Reduced Complexity Turbo Detection for Coded DS-CDMA Systems Employing BPSK Modulations

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Abstract

In this paper, a reduced complexity turbo detection receiver for coded DS-CDMA signals employing BPSK modulation technique is presented. The new scheme is based on a new family of MMSE filter whose coefficients are thought to be the solution of a (forced) real valued cost function in the bit rather than a complex one as in conventional MMSE receivers. The new receiver provides on the average 2dB gain with less number of iterations. Simulation results for performance evaluation are conducted under most interesting scenarios including asynchronous multipath channels, Time varying channels and multirate systems.

I. Introduction

Optimal decoding scheme for convolutionally coded CDMA system combines the trellises of both the asynchronous multiuser detector and the convolutional code, resulting in a prohibitive computational complexity. Iterative decoding schemes for convolutionally or turbo coded CDMA systems were proposed in [3] [4] and [2], differing in the type of the SISO multiuser detector used. In [3] and [4], a MAP SISO multiuser detector was proposed for convolutional coded synchronous CDMA systems, resulting in a computational complexity of $O(2^K)$ for the multiuser detector; extension to asynchronous CDMA system was proposed in [4]. To avoid the exponential complexity of the latter, linear SISO detectors based on soft interference cancellation and residual interference suppression were proposed, e.g. in [6], [1] [2] [5] [7]. It is worth to mention that the idea of exploring the BPSK modulation in designing linear receiver is not a new issue [9] [10], but the idea of taking this into account in design turbo detectors is not covered yet.

In the present work, we derive a new efficient and reduced complexity turbo detection receiver for coded DS-CDMA signals in multipath channels. The new algorithm is designed for signals employing binary phase sift keying taking advantage of the a priori information about the signals being sent. A MMSE filter, whose coefficients are found by minimizing the cost function defined only for the real part of sent symbols, in contrast to the conventional MMSE receivers used in [7] (which is defined for a general case of complex symbols), is proposed. The paper is organized as follows; in Section II, a convolutionally coded DS-CDMA model for multipath channel is presented, followed in Section III by the SISO low complexity detector. Simulation results are presented in Section IV, and finally a conclusion is drawn.

II. Convolutionnally coded DS-CDMA model

Consider a convolutionally coded DS-CDMA system with K users, using binary phase shift keying (BPSK) modulation and signaling over their respective multipath channels with additive white Gaussian noise. The binary information $\{d_k(m)\}$ for user $k = 1, 2, ..., K$, are convolutionally encoded with rate R_k. An interleaver is used to reduce error burst effect at the input of the decoder. The interleaved code-bits of the k^{th} user are mapped into BPSK symbols $b_k(i) \in \{-1,+1\}$, next modulated by a spreading waveform $c_k(t)$ of duration T . The complex base band representation of the k^{th} user transmitted signal is given by

$$s_k(t) = \sqrt{2P_k}\sum_i b_k(i)\, c_k(t-iT)\,, \tag{1}$$

where P_k is the transmitted power, $b_k(i) \in \{-1,+1\}$ is the i^{th} transmitted symbol and $c_k(t)$ is a normalized spreading waveform given by

$$c_k(t) = \sum_{n=0}^{N-1} c_{k,n}\Pi_{T_c}(t-nT_c)\,\cdot \tag{2}$$

here, $c_{k,n} \in \{-1/\sqrt{N}, +1/\sqrt{N}\}$ and the chip pulse $\Pi_{T_c}(t)$ is a rectangular pulse of duration T_c=T/N where N is the spreading gain. The received signal at the base station, a superposition of the attenuated and delayed signals transmitted by all K users, is given by

$$r(t) = \sum_{k=1}^{K}\sum_{l_k=1}^{L_k} w_{k,l_k} s_k(t-\tau_{k,l_k}) + v(t) \tag{3}$$

where w_{k,l_k} is the l_k^{th} path's complex gain for user k. It includes phase offsets and user's power P_k ; τ_{k,l_k} 's are the respective delays, and $v(t)$ is a Gaussian white noise with zero mean and double side spectral density of $N_0/2$.

