EFFECT OF CHANNEL ESTIMATION ERRORS ON THE PERFORMANCE OF MMSE-SIC WITH EQUAL BER POWER CONTROL IN UPLINK MC-CDMA

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ABSTRACT

The performance of MC-CDMA is limited by multiple access interference (MAI). To mitigate this problem, equal BER power control (PC) was proposed for MMSE-SIC receiver, which provides a powerful solution for MAI suppression [1], wherein, the power distribution on different users was derived under the assumption of perfect channel state information (CSI) at the receiver. In practice, CSI is obtained from channel estimation, in which errors are inevitable. Therefore, the analysis of the robustness of the MMSE-SIC with the equal BER PC to channel estimation errors (CEE) is of great importance. In this paper, a method of second-order approximation is applied to estimate the mean excess MSE (MEMSE) of different users in a given decision order. The accuracy of the approximation is confirmed by simulation results. Furthermore, it is interesting to find out that the MMSE-SIC receiver with the equal BER PC presents significant robustness to CEE.

1. INTRODUCTION

Due to the robustness to frequency-selective fading and flexibility of handling multiple data rates, MC-CDMA has become a promising candidate for future wireless multimedia communications [2]. However, its performance is limited by multiple access interference (MAI). To mitigate this problem, equal BER power control (PC) algorithm was proposed for MMSE-SIC receiver, which suppresses MAI effectively, resulting in a performance of a fully-loaded system close to the single user bound (SUB) [1]. The power distribution for different users was derived under the assumption of perfect channel state information (CSI) at the receiver. In practise, CSI is obtained from estimation, thus, channel estimation errors (CEE) are inevitable. Therefore, it is very important to analyze of the robustness of the MMSE-SIC with the equal BER PC to CEE.

In this paper, a method of second-order approximation is applied to estimate the mean excess MSE (MEMSE) of different users in a given decision order [3]. A simple expression of the approximate MEMSE is obtained and its accuracy is confirmed by simulation results. Moreover, it is found that under small CEE, the equal BER PC still benefits MMSE-SIC, making it robust to CEE.

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2. MMSE-SIC WITH EQUAL BER PC

In quasi-synchronous uplink MC-CDMA, we assume total N sub-carriers and K active users. The k^{th} user is assigned with a spreading code $\mathbf{c}_k$ of length N.[1] We consider a frequency-selective Rayleigh fading channel with correlated fading across sub-carriers. By using a cyclic prefix of proper length, flat fading can be obtained over each sub-carrier. With the assumption of invariant channel response during each OFDM symbol interval, the channel for the k^{th} user, can be represented by an $(N \times 1)$ vector, $\mathbf{h}_k = [h_{k,1}, h_{k,2}, \cdots, h_{k,N}]^T$, where each element is a complex Gaussian random variable with unit variance.

At the receiver, the output of the DFT during the i^{th} OFDM symbol interval can be expressed in a compact matrix form as

$$\mathbf{x}(i) = \mathcal{H}\mathbf{b}(i) + \boldsymbol{\eta}(i). \tag{1}$$

In (1), $\mathcal{H} = \widetilde{\mathbf{C}} \cdot \mathbf{A}$, where $\widetilde{\mathbf{C}} = [\mathbf{h}_1 \odot \mathbf{c}_1, \mathbf{h}_2 \odot \mathbf{c}_2, \cdots, \mathbf{h}_K \odot \mathbf{c}_K]$ denotes the channel-modified spreading code matrix ($\odot$ denotes element-wise multiplication) and $\mathbf{A} = \mathbf{diag}(a_1, a_2, \cdots, a_K)$ is a diagonal matrix containing the transmit amplitudes of all users; $\mathbf{b}(i) = [b_1(i), b_2(i), \cdots, b_K(i)]^T$ contains all users' transmitted symbols, which are assumed BPSK modulated with normalized power; The $(N \times 1)$ white Gaussian noise vector $\boldsymbol{\eta}(i)$ has zero mean and covariance matrix $\sigma_n^2 \mathbf{I}_N$, where $\mathbf{I}_N$ denotes an $N \times N$ identity matrix.

