PREDICTING THE PERFORMANCE AND CONVERGENCE BEHAVIOR OF A TURBO-EQUALIZATION SCHEME

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ABSTRACT

This paper proposes a simple and time non-consuming method to predict at any iteration the performance of a turbo equalization scheme using a soft-in/soft-out (SISO) Minimum Mean Square Error (MMSE) / Interference Cancellation (IC) equalizer and a SISO decoder. Gaussianity of the extrinsic Log-Likelihood Ratios (LLRs) output by the equalizer as well as the decoder is assumed. This paper shows that the equalizer behavior may then be very reliably predicted only by calculations (no simulations are needed) whereas that of the decoder requires simulations for only one independent input parameter. Comparison between the proposed prediction method and plain simulations of the overall turbo equalization scheme shows that our method accurately determines the system performance at any iteration.

1. INTRODUCTION

Turbo equalization (references in [1]) is a powerful mean to perform joint equalization and decoding when considering coded data transmission over frequency selective channels. The association of the code and the discrete-time equivalent channel (separated by an interleaver) may be regarded as the serial concatenation of two codes. The turbo principle may then be used at the receiver : the system performance measured in terms of the Bit Error Rate (BER) is improved through the exchange of extrinsic information between a SISO equalizer and a SISO decoder.

The goal of this paper is to predict the performance and convergence behavior of such a turbo equalization scheme at any iteration using a simple and time non-consuming method. Perfect channel knowledge is assumed. We will restrict ourselves to BPSK data modulation. The SISO MMSE/IC equalizer presented in [2] for multiuser context will be here used in a single user case. However, an extension of the proposed prediction method to both multiuser scenario and other linear equalizers is straightforward. A SISO convolutional decoder using the BCJR algorithm (reference in [3]) will be considered in the sequel although any other SISO decoder might also be used.

Various methods for predicting the convergence behavior of turbo decoding schemes as well as parallel or serially (references in [3]) concatenated codes have been previously proposed. They are based e.g. on variance or signal-to-noise ratio transfer analysis or extrinsic information transfer chart (EXIT chart). Almost all of them require simulations letting vary one input parameter for each of the two devices involved in the turbo process. The only exception to our knowledge is the calculation made in [4] of the output variance transfer function of a parallel interference canceller assuming gaussianity of the non-cancelled multiuser interference.

The prediction method proposed in this paper requires simulations letting vary one parameter at the decoder input only. Indeed, as we will show in the sequel, the equalizer behavior may be accurately and totally predicted by calculations (so without any simulation) using simplifying approximations. One of these is the gaussianity of the extrinsic LLRs output by the equalizer as well as the decoder. This approximation may be proved to be reliable by simulations and has already often been used e.g. in [3]. Once the decoder has been simulated for some fixed parameters (constraint length, code rate,...), our method enables to foretell the performance and convergence behavior of the equalized system given that decoder without further simulations for any frequency selective channel. Performance with the reference AWGN channel or with perfect knowledge of a priori information at the equalizer input may then also be directly obtained.

Another interest of the proposed prediction method is the following. Simulations of our overall turbo equalization scheme reveal that unlike what is assumed in [3] with iterative decoding the ratio between the mean and the variance of the LLRs at the equalizer input is not exactly equal to $1/2$. The behavior of our MMSE/IC equalizer may thus be more accurately predicted using two independent input

[*]This work is partly funded by the Federal Office for Scientific, Technical and Cultural Affairs, Belgium, through IAP contract No P5/11. V. Ramon would like to thank the Belgian FNRS for its financial support.

parameters (the mean and the variance of the input LLRs) rather than one (e.g. the mutual information used in [3]). Fortunately the computational method proposed in this paper is able to deal with independent mean and variance of LLRs at the equalizer input. It is all the more interesting as simulations of the equalizer behavior with two independent input parameters would be quite time prohibitive.

The sequel of this paper will be organized as follows. Section 2 will introduce the system model. Our method for predicting the performance of the turbo equalization scheme will be explained in section 3. Section 4 will compare for two types of channels the performance obtained with our prediction method to that obtained by plain simulations of the turbo equalizer. It will be observed that our method accurately determines the system performance at any iteration.

2. SYSTEM MODEL

In this section, the transmitter model, the overall iterative receiver and the main equations of the considered MMSE/IC equalizer will be successively presented. As almost no novelty is introduced with respect to the equalizer of [2], only the equations which will be useful for section 3 will be given. The only change will be a more accurate expression of the symbol estimate variance at the equalizer output. It will appear in subsection 2.3.

Most notations used hereafter will be the same as in [1] which extended to multi-level/phase modulation the SISO MMSE/IC equalizer of [2]. However, as already said in the introduction, we will here limit ourselves to BPSK data modulation.

