TIME-FREQUENCY CHANNEL MODELING AND ESTIMATION OF MULTI-CARRIER SPREAD SPECTRUM COMMUNICATION SYSTEMS

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ABSTRACT

In wireless communications, the channel is typically modeled as a random linear time-varying system that spreads the transmitted signal in both time and frequency due to multipath and Doppler effects. In this paper, we show how time-frequency analysis can be used to model and estimate the channel of a multi-carrier spread spectrum (MC-SS) system with a complex quadratic spreading sequence. We will show that in this case the effects of time delays and Doppler frequency shifts can be characterized effectively as time-shifts. Using the discrete evolutionary transform (DET) we are able to estimate these effective time shifts via a spreading function and use them to equalize the channel. To illustrate the performance of the proposed method we perform several simulations with different levels of channel noise, jammer and Doppler frequency shifts.

1. INTRODUCTION

In high speed mobile communications, orthogonal frequency division multiplexing (OFDM), as a multi-carrier modulation technique, is well known for its high performance in multipath fading environments compared to single carrier systems. Likewise, spread spectrum systems are well known for their mitigation of intentional jamming or nonintentional co-channel interference. Combining these two has led to multi-carrier spread spectrum (MC-SS), multi-carrier direct sequence spread spectrum (MC-DS), and multi-tone spread spectrum (MT-SS) systems [1, 2, 3]. In MC-SS, the data is spread by complex coefficients and then modulated by carriers of different frequencies. To achieve desirable flat spectrum, while providing a constant envelope, complex quadratic sequences are used as the spreading functions [2]. In this paper, we will show that we can take advantage of the properties of these sequences in the estimation of the communication channel and in the design of coherent receivers.

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Given the time and frequency spreading caused by the time-varying nature of the communication channel, equalization is needed to be able to recover the sent data. Although the time-varying nature of the channel is due to Doppler shifts, in many practical situations they are not significant or not considered. For instance, the RAKE receiver used in CDMA spread spectrum works well under slow fading even though the Doppler shifts are not considered [4, 5]. However if the Doppler shifts are considered, the performance of the receiver is improved.

Transmission channels are modeled as random, time-varying systems [6, 7, 8]. In this paper, we propose an MC-SS channel model that is linear and time varying for the duration of a data bit. This provides a characterization of multi-path fast fading as well as slow fading. The complex quadratic sequence in [2] is a complex linear chirp of unit amplitude and its discrete Fourier transform is also a linear chirp of unit magnitude, which are orthogonal when cyclically shifted. We will show how to use these characteristics to estimate the parameters of the LTV model by means of the spreading function computed from the discrete evolutionary transform (DET) of the received signal. This permits an evaluation of the number of paths, delays, Doppler frequency shifts and the gains characterizing the channel for one or more data bits. This information is then used to estimate the data bit sent [9].

2. CHANNEL MODEL

The time-varying frequency response of the channel, also known as Zadeh's function [10], characterizes the channel in terms of time delays, Doppler frequency shifts and gains, all of which vary randomly in the modeling. The existing connection between the Zadeh's function and the evolutionary spectral theory can thus be exploited to estimate the channel parameters and provide a way in the receiver to detect the transmitted data [9]. An L-path fading channel with Doppler frequency shifts is generally modeled by a separable impulse response:

$$\begin{aligned} h(n,k) &= \sum_{\ell=0}^{L-1} h_\ell(n-k) f_\ell(n) \\ &= \sum_{\ell=0}^{L-1} \alpha_\ell \delta(n-N_\ell) e^{j\psi_\ell n} \end{aligned} \quad (1)$$

where we have replaced $h_\ell(n) = \delta(n-N_\ell)$, as the impulse response of the all-pass systems corresponding to delays $\{N_\ell\}$, and $f_\ell(n) = \alpha_\ell e^{j\psi_\ell n}$ where $\{\psi_\ell\}$ are the Doppler frequencies with gains $\{\alpha_\ell\}$. Without loss of generality, we assume that the above model is valid for one bit, thus the model needs to be adjusted when considering slow or fast fading. In either case, the channel model is characterized, for one or more bits, by the number of paths L, the delays $\{N_\ell\}$, and the Doppler frequency shifts $\{\psi_\ell\}$. We will now show how to estimate these parameters using the evolutionary spectral theory.