The front end of the receiver implements a chip-matched filter. The observation vector of length N at time instant i, $\mathbf{r}_i = \left[r(iN+N), r(iN+N-1), ..., r(iN+1)\right]^T$ ($\mathbf{r}_i \in C^{N\times 1}$) [9], is

$$\mathbf{r}_i = \mathbf{CHb}_i + \mathbf{v}_i \tag{4}$$

Where $\mathbf{v}_i \sim N(0, \sigma^2 \mathbf{I}_{N\times N})$, $\mathbf{b}_i = [b_{1,i-1}\, b_{1,i}\; b_{2,i-1}\, b_{2,i} ... b_{K,i-1}\, b_{K,i}]^T$, $\mathbf{C} = \left[\mathbf{C}_1^R, \mathbf{C}_1^L, \cdots, \mathbf{C}_K^R, \mathbf{C}_K^L\right]$ with $\mathbf{C}_k^R$, and $\mathbf{C}_k^L$ are $N\times N$ matrices containing (columnwise) the right and left shifted versions of the user k speading code of length N.

If we consider $\tau_{k,l} = q_{k,l}T_c + \gamma_{k,l}T_c$, $q_{k,l} \in [0 \;\; N-1]$ and $\gamma_{k,l} \in [0 \;\; 1)$ then $\mathbf{H} = diag(\mathbf{h}_1, \mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_2, \cdots, \mathbf{h}_K, \mathbf{h}_K)$ where $\mathbf{h}_k$ is $N\times 1$ all zero vector with $\omega_{k,L_k}(1-\gamma_{k,L_k})$ at q_{k,L_k} position and $\omega_{k,L_k}\gamma_{k,L_k}$ at $q_{k,L_k}+1$.

Without loss of generality we consider $\max_{l,k}(\tau_{k,l}) \le T$, the asynchronous model presented above is introduced in [8] and extended in [2] to a more general case.

The asynchronous model in (4) as stated is not very useful for detection as it may be noticed that at any instant i the observations $\mathbf{r}_i$ does contain partially the information about the users' bits $b_{k,i}, k = 1, ..., K, \; i = 1, ...$, hence the observations $\mathbf{r}_{i+1}$ at $i+1$ are needed. The extended signal model will be

$$\begin{bmatrix} \mathbf{r}_i \\ \mathbf{r}_{i+1} \end{bmatrix} = \begin{bmatrix} [\mathbf{CH}]_{odd} & [\mathbf{CH}]_{even} & \mathbf{0}_{N\times K} \\ \mathbf{0}_{N\times K} & [\mathbf{CH}]_{odd} & [\mathbf{CH}]_{even} \end{bmatrix} \mathbf{b}_{i/i+1} + \mathbf{v}_{i/i+1} = \mathbf{r}_{i/i+1} \tag{5}$$

where $[\mathbf{CH}]_{odd}$ and $[\mathbf{CH}]_{even}$ are matrices constructed from the odd and even numbered columns of $\mathbf{CH}$, respectively, $\mathbf{v}_{i/i+1} = [\mathbf{v}_i^T \;\; \mathbf{v}_{i+1}^T]^T$ and

$$\mathbf{b}_{i/i+1} = [b_{1,i-1}, b_{2,i-1}, \cdots b_{k,i-1}, \cdots, b_{K,i-1},\; b_{1,i}, b_{2,i}, \cdots b_{k,i}, \cdots, b_{K,i}, b_{1,i+1}, b_{2,i+1}, \cdots b_{k,i+1}, \cdots, b_{K,i+1}]^T.$$

In the following let $\mathbf{A} = \begin{bmatrix} [\mathbf{CH}]_{odd} & [\mathbf{CH}]_{even} & \mathbf{0}_{N\times K} \\ \mathbf{0}_{N\times K} & [\mathbf{CH}]_{odd} & [\mathbf{CH}]_{even} \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} \mathbf{A} \\ \mathbf{A}^* \end{bmatrix}$.

III. Iterative turbo detector

An iterative receiver consists of the iterative SISO MMSE receiver and K-SISO channel decoder. The SISO MMSE design problem relies on the instantaneous design of feed forward and feedback filters using updates on the soft information provided by SISO channel decoder and a new family of cost functions rather than the conventional MMSE design criteria.