The MMSE-SIC can be performed using Cholesky factorization (CF) of the positive definite matrix $\mathbf{R}_m = \mathbf{R}_c + \sigma_n^2 \mathbf{A}^{-2}$, where $\mathbf{R}_c = \widetilde{\mathbf{C}}^H \widetilde{\mathbf{C}}$. The CF results in $\mathbf{R}_m = \mathbf{\Gamma}^H \mathbf{D}^2 \mathbf{\Gamma}$, with $\mathbf{\Gamma}$ upper triangular and monic (having all ones along the diagonal) and $\mathbf{D}^2 = \mathbf{diag}\left(d_1^2, d_2^2, \cdots, d_K^2\right)$, having all positive elements on its diagonal.

By assuming perfect SIC (no error propagations), to achieve equal BER after SIC for different users, the power allocation a_k^2 was derived in [1], which can be expressed in a successive form as

$$\begin{cases} a_k^2 = \frac{\lambda - \sigma_n^2}{r_{1,1}} & (k=1) \\ a_k^2 = \frac{\lambda - \sigma_n^2}{r_{k,k} - \sum_{j=1}^{k-1} \left|\gamma_{j,k}\right|^2 a_j^{-2} \lambda} & (k = 2, \cdots, K) \end{cases}, \tag{2}$$

where $\lambda \triangleq a_k^2 d_k^2$, $r_{k,k}$ and $\gamma_{j,k}$ denotes the $(k,k)^{th}$ and $(j,k)^{th}$ element of $\mathbf{R}_c$ and $\mathbf{\Gamma}$, respectively. With perfect CSI, $\mathbf{R}_c$ is completely known, while $\gamma_{j,k}$ can be solved successively.

Since $a_k^2 \in [0, +\infty)$ $(k = 1, 2, \cdots, K)$ are proved to be monotonically increasing with $\lambda \in \left[\sigma_n^2, +\infty\right)$, then under a short-term

[1] The spreading codes are assumed linear independent, i.e., $K \leq N$.

power constraint $\overline{\mathcal{P}} \in (0, +\infty)$, by using a simple search algorithm, a unique power distribution $\mathbf{a}^{2\dagger} = \left[a_1^{2\dagger}, a_2^{2\dagger}, \cdots, a_K^{2\dagger}\right]$ can be found, which satisfies $\overline{\mathcal{P}} = \frac{1}{K}\sum_{k=1}^{K} a_k^{2\dagger}$. With error-free feedback of the calculated power allocation from the receiver to the transmitter, the k^{th} user will transmit with power $a_k^{2\dagger}$, which benefits SIC significantly.

3. MMSE UNDER PERFECT CSI

In this section, the expression of the MMSE under the perfect CSI will be first introduced. With the assumption of perfect cancellation, under SIC, the decision error of the k^{th} user can be expressed as[2]

$$\begin{aligned} e_k &= b_k - \left(\mathbf{w}_k^H \mathbf{x} - \sum_{i=1}^{K-k} f_{K-i+1} b_{K-i+1}\right) \\ &= \widetilde{\mathbf{f}}_k^H \mathbf{b} - \mathbf{w}_k^H \mathbf{x}. \end{aligned} \quad (3)$$

In the above equation, $\mathbf{w}_k \triangleq [w_{k,1}, w_{k,2}, \cdots, w_{k,N}]^H$ and $\widetilde{\mathbf{f}}_k \triangleq \left[\mathbf{0}_{(k-1)\times 1}^T, \mathbf{f}_k^H\right]^H$ represent the feedforward and feedback vectors of the k^{th} user, with $\mathbf{f}_k = [1, f_{k+1}, \cdots, f_K]^H$ and $\mathbf{0}_{m\times n}$ denoting an $(m \times n)$ zero matrix. The MSE of the k^{th} user can be expressed as

$$\begin{aligned} \mathbf{MSE}_k &\triangleq E\left[e_k \cdot e_k^*\right] \\ &= \widetilde{\mathbf{f}}_k^H \mathbf{R_{bb}} \widetilde{\mathbf{f}}_k - \widetilde{\mathbf{f}}_k^H \mathbf{R_{bx}} \mathbf{w}_k \\ &\quad - \mathbf{w}_k^H \mathbf{R_{xb}} \widetilde{\mathbf{f}}_k + \mathbf{w}_k^H \mathbf{R_{xx}} \mathbf{w}_k, \end{aligned} \quad (4)$$

