{n=0}^{N-1} p(\tilde{s}_n(i) = a_n)$ is obtained by exploiting the bit-probabilities $b_l(a_n)$ of the soft-decoder outputs: $p(\tilde{s}_n(i) = a_n) = \prod_{l=0}^{log_2(Q)-1} p(b_l(a_n))$ assuming that the bits are independent. This property is usually assured by a large interleaver. $p(\tilde{\mathbf{y}}_N^{k-1}(i) | \tilde{\mathbf{s}}_N(i) = \mathbf{a}_N)$ is given by a multivariate Gaussian probability density function (PDF) with $\mathbf{R}_{\tilde{\mathbf{w}}_N^{k-1},\tilde{\mathbf{w}}_N^{k-1}} = \mathrm{E}[\tilde{\mathbf{w}}_N^{k-1}(\tilde{\mathbf{w}}_N^{k-1})^H]$:

$$p(\tilde{\mathbf{y}}_N^{k-1}(i)|\tilde{\mathbf{s}}_N(i) = \mathbf{a}_N) = \pi^{-N}\det{}^{-1}\left\{\mathbf{R}_{\tilde{\mathbf{w}}_N^{k-1},\tilde{\mathbf{w}}_N^{k-1}}\right\}$$
$$\exp\left\{-(\tilde{\mathbf{y}}_N^{k-1}(i) - \mathbf{a}_N)^H \mathbf{R}^{-1}_{\tilde{\mathbf{w}}_N^{k-1},\tilde{\mathbf{w}}_N^{k-1}}(\tilde{\mathbf{y}}_N^{k-1}(i) - \mathbf{a}_N)\right\}$$

The expression $p(\tilde{\mathbf{y}}_N^{k-1}(i))$ is calculated according to (6) by exploiting $\sum_{\mathbf{a}\in[a_0,\cdots,a_{Q-1}]^N} p(\tilde{\mathbf{s}}_N(i) = \mathbf{a}_N|\tilde{\mathbf{y}}_N^{k-1}(i)) = 1$.

If $\mathbf{R}_{\tilde{\mathbf{w}}_N^{k-1},\tilde{\mathbf{w}}_N^{k-1}}$ is diagonal (or approximated by a matrix containing its diagonal elements only), (5) can be considerably simplified, since $p(\tilde{\mathbf{s}}_N(i) = \mathbf{a}_N|\tilde{\mathbf{y}}_N^{k-1}(i)) = \prod_{n=0}^{N-1} p(\tilde{s}_n = a_n|\tilde{y}_n^{k-1}(i))$:

$$\mathbf{u}_P(i) = \sum_{n=0}^{N-1} \sum_{a_n\in[a_0,\cdots,a_{Q-1}]} p(\tilde{s}_n = a_n|\tilde{y}_n^{k-1}(i))\mathbf{H}_{\beta_\mathbf{i}}^{k-1}\mathbf{F}_{\mathrm{ZP}}^H\mathbf{a}_N^{(n)}$$

with $\mathbf{a}_N^{(n)} = (0,\cdots,0,a_n,0,\cdots,0)^T$ is derived from vector $\mathbf{a}_N$ in which only the $n$th element is non-zero.

**Proof of theorem 1.1:**

Define $\check{\mathbf{r}}_P^k(i) = \mathbf{r}_P(i) - \mathbf{H}_{\beta_\mathbf{i}}^{k-1}(i)\alpha(i)\mathbf{c}_P$ as the received vector after subtraction of the zero-forcing PRP interference estimate based on the previous CIR estimate $\mathbf{H}_{\beta_\mathbf{i}}^{k-1}(i)$. Assuming that $\mathbf{u}_P^k(i)$ is a vector used in order to reduce the interference onto the postfix convolved by the channel in $\mathbf{r}_P(i)$, the remaining total square error is given by

$$\epsilon^2(i) = \sum_{\mathbf{a}_N\in[a_0,\cdots,a_{Q-1}]^N} p(\tilde{\mathbf{s}}_N(i) = \mathbf{a}_N|\tilde{\mathbf{y}}_N^{k-1}(i))$$
$$\mathrm{E}\,\tilde{r}\left\{\left(\check{\mathbf{r}}_P(\tilde{\mathbf{s}}_N(i) = \mathbf{a}_N) - \mathbf{u}_P^k(i)\right)\left(\check{\mathbf{r}}_P(\tilde{\mathbf{s}}_N(i) = \mathbf{a}_N) - \mathbf{u}_P^k(i)\right)^H\right\}$$

where $\tilde{r}\{\cdot\}$ is the trace matrix operator. The optimum $\mathbf{u}_P^k(i)$ is found by setting

$$\frac{\partial\epsilon^2(i)}{\partial(\mathbf{u}_P^k(i))^\star} = \sum_{\mathbf{a}_N\in[a_0,\cdots,a_{Q-1}]^N} p(\tilde{\mathbf{s}}_N(i) = \mathbf{a}_N|\tilde{\mathbf{y}}_P^{k-1}(i))$$
$$(\mathbf{u}_P^k(i) - \mathbf{H}_{\beta_\mathbf{i}}^{k-1}\mathbf{F}_{\mathrm{ZP}}^H\mathbf{a}_N) = 0$$

which leads to the expression given by theorem 1.1. Q.E.D.