Before deriving the new family of SISO MMSE structure, one can notice that any linear receiver makes a decision about the bit $b_k(i)$ according to the rule

$$\hat{b}_k(i) = \mathrm{sgn}(\Re(\mathbf{w}_k^H(i)\mathbf{r}_{i/i+1})) \tag{6}$$

Where $\mathrm{sgn}(\bullet)$ denotes the sign function, $\Re(\bullet)$ denotes the real part, and $(\bullet)^H$ conjugate transpose. The conventional MMSE detector selects $\mathbf{w}_k(i)$ according to

$$\mathbf{w}_k(i) = \arg\min_{\mathbf{w}\in C^{2N\times 1}} E\left\{|\mathbf{w}^H(i)\mathbf{r}_{i/i+1} - b_k(i)|^2\right\} \tag{7}$$

On the other hand, since

$$\min_{\mathbf{w}\in C^{2N\times 1}} E\left\{|\mathbf{w}^H(i)\mathbf{r}_{i/i+1} - b_k(i)|^2\right\} \ge \min_{\mathbf{w}\in C^{2N\times 1}} E\left\{(\Re(\mathbf{w}^H(i)\mathbf{r}_{i/i+1}) - b_k(i))^2\right\} \tag{8}$$

It is understood that defining

$$\mathbf{w}_k(i) = \arg\min_{\mathbf{w}\in C^{2N\times 1}} E\left\{(\Re(\mathbf{w}^H(i)\mathbf{r}_{i/i+1}) - b_k(i))^2\right\} \tag{9}$$

The decision rule (9) is necessarily not inferior to the conventional MMSE. To solve (9), first notice that [10]

$$\Re(\mathbf{w}^H(i)\mathbf{r}_{i/i+1}) = \frac{1}{2}(\mathbf{w}^H(i)\mathbf{r}_{i/i+1} + \mathbf{w}^T(i)\mathbf{r}_{i/i+1}^*) = \frac{1}{2}\begin{pmatrix} \mathbf{w}\;(i) \\ \mathbf{w}^*(i) \end{pmatrix}^H \begin{pmatrix} \mathbf{r}_{i/i+1} \\ \mathbf{r}_{i/i+1}^* \end{pmatrix} = \mathbf{w}_a(i)\mathbf{r}_{a,i/i+1} \tag{10}$$

As a consequence, solving (9) is equivalent to solving the problem

$$\mathbf{w}_{a,k}(i) = \arg\min_{\mathbf{w}_a\in C_a^{4N\times 1}} E\left\{(\mathbf{w}_a^H(i)\mathbf{r}_{a,i/i+1} - b_k(i))^2\right\} \tag{11}$$

In (11), $C_a^{4N\times 1}$ is a vector space in the field C. Its elements are the augmented $4N$-dimensional complex vector whose first $2N$ entries ate the complex conjugate of the last $2N$. The internal operation is the usual component-wise vector sum in $C^{4N\times 1}$ and the external operation is $\times : \alpha \in C, \mathbf{w}_a \in C_a^{4N\times 1} \to \alpha\mathbf{w}_a = \begin{pmatrix} \alpha\mathbf{w} \\ \alpha^*\mathbf{w}^* \end{pmatrix}$.

We extend the newly defined MMSE problem in (11) by defining the output $y_k(i)$ of the new MMSE by

$$y_k(i) = \Re\{\mathbf{w}_{fk}^H(i)\mathbf{r}_{i/i+1} + \mathbf{w}_{bk}^H(i)\tilde{\mathbf{b}}_{i/i+1}^{(k)}\} \tag{12}$$

Where $\mathbf{w}_{fk}(i)$ is an $(2N\times 1)$ optimized feed forward coefficients vector, $\mathbf{w}_{bk}(i)$ is $(3K-1\times 1)$ optimized feedback coefficients vector, and $\tilde{\mathbf{b}}_{i/i+1}^{(k)}$ is obtained from $E\{\mathbf{b}_{i/i+1}\}$ eliminating the $(K+k)^{th}$ element.