where $\mathbf{R_{bb}} = E\left[\mathbf{b}\cdot\mathbf{b}^H\right] = \mathbf{I}_K$, $\mathbf{R_{bx}} = E\left[\mathbf{b}\cdot\mathbf{x}^H\right] = \mathcal{H}^H$, $\mathbf{R_{xb}} = E\left[\mathbf{x}\cdot\mathbf{b}^H\right] = \mathbf{R_{bx}}^H$ and $\mathbf{R_{xx}} = E\left[\mathbf{x}\cdot\mathbf{x}^H\right] = \mathcal{H}\mathcal{H}^H + \boldsymbol{\sigma}_n^2 \mathbf{I}_K$.

To minimize the $\mathbf{MSE}_k$, the optimal forward and feedback vectors are [4]

$$\mathbf{w}_{k,o} = \mathbf{R_{xx}}^{-1} \mathbf{R_{xb}} \widetilde{\mathbf{f}}_{k,o} \quad (5)$$

and

$$\mathbf{f}_{k,o} = \frac{\mathbf{R}_{\Delta,k}^{-1} \mathbf{u}_k}{\mathbf{u}_k^T \mathbf{R}_{\Delta,k}^{-1} \mathbf{u}_k}, \quad (6)$$

where $\mathbf{R}_{\Delta,k} = \begin{bmatrix} \mathbf{0}_{(K-k+1)\times(k-1)} \\ \mathbf{I}_{K-k+1} \end{bmatrix}^H \mathbf{R} \begin{bmatrix} \mathbf{0}_{(K-k+1)\times(k-1)} \\ \mathbf{I}_{K-k+1} \end{bmatrix}$ with $\mathbf{R} = \mathbf{R_{bb}} - \mathbf{R_{bx}} \mathbf{R_{xx}}^{-1} \mathbf{R_{xb}}$ and $\mathbf{u}_k = \left[1, \mathbf{0}_{(K-k)\times 1}^T\right]^T$. The resulting MMSE of the k^{th} user can be expressed as

$$\mathbf{MMSE}_k = \widetilde{\mathbf{f}}_{k,o}^H \mathbf{R} \widetilde{\mathbf{f}}_{k,o} = \mathbf{f}_{k,o}^H \mathbf{R}_{\Delta,k} \mathbf{f}_{k,o} = \frac{1}{\mathbf{u}_k^T \mathbf{R}_{\Delta,k}^{-1} \mathbf{u}_k} \quad (7)$$

with $\widetilde{\mathbf{f}}_{k,o} \triangleq \left[\mathbf{0}_{(k-1)\times 1}^T, \mathbf{f}_{k,o}^H\right]^H$.

4. MEAN EXCESS MSE (MEMSE) UNDER CEE

4.1. Excess MSE (EMSE) under a given channel realization with a certain CEE

After a certain channel estimation procedure, estimates $\widehat{\mathbf{h}}_k$ can be obtained for the k^{th} user. Then, the CEE of the k^{th} user is denoted as $\Delta\mathbf{h}_k \triangleq \widehat{\mathbf{h}}_k - \mathbf{h}_k$ with the assumption of $\|\Delta\mathbf{h}_k\|_2 \ll \|\mathbf{h}_k\|_2$ $(k = 1, 2, \cdots, K)$. Under CEE, the "equal BER" power distribution results in a distribution $\widetilde{\mathbf{A}}^\dagger$, different from $\mathbf{A}^\dagger$, that is used under the perfect CSI. Thus, $\widehat{\mathcal{H}} \triangleq \widehat{\mathbf{C}} \cdot \widetilde{\mathbf{A}}^\dagger$, where $\widehat{\mathbf{C}} = \left[\widehat{\mathbf{h}}_1 \odot \mathbf{c}_1, \cdots, \widehat{\mathbf{h}}_K \odot \mathbf{c}_K\right]$ and $\widetilde{\mathbf{A}}^\dagger = \mathbf{diag}\left(\widetilde{a}_1^\dagger, \widetilde{a}_2^\dagger, \cdots, \widetilde{a}_K^\dagger\right)$ with $\widetilde{a}_k^\dagger = a_k^\dagger + \Delta a_k$ (Δa_k denotes the amplitude difference for the k^{th} user.) By defining $\Delta\mathcal{H} \triangleq \widehat{\mathcal{H}} - \mathcal{H}$, it is clear that $\Delta\mathcal{H}_{i,j} = \widetilde{a}_i^\dagger c_{j,i} \Delta h_{j,i} + \Delta a_i c_{j,i} h_{j,i}$ $(i = 1, 2, \cdots, N$ and $j = 1, 2, \cdots, K)$.

