LOW-COMPLEXITY BERNOULLI-GAUSSIAN DETECTION OF BLINDLY PRECODED OFDM

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ABSTRACT

Multicarrier transmission over a frequency selective channel implies large differences between the Signal to Noise Ratios (SNR) on the transmitted tones. For independently detected tones, the best performance in terms of average bit error rate (BER) is obtained when conditions are equally good on all subcarriers, as proved by Jensen's inequality. In the case of blindly precoded Orthogonal Frequency Division Multiplexing (OFDM), independent minimum square error (MSE) detection over each tone and joint-maximum-likelihood (ML) detections are not equivalent, as tones are no longer independent. Jensen's bound, which is reached by MSE detection, can then be outperformed. However, the computational complexity of joint-ML detection makes it unrealistic in practical systems. In this paper we present a low complexity detection scheme based on Bernoulli-Gaussian (B-G) evaluation over the complex field that outperforms Jensen's MSE bound typically by several dB, without a need for feedback to the transmitter.

1. INTRODUCTION

Recent work on multicarrier systems ([1], [2]) has produced several tools that improve the average BER/SNR performance if subchannel conditions present large differences between tones, due to selective fading and/or colored noise. The average BER performance of the system is, considering the BER $= f(\text{SNR})$ function for a given constellation

$$\text{BER} = \frac{1}{N}\sum_{n=1}^{N}\text{BER}_n = \frac{1}{N}\sum_{n=1}^{N} f(\text{SNR}_n) \tag{1}$$

When the tones SNRs are in the region of cup ($\cup$) convexity of function f, Jensen's inequality ([3] [1]) proves that in the case of independently demodulated tones

$$\text{BER} \geq f\left(\frac{1}{N}\sum_{n=1}^{N}\text{SNR}_n\right) \tag{2}$$

with equality if all SNRs are equal. The exact boundary of the $\cup$ convexity region depends on the constellation and can be found analytically or numerically. Adaptive power distribution, and the use of 'minimum BER' precoders, have been shown to significantly improve performance. Both schemes allow one to obtain equal conditions (SNR) on each subchannel, hence reaching Jensen's bound on BER when tones are detected using equlizing technics (like MSE). Power allocation requires feedback to the transmitter, which is not always doable in practice. It also dramatically modifies the spectrum of the transmitted signal, which may produce power regulation issues. Blind precoders do not require feedback,

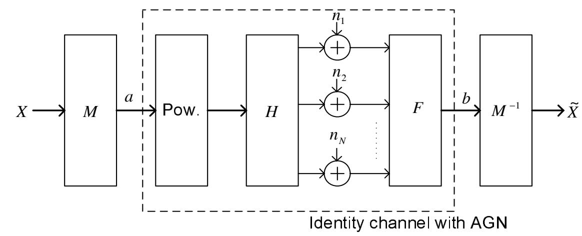

Fig. 1. Equivalent block diagram

and can be conditioned to keep the spectrum unmodified, but their ML detection is prohibitively expensive, often restricting the user to sub-optimal detection schemes. We derive a low complexity algorithm based on the B-G [4] deconvolution approach, that outperforms MSE detection by several dB in the case of dense multipath channels.

A OFDM system can be viewed, at the receiver, as N parallel subchannels, each of them with its own SNR. We consider the symbol-space baseband model as shown in Fig. 1.

If there is not inter-symbol interference (ISI) and a zero forcing equalizer is used, the equivalent channel between points a and b is considered as identity with additive gaussian noise. Each subchannel produces a subsequence of data with specific BER, which is a function of the constellation that is used and the noise conditions over this subchannel. The channel noise is assumed to be white over the bandwidth of each tone, but is in general colored if considered at the scale of the whole bandwidth of the system. The noise at the receiver is globally white if all tones present the same SNR, which is extremely unlikely for almost all OFDM systems. For simplicity, all derivations in this paper are based on symbol space considerations with this model. A more general approach can be obtained by detailing all components of the channel.

The rest of this paper is organized as follows, section 2 summarizes results on some linear precoders. Section 3 presents the new scheme and its performance. Finally, section 4 summarizes the main results and concludes the paper.