To ease the design let

$$\xi_k(i) = \Re(\mathbf{w}_{bk}^H(i)\tilde{\mathbf{b}}_{i/i+1}^{(k)}) \tag{13}$$

then (12) becomes

$$y_k(i) = \mathbf{w}_{a,fk}^H(i)\mathbf{r}_{a,i/i+1} + \xi_k(i) \tag{14}$$

Finding $\mathbf{w}_{a,fk}(i)$ and $\xi_k(i)$ consists in solving the following optimization problem:

$$\left\{\mathbf{w}_{a,fk}(i), \xi_k(i)\right\} = \min_{\substack{\mathbf{w}_a(i)\in C^{4N\times 1},\\ \xi(i)\in R,\\ E\{\mathbf{b}_{i/i+1}\}\neq \mathbf{0}}} E\left\{(y_k(i) - b_k(i))^2\right\} = \min_{\substack{\mathbf{w}_a(i)\in C^{4N\times 1},\\ \xi(i)\in R,\\ E\{\mathbf{b}_{i/i+1}\}\neq \mathbf{0}}} E\left\{\begin{matrix}(\mathbf{w}_a^H(i)\left(b_k(i)\mathbf{B}_{K+k} + \mathbf{B}^{(k)}\mathbf{b}_{i/i+1}^{(k)} + \mathbf{v}_{a,i/i+1}\right) \\ +\xi\,(i) - b_k(i))^2\end{matrix}\right\} \tag{15}$$

where $\mathbf{B}_{K+k}$ is the $(K+k)^{th}$ column of the ($4N \times 3K$) matrix $\mathbf{B}$, $\mathbf{B}^{(k)}$ is obtained from $\mathbf{B}$ by eliminating the $(K+k)^{th}$ column, and $\mathbf{b}^{(k)}_{i/i+1}$ is obtained from the vector $\mathbf{b}_{i/i+1}$ by omitting the $(K+k)^{th}$ element.

The MMSE optimization problem reduces to solving the following equations

$$E\left\{\mathbf{b}^{(k)}_{i/i+1}\right\}^T \mathbf{B}^{(k)H}\mathbf{w}_a(i) + \xi\ (i) = 0 \tag{16}$$

$$\left\{\mathbf{B}_{K+k}\mathbf{B}_{K+k}{}^H + \mathbf{B}^{(k)}E\left\{\mathbf{b}^{(k)}_{i/i+1}\mathbf{b}^{(k)}_{i/i+1}{}^T\right\}\mathbf{B}^{(k)H} + \sigma^2\mathbf{I}_{4N\times4N}\right\}\times \tag{17}$$
$$\mathbf{w}_a(i) + \mathbf{B}^{(k)}E\left\{\mathbf{b}^{(k)}_{i/i+1}\right\}\xi\ (i) = \mathbf{B}_{K+k}$$

Solving (16) and (17) leads to

$$\mathbf{w}_{a,fk}(i) = \left(\boldsymbol{\psi}_k + \boldsymbol{\varphi}_k(i) + \sigma^2\mathbf{I}_{4N\times4N} - \boldsymbol{\chi}_k(i)\boldsymbol{\chi}_k^H(i)\right)^{-1}\mathbf{B}_{K+k} \tag{18}$$

$$\xi_k(i) = -\boldsymbol{\chi}_k^H(i)\mathbf{w}_{a,fk}(i) \tag{19}$$

Using (13), it will be straightforward to deduce that

$$\mathbf{w}^H_{bk}(i) = \xi_k(i)\left(\tilde{\mathbf{b}}^{(k)}_{i/i+1}\tilde{\mathbf{b}}^{(k)}_{i/i+1}{}^T\right)^{-1} \tag{20}$$

with $\boldsymbol{\psi}_k = \mathbf{B}_{K+k}\mathbf{B}^H_{K+k}$

$\boldsymbol{\varphi}_k(i) = \mathbf{B}^{(k)}(\tilde{\mathbf{b}}^{(k)}_{i/i+1}\tilde{\mathbf{b}}^{(k)}_{i/i+1}{}^T + \boldsymbol{\Delta}_k(i))\mathbf{B}^{(k)H}$ and $\boldsymbol{\chi}_k(i) = \mathbf{B}^{(k)}\tilde{\mathbf{b}}^{(k)}_{i/i+1}$

where

$$\boldsymbol{\Delta}_k(i) = \sum_j (1 - E\{\mathbf{b}_{i/i+1}(j)\}^2)\mathbf{e}_j\mathbf{e}_j{}^T + \sum_{j\neq k}(1 - E\{\mathbf{b}_{i/i+1}(K+j)\}^2)\mathbf{e}_{K+j}\mathbf{e}_{K+j}{}^T$$
$$+ \sum_j (1 - E\{\mathbf{b}_{i/i+1}(2K+j)\}^2)\mathbf{e}_{2K+j}\mathbf{e}_{2K+j}{}^T$$

and $\mathbf{e}_l$ denotes a ($3K-1\times1$) all-zeroes vector with "1" at the l^{th} element. In the first iteration, we set $\tilde{\mathbf{b}}^{(k)}_{i/i+1} = \mathbf{0}_{3K-1\times1}$, it is equivalent by assuming that the code bits are uniformly distributed and equiprobable. At each iteration, $\tilde{\mathbf{b}}^{(k)}_{i/i+1}$ are calculated using the soft information in the form of LLR obtained from the decoder.