In this case, the resulting "optimal" feedforward and feedback vectors can be expressed as

$$\widehat{\mathbf{w}}_{k,o} = \widehat{\mathbf{R}}_{\mathbf{xx}}^{-1} \widehat{\mathbf{R}}_{\mathbf{xb}} \widehat{\widetilde{\mathbf{f}}}_{k,o} \quad (8)$$

and

$$\widehat{\mathbf{f}}_{k,o} = \frac{\widehat{\mathbf{R}}_{\Delta,k}^{-1} \mathbf{u}_k}{\mathbf{u}_k^T \widehat{\mathbf{R}}_{\Delta,k}^{-1} \mathbf{u}_k} \quad (9)$$

where

$$\begin{aligned} \widehat{\mathbf{R}}_{\mathbf{xx}} &= \widehat{\mathcal{H}}\widehat{\mathcal{H}}^H + \boldsymbol{\sigma}_n^2 \mathbf{I}_K \\ &\approx \underbrace{\mathcal{H}\mathcal{H}^H + \boldsymbol{\sigma}_n^2 \mathbf{I}_K}_{\mathbf{R_{xx}}} + \underbrace{\mathcal{H}\Delta\mathcal{H}^H + \Delta\mathcal{H}\mathcal{H}^H}_{\Delta\mathbf{R_{xx}}}, \end{aligned} \quad (10)$$

$$\begin{aligned} \widehat{\mathbf{R}}_{\mathbf{bx}} &= \widehat{\mathcal{H}}^H \\ &= \underbrace{\mathcal{H}^H}_{\mathbf{R_{bx}}} + \underbrace{\Delta\mathcal{H}^H}_{\Delta\mathbf{R_{bx}}}, \end{aligned} \quad (11)$$

$$\widehat{\mathbf{R}}_{\mathbf{xb}} = \widehat{\mathbf{R}}_{\mathbf{bx}}^H \quad (12)$$

and

$$\widehat{\mathbf{R}}_{\Delta,k} = \left[\mathbf{0}_{(K-k+1)\times(k-1)}, \mathbf{I}_{K-k+1}\right] \widehat{\mathbf{R}} \begin{bmatrix} \mathbf{0}_{(k-1)\times(K-k+1)} \\ \mathbf{I}_{K-k+1} \end{bmatrix} \quad (13)$$

with $\widehat{\mathbf{R}} = \widehat{\mathbf{R}}_{\mathbf{bb}} - \widehat{\mathbf{R}}_{\mathbf{bx}} \widehat{\mathbf{R}}_{\mathbf{xx}}^{-1} \widehat{\mathbf{R}}_{\mathbf{xb}}$. Similarly, by defining $\widehat{\mathbf{R}}_{\Delta,k} = \mathbf{R}_{\Delta,k} + \Delta\mathbf{R}_{\Delta,k}$ and $\widehat{\mathbf{R}} = \mathbf{R} + \Delta\mathbf{R}$ and using the well-known first-order expansion

$$(\mathbf{X} + \Delta\mathbf{X})^{-1} \approx \mathbf{X}^{-1} - \mathbf{X}^{-1} \Delta\mathbf{X} \mathbf{X}^{-1}, \quad (14)$$

it can be shown that

$$\Delta\mathbf{R}_{\Delta,k} = \left[\mathbf{0}_{(K-k+1)\times(k-1)}, \mathbf{I}_{K-k+1}\right] \Delta\mathbf{R} \begin{bmatrix} \mathbf{0}_{(k-1)\times(K-k+1)} \\ \mathbf{I}_{K-k+1} \end{bmatrix} \quad (15)$$

with

$$\Delta\mathbf{R} = -\mathbf{R_{bx}} \mathbf{R_{xx}}^{-1} \Delta\mathcal{H} \mathbf{R} - \mathbf{R} \Delta\mathcal{H}^H \mathbf{R_{xx}}^{-1} \mathbf{R_{xb}}. \quad (16)$$