As shown in [9], the frequency response of the LTV channel $H(n,\omega_k)$ is given by;

$$H(n,\omega_k) = \sum_{\ell=0}^{L-1} \alpha_\ell e^{j\psi_\ell n} e^{-j\omega N_\ell}, \quad (2)$$

which can be easily verified to be the Fourier Transform of the separable impulse response $h(n,k)$ in (1). Now, the bi-frequency function $B(\Omega,\omega_k)$ is found by computing the Fourier transform of $H(n,\omega_k)$ with respect to the n variable:

$$B(\Omega,\omega) = 2\pi \sum_{\ell=0}^{L-1} \alpha_\ell e^{-j\omega N_\ell} \delta(\Omega-\psi_\ell) \quad (3)$$

Finally, from the inverse Fourier transform of $B(\Omega,\omega)$, with respect to ω, the spreading function is given by;

$$S(\Omega,k) = 2\pi \sum_{\ell=0}^{L-1} \alpha_\ell \delta(\Omega-\psi_\ell)\delta(k-N_\ell) \quad (4)$$

which displays peaks located at the delays and the corresponding Doppler frequencies, and with $2\pi\alpha_\ell$ as their amplitudes, which are used to estimate the bit sent.

3. MC-SS CHANNEL MODELING AND ESTIMATION

In MC-SS [1, 2, 3] the bit sequence $d(n)$ is spread by frequency domain spreading coefficients $\{G(k)\}$ which then modulate multiple carriers. The time-domain view of MC-SS corresponds to a direct sequence spread spectrum with a complex spreading function $g(n)$ which is the IDFT of the $G(k)$. The transmitted signal thus depends on the spreading sequence used, which typically has flat spectrum, but not a constant amplitude. The constant envelope in time and frequency is required for the spreading characteristics as well as technical reasons. In [2], the following complex quadratic sequences are used to spread the message in time and in frequency:

$$\begin{aligned} g(n) &= e^{-j\frac{\pi}{8}} e^{j\frac{2\pi}{N}\frac{1}{2}n^2}, \ n = 0,\cdots,N-1 \\ G(k) &= e^{j\frac{\pi}{8}} e^{-j\frac{2\pi}{N}\frac{1}{2}k^2}, \ k = 0,\cdots,N-1 \end{aligned}$$

These are linear chirps with the following properties:

- $g(n)$ and $G(k)$ are DFT pair and $G(k) = g(k)^*$, where $*$ denotes complex conjugate,
- $g(n)$ and $G(k)$ have constant envelope,
- Circularly shifted versions of $g(n)$ or $G(k)$ are orthogonal.

For the baseband case, the transmitted signal $s(n)$ is given by

$$\begin{aligned} s(n) &= \sum_{k=0}^{N-1} d\, G(k)\, e^{j\omega_k n}, \ 0 \le n \le N-1 \\ &= dg(n) \end{aligned} \quad (5)$$

therefore the output of the time-varying channel, $y(n)$ becomes

$$y(n) = d \sum_{\ell=0}^{L-1} \alpha_\ell g(n-N_\ell) e^{j\psi_\ell n} \quad (6)$$

where L corresponds to the number of paths, $\{N_\ell\}$ and $\{\psi_\ell\}$ are the corresponding time delays and Doppler frequency shifts. Note that we only consider integer time and Doppler frequency shifts. We assume the received signal $r(n)$ is corrupted by a white Gaussian channel noise $\eta(n)$, and a jamming interference $j(n)$ so that, $r(n) = y(n) + \eta(n) + j(n)$.

According to the properties of $g(n)$ we have : 1) Any time delay caused by the channel on $g(n)$ is equivalent to a Doppler frequency shift on $g(n)$; 2) Any Doppler frequency shift on $g(n)$ is equivalent to a time advance on $g(n)$. These can be shown as follows:

1. Delay by N_0:

$$\begin{aligned} g(n-N_0) &= e^{-j\frac{\pi}{8}} e^{j\frac{2\pi}{2N}(n-N_0)^2} \\ &= g(n) e^{-j\frac{2\pi}{N}N_0 n} e^{j\frac{\pi}{N}N_0^2} \end{aligned}$$

where $e^{-j(2\pi N_0 n/N)}$ corresponds to a Doppler shift $\psi_0 = -2\pi N_0/N$ and $e^{j(2\pi N_0^2/2N)}$ is a constant.