Vital to the turbo processing, the SISO detector proposed here should be amounted so that it provides LLR's instead of soft decisions $y_k(i)$. To do it we assume that the output of the soft MMSE $y_k(i)$ in (12) represents the output of an equivalent additive white Gaussian noise channel having $b_k(i)$ as its input symbol [2]

$$y_k(i) = \lambda_k(i)b_k(i) + z_k(i) \tag{21}$$

where $\lambda_k(i)$ is the equivalent amplitude at instant I for the k^{th} user DF-MMSE filter output, and $z_k(i) \sim N(0, \rho_k(i)^2)$ is a Gaussian noise [2]. Therefore, the extrinsic information is given by

$$L^{ext}_{\det}(b_k(i)) = \frac{4\Re(\lambda_k(i)y_k(i))}{\rho_k(i)^2}\,. \tag{22}$$

Using (5) and (21), the parameters $\lambda_k(i)$ and $\rho_k(i)^2$ can be computed taking expectation is taken with respect to the code bits and $\mathbf{v}_{i/i+1}$

$$\lambda_k(i) = E\{y_k(i)b_k(i)\} \tag{23}$$
$$= \mathbf{w}_{a,fk}(i)^H\mathbf{B}\hat{\mathbf{b}}_{i/i+1}{}^{(k)} + \xi_k(i)E\{b_k(i)\}$$

where $\hat{\mathbf{b}}_{i/i+1}{}^{(k)} = E\{\mathbf{b}_{i/i+1}\}\cdot E\{b_k(i)\} + (1 - E\{b_k(i)\}^2)\mathbf{e}_{K+k}$

$$\rho_k(i)^2 = E\left\{|y_k(i)|^2\right\} - \lambda_k(i)^2 \tag{24}$$
$$= \mathbf{w}_{a,fk}(i)^H E\left\{\mathbf{r}_{a,i/i+1}\mathbf{r}_{a,i/i+1}{}^H\right\}\mathbf{w}_{a,fk}(i) + \xi_k(i)\xi_k(i)^*$$
$$+ 2\Re(\xi_k(i)E\{\mathbf{b}_{i/i+1}\}^T\mathbf{B}^H\mathbf{w}_{a,fk}(i)) - \lambda_k(i)^2$$

Notice that the algorithm complexity obvious from (18) where the inverse of a $4N\times4N$ matrix, $\left(\boldsymbol{\psi}_k + \boldsymbol{\varphi}_k(i) + \sigma^2\mathbf{I}_{4N\times4N} - \boldsymbol{\chi}_k(i)\boldsymbol{\chi}_k^H(i)\right)$, is performed at each instant i, a complexity burden $\mathrm{O}\left(2^3(2N)^3\right)$ as compared to $\mathrm{O}\left((2N)^3\right)$ of [8]. We can reduce the computational complexity to the order of $O\left(P_{iteration}(4N)^2\right)$ using the following steepest descent algorithm.

At instant *i*, compute iteratively (locally) $P_{iteration}$ times,

$$\mathbf{w}^p_{a,k}(i) = \left(\mathbf{I}_{4N\times4N} - \mu\left(\boldsymbol{\psi}_k + \boldsymbol{\varphi}_k(i) + \sigma^2\mathbf{I}_{4N\times4N} - \boldsymbol{\chi}_k(i)\boldsymbol{\chi}_k^H(i)\right)\right) \tag{25}$$
$$\mathbf{w}^{p-1}_{a,k}(i) + \mu\mathbf{B}_{K+k},\quad p = 1,2,\ldots,P_{iteration}$$