Assuming perfect SIC, then under given CEE, the MSE of the k^{th} user can be expressed as

$$\begin{aligned} \widehat{\mathbf{MSE}}_k &= \widehat{\widetilde{\mathbf{f}}}_{k,o}^H \mathbf{R_{bb}} \widehat{\widetilde{\mathbf{f}}}_{k,o} - \widehat{\widetilde{\mathbf{f}}}_{k,o}^H \mathbf{R_{bx}} \widehat{\mathbf{w}}_{k,o} \\ &\quad - \widehat{\mathbf{w}}_{k,o}^H \mathbf{R_{xb}} \widehat{\widetilde{\mathbf{f}}}_{k,o} + \widehat{\mathbf{w}}_{k,o}^H \mathbf{R_{xx}} \widehat{\mathbf{w}}_{k,o}. \end{aligned} \quad (17)$$

With the assumption of small CEE, by defining the first-order perturbations as $\Delta\mathbf{w}_{k,o} = \widehat{\mathbf{w}}_{k,o} - \mathbf{w}_{k,o}$ and $\Delta\widetilde{\mathbf{f}}_{k,o} = \widehat{\widetilde{\mathbf{f}}}_{k,o} - \widetilde{\mathbf{f}}_{k,o}$, the $\widehat{\mathbf{MSE}}_k$ can be approximated as [3]

$$\begin{aligned} \widehat{\mathbf{MSE}}_k \approx\; & \mathbf{MMSE}_k + \text{first-order error terms} \\ & + \underbrace{\begin{bmatrix} \Delta\widetilde{\mathbf{f}}_{k,o}^H & \Delta\mathbf{w}_{k,o}^H \end{bmatrix} \boldsymbol{\Lambda} \begin{bmatrix} \Delta\widetilde{\mathbf{f}}_{k,o} \\ \Delta\mathbf{w}_{k,o} \end{bmatrix}}_{\text{sec ond-order error term}}, \end{aligned} \quad (18)$$

[2]Notice that the symbol with larger index will be detected earlier. Also, for simplicity, the time index i is omitted.

where $\Lambda = \begin{bmatrix} \mathbf{R_{bb}} & -\mathbf{R_{bx}} \\ -\mathbf{R_{xb}} & \mathbf{R_{xx}} \end{bmatrix}$. In equation (18), the "first-order error terms" is identically zero due to the optimality of the point $\left(\mathbf{w}_{k,o}^H, \widetilde{\mathbf{f}}_{k,o}^H\right)$. Therefore, the excess MSE (EMSE) under CEE can be estimated by the second-order error term (SOT), which is termed as a second-order approximation [3]. With mathematical manipulations, the SOT of the k^{th} user, can be expressed as

$$\begin{aligned} \mathbf{SOT}_k &= \left(\widetilde{\mathbf{f}}_{k,o}^H \Delta\mathbf{R_{bx}} - \mathbf{w}_{k,o}^H \Delta\mathbf{R_{xx}}\right) \mathbf{R_{xx}}^{-1} \cdot \\ &\quad \left(\Delta\mathbf{R_{xb}} \widetilde{\mathbf{f}}_{k,o}^H - \Delta\mathbf{R_{xx}} \mathbf{w}_{k,o}\right) \\ &\quad + \mathbf{f}_{k,o}^H \Delta\mathbf{R}_{\Delta,k} \mathbf{R}_{\Delta,k}^{-1} \Delta\mathbf{R}_{\Delta,k} \mathbf{f}_{k,o} \\ &\quad - \frac{\mathbf{f}_{k,o}^H \Delta\mathbf{R}_{\Delta,k} \mathbf{f}_{k,o}}{\mathbf{MMSE}_k}. \end{aligned} \quad (19)$$

With the above equation, the EMSE of the k^{th} user under a given channel realization and a certain small CEE can be approximated by $\mathbf{SOT}_k$.