2. LINEAR PRECODER

Jensen's inequality [3] [1] states that performance is optimal for independently detected tones if all tones present the same SNR, or equivalently, the same residual noise power. However, this result is true only if all sub-channels show SNRs that are in the region of convexity $\cup$ of the BER vs. SNR function for this constellation [3] [1]. We define the linear precoder $\mathbf{M}$ (square complex-valued matrix of size NxN), and the precoding outputs X' obtained from a data vector X (at point a in fig. 1) as:

$$X' = \mathbf{M}X \tag{3}$$

The precoding matrix $\mathbf{M}$ is assumed to be invertible in order to be able to recover the data (Zero Forcing). The vector of received signals Y is then defined at point b in fig. 1 as

$$Y = X' + W \tag{4}$$

where W is the vector of the noise samples. Hence

$$\mathrm{E}\left[WW^\dagger\right] = \mathbf{R} = \mathrm{diag}\{\sigma_1^2, \ldots, \sigma_N^2\} \tag{5}$$

The precoding is inverted at the reception, and we get the received data vector

$$\tilde{X} = \mathbf{M^{-1}}Y = X + \mathbf{M^{-1}}W \tag{6}$$

As detailed in [1], precoders that guarantee that the received noise correlation matrix has a constant diagonal include normalized Hadamard matrices (when they exist), along with DFT and IDFT matrices (which are defined for all N). We do not detail here the construction and all properties of the precoder. For details, refer to listed reference [1]. However, it must be remembered that, in all cases, the precoder is fixed, does not depend on the channel conditions, and is known at the receiver. It can also easily be checked that these three types of precoders do not modify the spectrum of the transmitted signal. We also point out that all three types of matrices are unitary and hence $\mathbf{M}^{-1} = \mathbf{M}^\dagger$. The DFT matrix will be used as the default precoder.

3. BERNOULLI-GAUSSIAN DETECTION

The noise covariance matrix at the reception is obtained from (6).

$$\mathbf{R}_M = \mathrm{E}\left[\left(\mathbf{M^{-1}}W\right)\left(\mathbf{M^{-1}}W\right)^\dagger\right] = \mathbf{M}^\dagger\mathbf{R}\mathbf{M} \tag{7}$$

Clearly, $\mathbf{R}_M$ is not diagonal, except if all elements of the diagonal matrix $\mathbf{R}$ are equal. Hence the equalizer-based (MSE or ZF) decision, tone by tone, on the received symbols is not optimal, even though it reaches Jensen's bound. The joint maximum likelihood decision $\hat{X}$ for a received vector $\tilde{X}$ is the one, among the constellation symbol vectors, minimizing

$$L_m(\hat{X}) = \left(\tilde{X} - \hat{X}\right)^\dagger \mathbf{R}_M^{-1} \left(\tilde{X} - \hat{X}\right) \tag{8}$$

This is obtained by straightforward derivation of the log-likelihood function assuming complex circular noise. We point out here that $\mathbf{R}_M$ and $\mathbf{R}_M^{-1}$ can be built from the measured SNRs on each tones (which can be obtained at no extra cost while computing the FEQ $\mathbf{F}$ coefficients) and the knowledge of $\mathbf{M}$. We thus get

$$\mathbf{R}_M^{-1} = \mathbf{M}^\dagger\mathbf{R}^{-1}\mathbf{M} \tag{9}$$

A brute force solution to the minimization of (8) is not realistic, as the number of possibilities is P^N where P is the number of symbols in the constellation. Algorithmic solutions like sphere decoding reduce this complexity a lot, but not enough for practical implementation. We now restrict ourselves to the case of QAM. The objective is to derived a procedure that reaches or approximates the result of extensive testing, without the complexity of complete joint minimization. If, in each tone of the received vector $\tilde{X}$, we limit the tests to the 4 symbols surrounding the (soft) received point $\tilde{X}(n)$ (somehow like CHASE algorithm [5]), the total number of possible combinations drops down to 4^N, which is still challenging and not doable in practical systems as soon as N is not trivially small. In order to further reduce the complexity of the test procedure, an iterative algorithm avoiding extensive testing has to be devised.

3.1. One-tone perturbation

(8) develops into the 'likelihood metric' for an estimated vector $\hat{X}_0$

$$L_m(\hat{X}_0) = \tilde{X}^\dagger\mathbf{R_M}^{-1}\tilde{X} - 2\mathrm{Re}\left[\hat{X}_0^\dagger\mathbf{R_M}^{-1}\tilde{X}\right] + \hat{X}_0^\dagger\mathbf{R_M}^{-1}\hat{X}_0 \tag{10}$$