2.1. Transmitter model

The transmission scheme is the same as in [1]. A frame of information bits u_k is encoded by a rate-r convolutional encoder. The resulting encoded bits x_l are interleaved using a random permutation function to give the interleaved coded bits x_i. These bits are then mapped onto BPSK symbols $s_i \in \{+1, -1\}$ according to $s_i = 2\,x_i - 1$. These BPSK symbols are transmitted over the channel, which is assumed to be static and perfectly known. At the receiver, matched filtering to the whole transmission chain, symbol-rate sampling and discrete-time noise whitening are successively performed. The channel may thus be represented by its equivalent discrete-time white noise filter model, i.e. a causal discrete-time filter with coefficients $h_j\,(j = 0, \ldots, L)$ corrupted by white gaussian noise samples n_i of variance σ_n^2. The symbols r_i at the output of the channel may thus be expressed as $r_i = \sum_{j=0}^{L} h_j\, s_{i-j} + n_i$.

2.2. Overall iterative receiver

The block scheme of the iterative receiver is more described in [1]. It is a classical turbo-equalizer which consists of two stages : a SISO equalizer and a SISO decoder separated by a bit-deinterleaver and a bit-interleaver. Those two stages exchange extrinsic information, on iterative fashion, in order to improve the system performance. The decoder considered in the sequel will be implemented using the well-known BCJR algorithm although any other SISO decoder might be used.

2.3. Main equations of equalizer

As already said, the SISO MMSE/IC equalizer presented in [2] for multiuser context will be here used in a single user case. Only its main equations will be reminded. Defining the equalizer length as $N \triangleq N_1 + N_2 + 1$, we introduce a sliding-window model using the vectors

$$\begin{aligned}
\mathbf{r}_i &\triangleq [r_{i-N_1}\ \ldots\ r_i\ \ldots\ r_{i+N_2}]^T_{N\times 1}\\
\mathbf{s}_i &\triangleq [s_{i-N_1-L}\ \ldots\ s_i\ \ldots\ s_{i+N_2}]^T_{(N+L)\times 1}\\
\mathbf{n}_i &\triangleq [n_{i-N_1}\ \ldots\ n_i\ \ldots\ n_{i+N_2}]^T_{N\times 1}
\end{aligned} \tag{1}$$

and the $(N \times (N+L))$-channel matrix

$$\mathbf{H} = \begin{bmatrix} h_L & \ldots & h_0 & 0 & \ldots & \ldots & 0\\ 0 & h_L & \ldots & h_0 & 0 & \ldots & 0\\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots\\ 0 & \ldots & \ldots & 0 & h_L & \ldots & h_0 \end{bmatrix}. \tag{2}$$

At each time step i, we may then write

$$\mathbf{r}_i = \mathbf{H}\,\mathbf{s}_i + \mathbf{n}_i \tag{3}$$

where $\mathbf{n}_i \sim \mathcal{N}_c(0, \sigma_n^2\mathbf{I})$, $\mathbf{I}$ being the $N \times N$ identity matrix.

Let $\bar{s}_i \triangleq E\{s_i\}$ and $\mathrm{var}\{s_i\} \triangleq 1 - \bar{s}_i^2$ denote respectively the so-called a priori mean and variance of symbol s_i which are computed as explained in [1] from the a priori LLRs at the equalizer input. Using the following definitions

$$\bar{\mathbf{s}}_i = [\bar{s}_{i-N_1-L}\ \ldots\ \bar{s}_{i-1}\ \ 0\ \ \bar{s}_{i+1}\ \ldots\ \bar{s}_{i+N_2}]^T \tag{4}$$

$$\begin{aligned}
\mathbf{R}_{\mathbf{ss},i} &= \mathrm{diag}[\mathrm{var}\{s_{i-N_1-L}\}\ \ldots\ \mathrm{var}\{s_{i-1}\}\ \ 1\\
&\qquad \mathrm{var}\{s_{i+1}\}\ \ldots\ \mathrm{var}\{s_{i+N_2}\}]\\
\mathbf{w}_i &= [\mathbf{H}\mathbf{R}_{\mathbf{ss},i}\mathbf{H}^H + \sigma_n^2\mathbf{I}]^{-1}\mathbf{H}\mathbf{e}
\end{aligned} \tag{5}$$

the symbol estimate $\hat{s}_i$ is given by

$$\hat{s}_i = \mathrm{Re}\left\{\mathbf{w}_i^H[\mathbf{r}_i - \mathbf{H}\,\bar{\mathbf{s}}_i]\right\} \tag{6}$$

where $\mathbf{e}$ denotes a length-$(N+L)$ vector of all zeros except for the $(N_1 + L + 1)$th element, which is 1.