4.2. MEMSE under a given channel realization

To derive a close form of the MEMSE (i.e., averaged over all CEE) under a given channel realization, a similar mathematical manipulation is used as proposed in [3].

By defining $\Delta\mathfrak{h} \triangleq \left[[\Delta\mathcal{H}]_1^H, [\Delta\mathcal{H}]_2^H, \cdots, [\Delta\mathcal{H}]_K^H\right]^H$, where $[\Delta\mathcal{H}]_j$ denotes the j^{th} column of $\Delta\mathcal{H}$, the covariance matrix of $\Delta\mathfrak{h}$ can be expressed as

$$\mathbf{R}_{\Delta\mathfrak{h}} = E\left[\Delta\mathfrak{h}\Delta\mathfrak{h}^H\right]. \quad (20)$$

If a matrix $\mathcal{S}_k$ could be found, which satisfies the following equation

$$\mathbf{SOT}_k = \Delta\mathfrak{h}^H \mathcal{S}_k \Delta\mathfrak{h} = tr\left(\Delta\mathfrak{h}^H \mathcal{S}_k \Delta\mathfrak{h}\right), \quad (21)$$

then, using the well-known property $Tr(\mathbf{AB}) = Tr(\mathbf{BA})$, the following relationship between the MEMSE and $\mathbf{R}_{\Delta\mathfrak{h}}$ can be constructed, which is given by

$$E[\mathbf{SOT}_k] = E_{\Delta\mathfrak{h}}\left[tr\left(\Delta\mathfrak{h}^H \mathcal{S}_k \Delta\mathfrak{h}\right)\right] = Tr\left(\mathcal{S}_k \mathbf{R}_{\Delta\mathfrak{h}}\right). \quad (22)$$

To solve $\mathcal{S}_k$, a new matrix $\mathcal{W}_{k,o}$ is introduced, which satisfies

$$\mathbf{w}_{k,o}^H \Delta\mathcal{H} = \Delta\mathfrak{h}^H \mathcal{W}_{k,o}. \quad (23)$$

It can be shown that $\mathcal{W}_{k,o}$ is a $(KN \times K)$ matrix, which is given by

$$\mathcal{W}_{l,o} = \begin{bmatrix} \mathbf{w}_{k,o}^* & \mathbf{0}_{N\times1} & \cdots & \mathbf{0}_{N\times1} \\ \mathbf{0}_{N\times1} & \mathbf{w}_{k,o}^* & \cdots & \mathbf{0}_{N\times1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}_{N\times1} & \mathbf{0}_{N\times1} & \cdots & \mathbf{w}_{k,o}^* \end{bmatrix}. \quad (24)$$

Also, by defining

$$\begin{aligned} \mathbf{g}_{k,o} &\triangleq [g_{k,o,1}, g_{k,o,2}, \cdots, g_{k,o,K}]^H \\ &\triangleq \widetilde{\mathbf{f}}_{k,o} - \mathcal{H}^H \mathbf{w}_{k,o}, \end{aligned} \quad (25)$$

it can be shown that

$$\mathbf{g}_{k,o} = \mathbf{R}\widetilde{\mathbf{f}}_{k,o}. \quad (26)$$

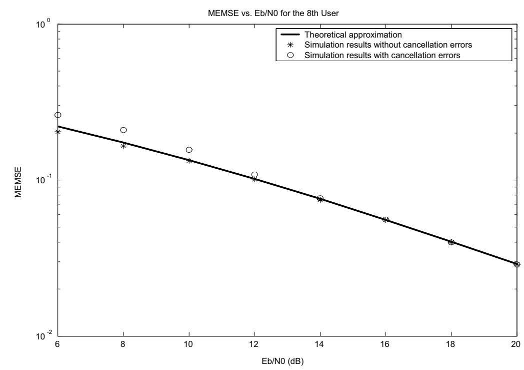

Fig. 1. MEMSE versus E_b/N_0 of the 8^{th} user.