Where Re stands for 'real part'. If we wish to test a one-tone perturbation on $\hat{X}_0$, then we note

$$\Delta_n = \begin{bmatrix} 0 \\ \vdots \\ \delta \\ \vdots \\ 0 \end{bmatrix} \tag{11}$$

where the subscript n means that $\delta = \delta_{\mathrm{re}} + j\delta_{\mathrm{im}}$ is the n^{th} component of the Nx1 vector Δ_n. Replacing $\hat{X}_0$ by $\hat{X}_1 = \hat{X}_0 + \Delta_n$ let

$$L_m(\hat{X}_1) = L_m(\hat{X}_0) + L_{\Delta_n} \tag{12}$$

with

$$L_{\Delta_n} = 2\mathrm{Re}\left[\left(\hat{X}_0 - \tilde{X}\right)^\dagger\mathbf{R_M}^{-1}\Delta_n\right] + \Delta_n^\dagger\mathbf{R_M}^{-1}\Delta_n \tag{13}$$

The perturbation Δ_n hence reduces the likelihood metric if

$$L_{\Delta_n} < 0 \tag{14}$$

Knowing that the perturbation Δ_n has only one non-zero component, we obtain

$$b = \Delta_n^\dagger\mathbf{R_M}^{-1}\Delta_n = |\delta|^2\mathbf{R_M}_{n;n}^{-1} = |\delta|^2\alpha \tag{15}$$

with

$$\alpha = \frac{1}{N}\sum_{n=1}^{N}\frac{1}{\sigma_n^2} \tag{16}$$

This result on α is guaranteed when any one of the previously mentioned precoding/decoding matrices is used. α is a constant. In the case of a regularly spaced constellation, such as standard normalized QAM, δ_{re} and $\delta_{\mathrm{im}} \in \{0; \pm 2\}$ (see Fig. 2 for an example). Define the Nx1 vector

$$A = \left[\left(\hat{X}_0 - \tilde{X}\right)^\dagger\mathbf{R_M}^{-1}\right]^T = A_{\mathrm{re}} + jA_{\mathrm{im}} \tag{17}$$

the other term of (13) is

$$\begin{aligned} 2\mathrm{Re}\left[\left(\hat{X}_0 - \tilde{X}\right)^\dagger\mathbf{R_M}^{-1}\Delta_n\right] &= 2\mathrm{Re}\left[\delta A(n)\right] \\ &= 2\left(\delta_{\mathrm{re}}A_{\mathrm{re}}(n) - \delta_{\mathrm{im}}A_{\mathrm{im}}(n)\right) \end{aligned} \tag{18}$$

The modification Δ_n is of interest if and only if

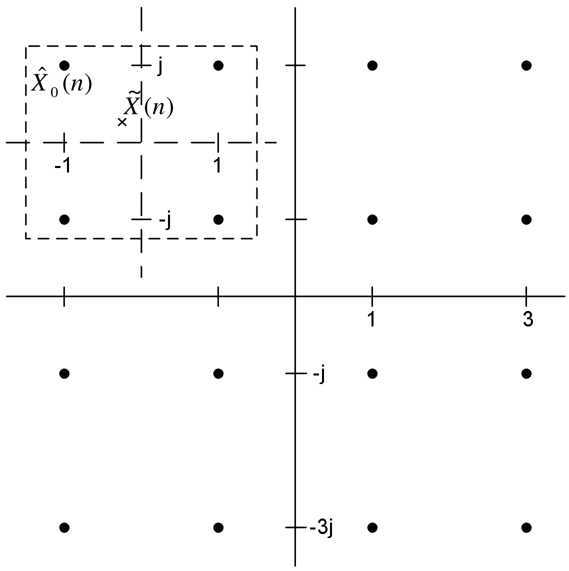

Fig. 2. Example of tested 4-PSK in the case of 16-QAM

$$2\left(\delta_{\text{re}} A_{\text{re}}(n) - \delta_{\text{im}} A_{\text{im}}(n)\right) + \left(\delta_{\text{re}}^2 + \delta_{\text{im}}^2\right)\alpha < 0 \tag{19}$$

To get the most negative metric modification, extensive testing of all combinations of real and imaginary parts ($\delta_{\text{re}}, \delta_{\text{im}} \in \{0, \pm 2\}$), and independent testing on those two parts let the same result. Thus (19) splits without loss of performance into

$$\begin{cases} 2\delta_{\text{re}} A_{\text{re}}(n) + \delta_{\text{re}}^2 \alpha < 0 \\ -2\delta_{\text{im}} A_{\text{im}}(n) + \delta_{\text{im}}^2 \alpha < 0 \end{cases} \tag{20}$$

Those separate evaluations reduce the total number of tests to be performed, hence the global complexity. The improvement can be tested independently on each tone (from (19) and (20)), thus removing the need for joint optimization.