2. Doppler frequency shift by $\psi_1 = \frac{2\pi}{N} N_1$:

$$\begin{aligned} g(n)e^{j\psi_1 n} &= e^{-j\frac{\pi}{8}} e^{j\frac{2\pi}{2N}n^2} e^{j\psi_1 n} \\ &= g(n+N_1)e^{-j\frac{2\pi}{2N}N_1^2} \end{aligned}$$

where $g(n+N_1)$ is $g(n)$ advanced by N_1 samples and $e^{-j(2\pi N_1^2/2N)}$ is a constant. Considering a third case:

3. Delay N_0 and Doppler frequency shift $\psi_1 = \frac{2\pi}{N} N_1$:

$$\begin{aligned} g(n-N_0)e^{j\psi_1 n} &= g(n)e^{-j\frac{2\pi(N_0-N_1)n}{N}} e^{j\frac{\pi N_0^2}{N}} \\ &= g(n-N_0+N_1)e^{-j\frac{\pi(N_1^2-2N_0N_1)}{N}} \end{aligned}$$

The last item indicates that when $g(n)$ is delayed in time and shifted in frequency the result is that $g(n)$ is shifted in time only by an effective value $N_e = -N_0 + N_1$ and multiplied by a complex constant. Considering when there are no noise and jammer, the received signal is $y(n) = \sum_{\ell=0}^{L-1} \alpha_\ell g(n - N_{e,\ell})$, after taking care of the Doppler effects of the channel. Although in this case the method in [9], based on the DET, will be used to estimate the $N_{e,\ell}$ from $S(0,k)$, we can show that the LTI nature of the effective model makes the computations easier. In fact, the transfer function of the channel becomes

$$\tilde{H}(\omega_k) = \sum_{\ell=0}^{L-1} \alpha_\ell e^{-j\omega_k N_{e,\ell}} \tag{7}$$

and its inverse Fourier transform gives

$$\tilde{h}(n) = \sum_{\ell=0}^{L-1} \alpha_\ell \delta(n - N_{e,\ell}) \tag{8}$$

Both of these functions are special cases of $H(n,\omega_k)$ and $S(\Omega_s,k)$. In fact, $\tilde{h}(n)$ coincides with $S(0,n)$ as defined in equation (4). When the channel noise $\eta(n)$ and the jammer $j(n)$ are considered in the received signal, the effective shifts $\{N_{e,\ell}\}$ found before are only estimates, but can be used to detect the data bit d that was sent when the noise and the jammer have relatively low power. If the estimated effective shift is $\hat{N}_e$, corresponding to the shortest path (presumably having the smallest attenuation), and using the circular shift orthogonality of the $g(n)$, we obtain the following decision variable

$$\begin{aligned} \rho &= \sum_{n=0}^{N-1} r(n) \frac{g^*(n-\hat{N}_e)}{\hat{\alpha}_e} \\ &= d\frac{\alpha_{\ell_0}}{\hat{\alpha}_e} + \sum_{n=0}^{N-1} [\eta(n) + j(n)] \frac{g^*(n-\hat{N}_e)}{\hat{\alpha}_e} \end{aligned}$$

when $\hat{N}_e$ coincides with one of the actual effective values $N_{e,\ell}$. Considering the channel noise is zero mean, and the jammer has a small average, then the expected value of the decision variable is $E[\rho] = d\alpha_{\ell_0}/\hat{\alpha}_e$, which is close to d.

4. EXPERIMENTAL RESULTS

We tested the performance of our method by using Monte Carlo simulations. The wireless channel is simulated randomly, i.e, the number of paths varies between $1 \leq L \leq 5$, the delays, N_i and the doppler frequency shift $0 \leq \psi_i \leq \psi_{\text{max}}$ of each path are chosen randomly. Input data is modulated by $K = 100$ sub-carriers. Assuming a bandwidth of $500KHz$, frequency spacing between the sub-carriers is $F = 5KHz$. Estimation of the effective shifts is illustrated in Fig. 1. For a first set of simulations, the SNR of the channel noise changes between -4 and $6dB$, for normalized Doppler frequency shift less than $\psi_{\text{max}} = 0.001\pi$ and the BER is calculated in four different ways: 1) No Channel Estimation, 2) Proposed Method, 3) LTI known channel model, not considering Doppler effects, and 4) Known Channel parameters (see Fig. 2). We found that for SNR values larger than 2 dB, the BER values were close to zero.

In the second set of simulations, we test the effect of Doppler in the BER calculation. We find that for Doppler frequencies larger than 0.002π rad the effect is very significant in the channel estimation (see Fig. 3). Finally, we tested the robustness of the channel estimation to the presence of wide-band jammers. Figure 4 illustrates that for chirp jammers with a jammer to signal ratio bigger than 2 dB, estimation of the channel parameters is significantly hampered by the jammer.