Similarly, another $(KN \times K)$ matrix $\mathcal{G}_{k,o}$ can be constructed, which satisfies the following relationship

$$\mathbf{g}_{k,o}^H \Delta\mathcal{H} = \Delta\mathfrak{h}^H \mathcal{G}_{k,o}, \quad (27)$$

where, the j^{th} column of $\mathcal{G}_{k,o}$ can be expressed as $[\mathcal{G}_{k,o}]_j = \left[\mathbf{0}_{(j-1)\times1}^T, g_{k,o,1}, \mathbf{0}_{(N-1)\times1}^T, g_{k,o,2}, \mathbf{0}_{(N-1)\times1}^T, \cdots, g_{k,o,K}, \mathbf{0}_{(N-j)\times1}^{TH}\right]^H$.

By using $\mathcal{W}_{k,o}$ and $\mathcal{G}_{k,o}$ and previous obtained results, the MEMSE, under a given channel realization, can be expressed as:[3]

$$\mathbf{MEMSE}_{k|\mathcal{H}} = Tr\left(\mathcal{S}_k \cdot \mathbf{R}_{\Delta\mathfrak{h}}\right). \quad (28)$$

In the above equation, $\mathcal{S}_k \triangleq \Psi_1 \mathbf{R_{xx}}^{-1} \Psi_1^H + \Psi_2 \Xi \Psi_2^H - \frac{\Psi_3}{\mathbf{MMSE}_k}$, in which $\Psi_1 = \mathcal{G}_{k,o} - \mathcal{W}_{k,o}\mathcal{H}^H$, $\Psi_2 = \mathcal{W}_{k,o}\mathbf{R} - \mathcal{G}_{k,o}\mathbf{R_{xx}}^{-1}\mathbf{R_{xb}}$, $\Xi = \begin{bmatrix} \mathbf{0}_{(k-1)\times(K-k+1)} \\ \mathbf{I}_{K-k+1} \end{bmatrix} \mathbf{R}_{\Delta,k}^{-1} \left[\mathbf{0}_{(K-k+1)\times(k-1)}, \mathbf{I}_{K-k+1}\right]$ and $\Psi_3 = 4\mathcal{W}_{k,o}\mathbf{g}_{k,o}\mathbf{g}_{k,o}^H \mathcal{W}_{k,o}^H$.

4.3. MEMSE averaged of all channel realizations

With equation (28), the MEMSE averaged over all channel realizations can be calculated as

$$\mathbf{MEMSE}_k = E_{\mathcal{H}}\left[Tr\left(\mathcal{S}_k \cdot \mathbf{R}_{\Delta\mathfrak{h}}\right)\right]. \quad (29)$$

In equation (29), $\mathbf{R}_{\Delta\mathfrak{h}}$ is difficult to know in advance, since the statistic property of Δa_k $(k = 1, 2, \cdots, K)$ is hard to estimate due to the nonlinear calculation. However, as it will be shown later from the simulation results that under small CEE, Δa_k could be assumed negligible.

Therefore, $\mathbf{R}_{\Delta\mathfrak{h}}$ depends mainly on CEE and the power distribution $\left(\mathbf{A}^\dagger\right)^2$, derived under the perfect CSI. With the assumption of i.i.d CEE among different users and sub-carriers, $\mathbf{R}_{\Delta\mathfrak{h}}$ can be expressed as

$$\mathbf{R}_{\Delta\mathfrak{h}} \approx \frac{1}{N}\sigma_e^2 \left(\mathbf{I}_N \otimes \left(\mathbf{A}^\dagger\right)^2\right), \quad (30)$$

where σ_e^2 denotes the variance of CEE and $\otimes$ denotes the Kronecker product. Therefore, the MEMSE, for the k^{th} user, averaged

[3]Notice although the final results are similar as obtained in [3], the formats of matrices are quite different due to the different problems.

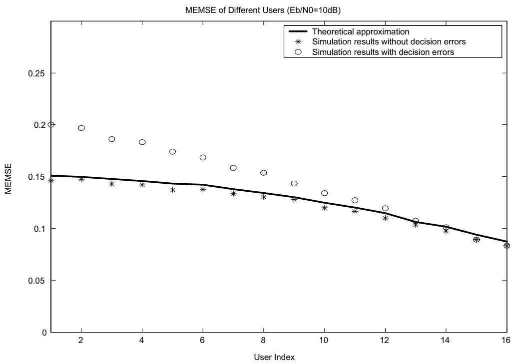

Fig. 2. MEMSE of different users ($E_b/N_0 = 10dB$).