3.2. Detection algorithm

We now develop the simplified B-G detection algorithm of complex-valued signals. We assume Gray mapping and ± 1 bit labeling. Using Gray mapping (usually done as it limits the BER), allows us to consider that any 4-point neighborhood surrounding $\tilde{X}$ is strictly equivalent to a 4-PSK. Labeling bits with ± 1 instead of $\{0; 1\}$ will allow to simplify test expressions.

We set the initial detected vector to $\hat{X}_0 = \text{MSE}\left[\tilde{X}\right]$. That is, we decide, on each tone, on the closest symbol. This basic detection scheme is the one reaching Jensen's bound [1]. At this point, if the used constellation is larger than 4-PSK, we consider only the 4-PSK surrounding $\tilde{X}$. Fig.2 illustrates this procedure, in the case of a 16-QAM. We note $b_{\text{re}}(n)$ and $b_{\text{im}}(n)$ the bits coded respectively on the real and imaginary parts of the neighboring 4-PSK, $n \in \{1 \ldots N\}$. Fig.2 shows one tone using 16-QAMs. The considered 4-PSK to be tested is the one in the second quadrant (denoted by the dashed box). In this example, the received signal $\tilde{X}(n)$ will create an MSE estimate $\hat{X}_0(n) = -3 + 3j$ in the 16-QAM, which corresponds to $b_{\text{re}}(n) = -1$ and $b_{\text{im}}(n) = 1$ in the tested 4-PSK. In the case of a 16-QAM, only 2 of the 4 coded bits are considered for modification.

The conditions on the permutation of $b_{\text{re}}(n)$ and $b_{\text{im}}(n)$ are, once A is computed, obtained directly from (20) by inspection of the possible cases. If $b_{\text{re,im}}(n) = 1$, then the only reasonable perturbation of the (Re,Im)-part of this tone is $\delta_{\text{re,im}} = -2$. Conversely, if $b_{\text{re,im}}(n) = -1$, then $\delta_{\text{re,im}} = 2$ is the only perturbation that allows to stay in the considered 4-PSK. The permutation of the detected bits occurs if:

$$\begin{cases} b_{\text{re}}(n) = -b_{\text{re}}(n) \text{ iif } A_{\text{re}}(n) b_{\text{re}}(n) > \alpha \\ b_{\text{im}}(n) = -b_{\text{im}}(n) \text{ iif } -A_{\text{im}}(n) b_{\text{im}}(n) > \alpha \end{cases} \tag{21}$$

We used the hypothesis that constellation points are spaced like in Fig.2. Using a different constellation may result in minor modification of this test. This is a simple threshold test. After updating all tones, we get the new estimate $\hat{X}_1$. In this procedure, A is never updated, as there is no true joint detection.

The algorithm can be run iteratively, after updating A using

$$A_{i+1} = \left(\hat{X}_i + \sum_{\substack{n=1 \\ n \in D}}^{N} \Delta n - \tilde{X}\right)^{\dagger} \mathbf{R_M}^{-1} \tag{22}$$

D is the set of the perturbations that reduce the likelihood cost function. However, the additional gain is typically low compared to the benefit of the first iteration, as the algorithm saturates quickly. The steps of the algorithm, in the case of QAM, are the following:

1. Compute $\mathbf{R}_M^{-1}$ as in (9), compute α as in (16)
2. Compute $\hat{X}_0 = \text{MSE}\left[\tilde{X}\right]$
3. Compute A as in (17) (or (22) after the first iteration)
4. In the case of constellation greater than 4-PSK, isolate the 4-PSK to be tested
5. Determine b_{re} and b_{im}
6. Perform (21) for $n = 1 \ldots, N$, obtain $\hat{X}_1$
7. If one more iteration is required, $\hat{X}_0 = \hat{X}_1$, goto step 3

Eq. (21) shows that the detection rule is simpler than the initial negative log-likelihood minimization function (8). In (21), $4N$ tests are performed before a new matrix multiplication is required (only estimating vector A requires it). Steps 3 of the algorithm is the most expensive in terms of number of operations. Computing $\mathbf{R}_M^{-1}$ and α (step 1) happens only once for a given channel, so its contribution to global complexity depends on the channel coherence time.The remaining steps of the algorithm present relatively low complexity.

When all the residual noise variances are equal ($\sigma_n^2 = c\ \forall n$), $\mathbf{R}_M^{-1}$ reduces to a diagonal matrix (from (9)). In this case, there is no gain at all, as MSE, B-G and even ML detections are equivalent. Hence, the detection method can be selected at the receiver once the SNRs have been estimated.