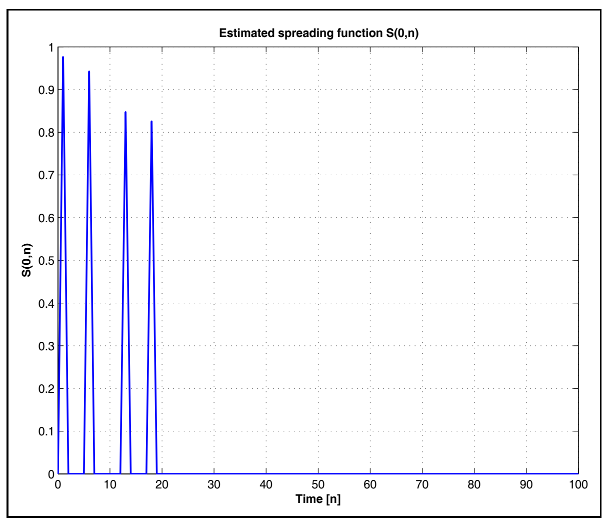

Fig. 1. Estimated spreading function $S(0,n)$

5. CONCLUSIONS

In this paper we show how time-frequency channel estimation can be used in multi-carrier spread spectrum. Using the effective shift property of quadratic sequences found here, we are able to develop an estimation procedure and

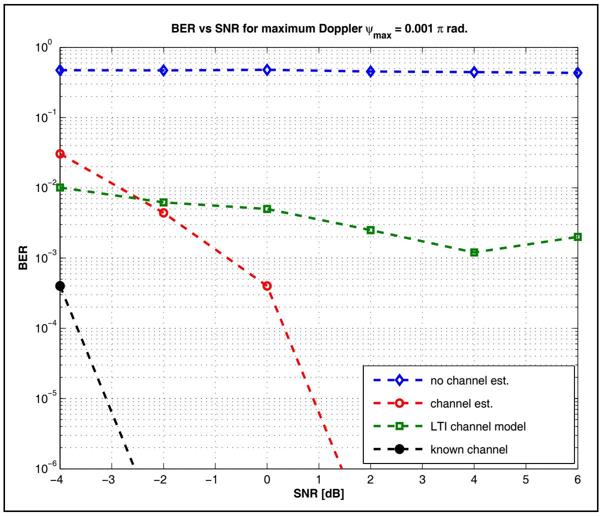

Fig. 2. BER vs SNR for $\psi_{max} = 0.001\pi$ rad.

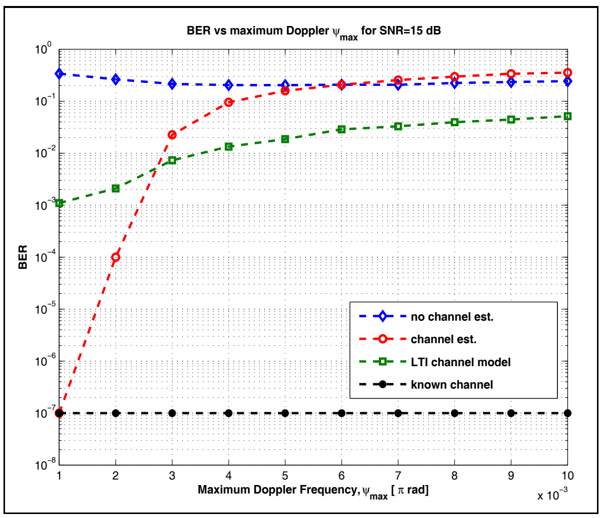

Fig. 3. BER vs maximum Doppler for SNR=15 dB.

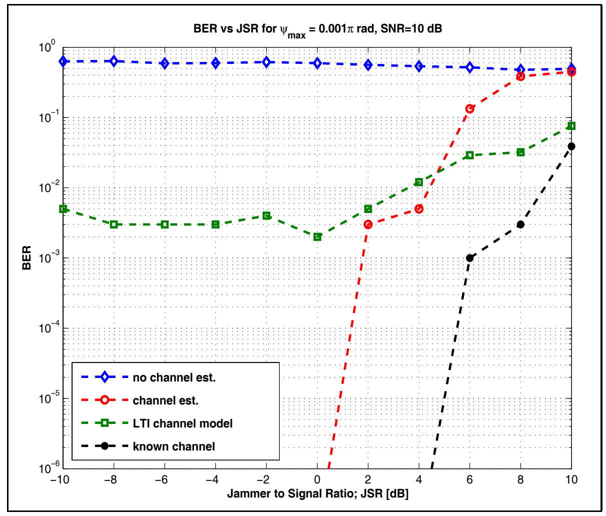

Fig. 4. BER vs JSR for SNR=10 dB.

an appropriate receiver. We illustrate the performance of our method for different channel noise, Doppler frequency shift, and jammer levels and find that the results are very encouraging.

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