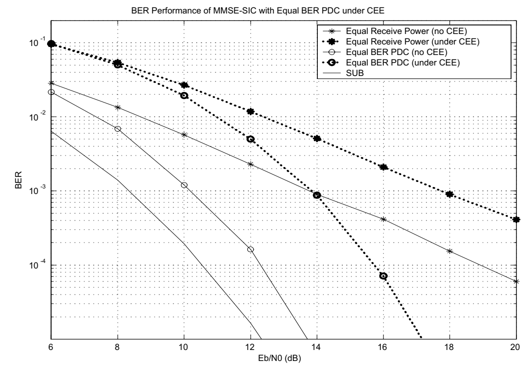

Fig. 3. Average BER performance over 16 users of the MMSE-SIC under CEE.

over all channel realizations, can be simplified as

$$\mathbf{MEMSE}_k \approx \frac{1}{N}\sigma_e^2 E_{\mathcal{H}}\left[Tr\left(\mathcal{S}_k\cdot\left(\mathbf{I}_N \otimes \left(\mathbf{A}^\dagger\right)^2\right)\right)\right]. \quad (31)$$

5. SIMULATION RESULTS AND DISCUSSIONS

An Indoor Rayleigh fading channel model, with total bandwidth 100MHz and RMS delay spread 25ns is considered. The number of sub-carriers N is chosen to be 16 and a fully-loaded MC-CDMA system with $K = 16$ is considered for all simulations. Orthogonal Walsh-Hadamard codes are used as spreading codes.

A CEE model described in [5] is employed for simulations, by which, the CEE among different users and sub-carriers are i.i.d.. The variance of the CEE on each sub-carrier depends on the SNR of the pilot symbols and the ICI caused by frequency offset.

In Figure 1, the MEMSE (averaged over 100 channel realizations) versus E_b/N_0 for the 8^{th} user is shown. Under each channel realization, 100 random CEE are produced and the results are averaged over all CEE. From this figure, it is clear that the theoretical derivation (equation (31)) matches the simulation result with perfect SIC very well, particularly in higher E_b/N_0. Therefore, the amplitude difference Δa_k caused by CEE does not contribute significant MEMSE, which confirms the assumption of ignoring it. The actual simulation result with cancellation errors is also plotted for comparison, from which, it is clear that for the 8^{th} user, under the equal BER PC, cancellation errors cause only slight extra MEMSE at low E_b/N_0.

In Figure 2, the MEMSE of different users is shown under $E_b/N_0 = 10dB$. From this figure, the accuracy of the second-order approximation is again confirmed. Also, it is very interesting to find that under the "equal BER" PC, earlier detected user (larger index) have smaller MEMSE than later detected ones (smaller index), therefore, the earlier detected users have a higher reliability than the later detected ones, which can still benefit SIC. The cancellation error propagation among different users can also be seen very clearly from this figure. For the later detected users, more cancellation errors result in higher additional MEMSE.

In Figure 3, the average BER performance over 16 users under CEE is compared with that under the perfect CSI by actual simulations. From this figure, it is clear that the performances of the MMSE-SIC with the equal BER PC or equal received power will both be degraded under CEE. However, under CEE, the MMSE-SIC with the equal BER PC still retains a significant performance advantage over the MMSE-SIC with the equal received power. For example, at a BER of 10^{-3}, the performance advantage is around 4dB, which is even slightly better than the 3.5dB, obtained under the perfect CSI. Therefore, from these observations, it can be concluded that the equal BER PC makes the MMSE-SIC more robust to CEE.

6. CONCLUSIONS

In this paper, the effects of CEE on the performance of the MMSE-SIC with the equal BER PC is investigated. By applying a method of second-order approximation, the MEMSE can be derived for different users, under a given decision order and power allocation. Under the assumption of ignoring cancellation errors, the accuracy of the approximation is confirmed by simulation results. Furthermore, it is very interesting to find out that under small CEE, the equal BER PC can still benefit MMSE-SIC, making it more robust to CEE than the equal received power. Therefore, the MMSE-SIC with the equal BER PC provides a very powerful solution for MAI suppression in a practical MC-CDMA system.

7. REFERENCES

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