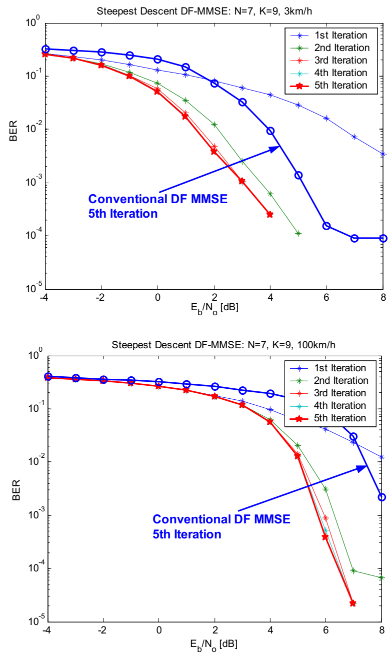
The complexity saving is obvious if at least $P_{iteration} < 4N$. The simulation results in mostly all cases show that steepest descent solution is as good as the direct inversion for a $P_{iteration}$ of 2 to 3 using a proper choice of adaptation step μ.

IV. Simulation results

To test the performance of the proposed DF-MMSE using steepest descent[1] (25) ($P_{iteration} = 3$ and $\mu = 0.1$) against the conventional DF-MMSE developed in [7], simulations are conducted in different scenarios. Unless otherwise stated we consider an over loaded system with processing gain N=7, using gold sequences of length 7. K=9 equal power users asynchronously access the channel with a multipath channel of 3 paths. The delays are uniformally distributed over $[0\ NT_c]$. The complex gains are Gaussian distributed. A convolutional encoder of rate ½ with constraint length 4 and generating polynomial $(13, 15)_8$ is used. Blocks of 300 symbols are transmitted. The SNR gain will be evaluated at 10^{-2} BER.

Time varying channel scenario: in this scenario the channel is time varying, we consider pedestrian speed of 3km/h and high speed of 100km/h. The 3 paths powers in dB's are 0, -3, and -9 dB, respectively. The carrier frequency is f_c=900MHz and the chip rate is 1.246Mcps.

[1] The simulation results show the reduced complexity algorithm performance using steepest descent. Direct matrix inversion provides similar (if not identical) performances. To make the figures more readable, the performance curves using direct matrix inversion are omitted.



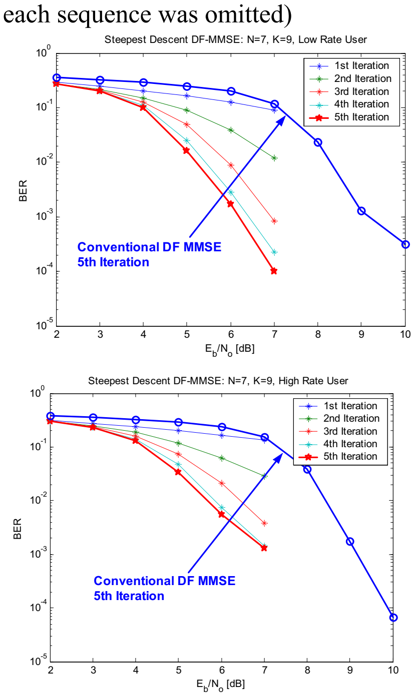
a)

b)

Fig. 1 Time varying channel, a) 3Km/h,b) 100Km/h.

Notice that (figure 1), at low speed (3km/h), the proposed Steepest Descent DF MMSE outperforms the conventional DF MMSE by more than 2dB, while at high speed as high as 100km/h, the gain exceeds even 2.5dB.

Multirate scenario: we consider high-rate users (HRUC) and low-rate users Classes (LRUC). Consider 9 users with spreading factor of 7 (178kb/s), (from the gold sequence of length 7), and 16 users with spreading factor of 14 (89kb/s), (from the subset of the Gold sequence of length 15 where the last chip of each sequence was omitted)



. a)

b)

Fig. 2 Multirate System, a) Low rate users, b) High rate users

Gains of 3dB for high rate user and 3.5dB for low rate users may be observed from the results presented in figure 2. Low rate users (analogous to weak users for a single rate system in a near far situation) perform better than high rate users (analogous to strong users in a single rate system).

V. Conclusion

In this paper, a reduced complexity turbo detection receiver for joint detection and decoding for coded DS-CDMA signals in multipath channels is derived. The new scheme is based on a new family of MMSE filter whose coefficients are found minimizing the cost function defined explicitly for real-valued symbols [13], in contrast to complex one as in conventional MMSE receivers. Complexity reduction is achieved by applying steepest descent technique. On the average a 2dB gain can be achieved at a quadratic complexity load.

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