## References


[1] IEEE 802.11a. Part 11:Wireless LAN Medium Access Control (MAC) and Physical Layer specifications (PHY) - High Speed Physical Layer in the 5GHz band. IEEE Std 802.11a-1999, IEEE Standards Department, New York, January 1999.

[2] European Telecommunications Standard . "Radio broadcast systems: Digital audio broadcasting (DAB) to mobile, portable, and fixed receivers". Technical report, preETS 300 401 , March 1994.

[3] M. Muck, M. de Courville, M. Debbah, and P. Duhamel. A Pseudo Random Postfix OFDM modulator and inherent channel estimation techniques. In *GLOBECOM conference records*, San Francisco, USA, December 2003.

[4] Markus Muck, Alexandre Ribeiro Dias, Marc de Courville, and Pierre Duhamel. A Pseudo Random Postfix OFDM based modulator for multiple antennae systems. In *Proceedings of the Int. Conf. on Communications*, 2004.

[5] Markus Muck, Marc de Courville, and Pierre Duhamel. Postfix Design for Pseudo Random Postfix OFDM Modulators. In *International OFDM Workshop*, Dresden, Germany, September 2004.

[6] Kai-Uwe Schmidt, Markus Muck, Jens Schoenthier, and Marc de Courville. Time or Frequency Domain Pilots for Channel Tracking in Wireless OFDM Systems? In *International OFDM Workshop*, Dresden, Germany, September 2004.

[7] A. Akansu, P. Duhamel, X. Lin, and M. de Courville. Orthogonal Transmultiplexers in Communication : A Review. *IEEE Trans. on Signal Processing*, 463(4):979–995, April 1998.

[8] S. Haykin. *Adaptive Filter Theory*. Prentice-Hall, Englewood Cliffs, New Jersey, USA, 1986.

[9] ETSI Normalization Committee. Channel Models for HIPERLAN/2 in different indoor scenarios. Norme ETSI, document 3ERI085B, European Telecommunications Standards Institute, Sophia-Antipolis, Valbonne, France, 1998.


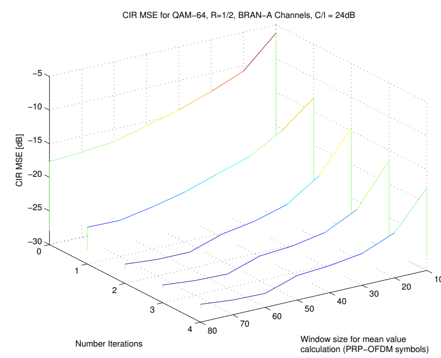


Fig. 1. CIR MSE for 64QAM, BRAN-A, C/I=24dB.

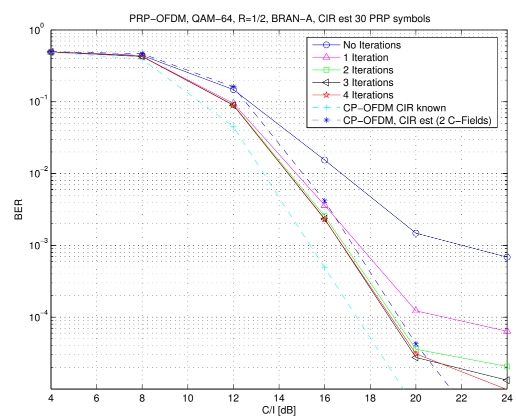


Fig. 2. Simulation results for 64QAM, R=1/2, BRAN-A, CIR estimation over 30 PRP-OFDM symbols.

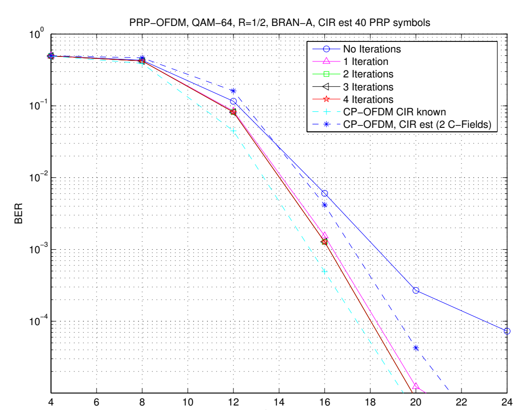


Fig. 3. Simulation results for 64QAM, R=1/2, BRAN-A, CIR estimation over 40 PRP-OFDM symbols.