3.3. Example

As an example, we show the performance over one of the indoor wireless channels prototyped by the IEEE group for Ultra Wide-Band systems (802.15.3a) [6]. Fig. 3 shows the residual noise variances on each tone without precoding, normalized by the largest variance. Fig. 4 shows the performance obtained without precoder, and with precoder and MSE (Jensen's bound) or B-G detections (one iteration). In this plot, the SNR is defined as SNR $= \sigma_s^2 \alpha$. In this example, fifty 4-PSKs are combined ($N = 50$). As predicted by Jensen's inequality, precoding improves the performance substantially in the region of $\cup$ convexity of the BER vs. SNR function. Additionally, B-G detection increases performance by about 2 dB for a BER $= 10^{-5}$, in one iteration. Fig.5 presents a close-up of Fig. 4, also showing the performance obtained with 2 and 5 iterations of the B-G algorithm. As expected, the extra performance obtained with additional iterations shrinks very fast. In this case, there is no need for more than 2 iterations.

4. CONCLUSION

Starting from the joint maximum likelihood detection of blindly precoded OFDM, we derived a permutation based iterative algorithm that improves the MSE detection with low complexity. This algorithm outperforms the performance of independently decoded systems typically by several dB over multipath channel, making this approach a good candidate to improve the performance of OFDM systems in difficult channel conditions. It can be noticed in Fig. 4 that performance is increased even outside of the region of convexity $\cup$ of the BER vs. SNR function. For low SNR, independent detection (Jensen's bound) is not as good as unprecoded transmission, while B-G dectection still creates an improvement. However, the exact boundary of the region of improvement is not known at this point and determining it would be of great interest. Low complexity implementation of the algorithm could lead to further optimization of each step. Ongoing work includes the generalization of this approach to any type of channel and precoder.

5. REFERENCES

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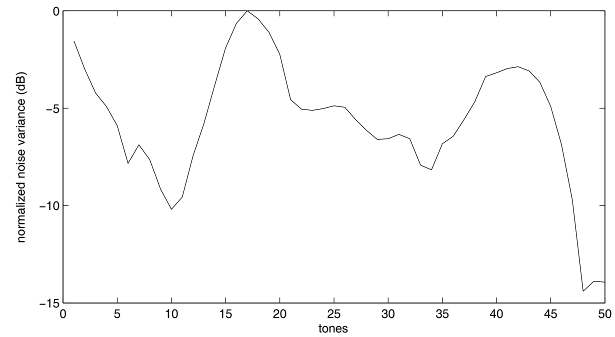

Fig. 3. Normalized residual noise variance

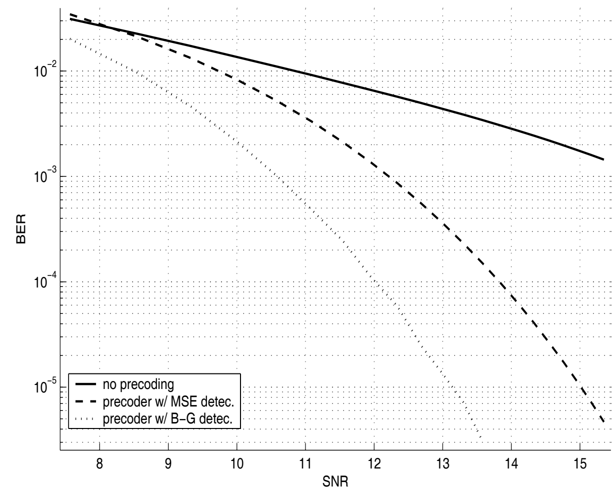

Fig. 4. Performance with MSE detection, B-G detection (1 iteration), and without precoding for 50 QPSKs

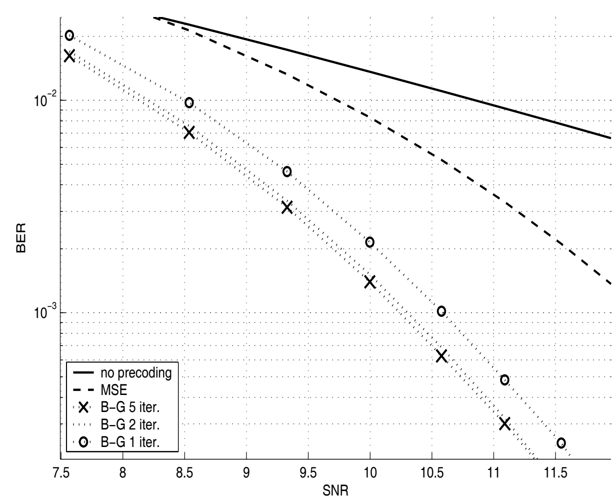

Fig. 5. Performance comparison for 1,2 and 5 iterations (detail)