An efficient approximation (see [5] and reference therein) leading to a complexity reduction of the equalizer and weak performance degradation may be obtained by computing the mean a priori variance over the L_s transmitted symbols in the frame

$$v \triangleq \frac{1}{L_s}\sum_{i=1}^{L_s} \mathrm{var}\{s_i\} \tag{7}$$

which enables to define a constant (i.e. independent of i) matrix $\mathbf{R_{ss}} = \text{diag}[v\ \ldots\ v\ 1\ v\ \ldots\ v]$. Consequently $\mathbf{w}_i$ calculated with $\mathbf{R_{ss}}$ instead of $\mathbf{R}_{\mathbf{ss},i}$ does not depend on i either and will be denoted by $\mathbf{w}$.

At the output of the equalizer, we assume that the estimate $\hat{s}_i$ is the output of an equivalent AWGN channel having s_i as its input : $\hat{s}_i = \mu\, s_i + \nu_i$ where ν_i is a real noise and $\nu_i \sim \mathcal{N}(0, \sigma_\nu^2)$. Parameters μ and σ_ν^2 may be easily calculated as follows

$$\mu = \mathbf{w}^H \mathbf{H} \mathbf{e} \tag{8}$$

$$\sigma_\nu^2 = \frac{\sigma_n^2}{2}(\mathbf{w}^H\mathbf{w}) + \text{Re}(\mathbf{w}^H\mathbf{H})\mathbf{R_{ss}}\text{Re}(\mathbf{H}^H\mathbf{w}) - \mu^2. \tag{9}$$

Equation (9) gives a more accurate expression of the symbol estimate variance at the equalizer output than [2], which turns out be essential for the prediction method described in section 3. Reference [2] assumed for simplicity that the real and imaginary parts of estimate $\hat{s}_i = \mathbf{w}_i^H[\mathbf{r}_i - \mathbf{H}\bar{\mathbf{s}}_i]$ (i.e. (6) before taking the real part) had the same variance which is not exactly the case. Reference [2] shows that in the case of BPSK data modulation the equalizer outputs extrinsic LLRs on the interleaved coded bits x_i given by

$$L_e^{EQ}(x_i) = \frac{2\,\mu}{\sigma_\nu^2}\hat{s}_i. \tag{10}$$

3. METHOD FOR PREDICTING THE TURBO-EQUALIZER PERFORMANCE

In this section, a simple method for predicting the performance of the turbo equalization scheme will be developed. The idea is to predict the behavior of the turbo equalizer by solely looking separately at the input/output relations of each constituent stage (equalizer, decoder). More precisely, the mean and variance of the extrinsic LLRs output by each of the two stages will be determined as a function of the mean and the variance of their input LLRs, called a priori LLRs. This will be done in the two following subsections. Interleaver and de-interleaver do not of course affect those input/output relations. Gaussianity of the extrinsic LLRs output by the two stages will be assumed like in [3]. Simulations may illustrate the validity of this assumption.

3.1. Predicting the equalizer behavior

The a priori equalizer input LLRs on the interleaved coded bits are assumed to be Gaussian-like distributed

$$L_a^{EQ}(x_i) = \mu_a^{EQ}\, s_i + n_{a,i}^{EQ} \tag{11}$$

with $n_{a,i}^{EQ} \sim \mathcal{N}(0, \sigma_a^{2,EQ})$. It may be easily shown from the definition of LLR that $\bar{s}_i = \tanh(0.5\, L_a^{EQ}(x_i))$ and $\text{var}\{s_i\} = 1 - \tanh^2(0.5\, L_a^{EQ}(x_i))$. Given this latter expression and (7), the mean of v calculated over the Gaussian distribution of the equalizer input LLRs may be expressed as

$$\bar{v}(\mu_a^{EQ}, \sigma_a^{2,EQ}) = 1 - \int_{-\infty}^{+\infty} \tanh^2\left(\frac{y}{2}\right)\frac{1}{\sqrt{2\pi\sigma_a^{2,EQ}}} \left(\frac{1}{2}\exp\left(\frac{-(y-\mu_a^{EQ})^2}{2\sigma_a^{2,EQ}}\right) + \frac{1}{2}\exp\left(\frac{-(y+\mu_a^{EQ})^2}{2\sigma_a^{2,EQ}}\right)\right) dy \tag{12}$$

where the two terms in the integral accounts respectively for $s_i = +1$ and $s_i = -1$. This integral may be computed numerically for any values of μ_a^{EQ} and $\sigma_a^{2,EQ}$.

If we assume long enough frames, the variance of v around its mean may be neglected. Then we approximate v by a constant value, namely its mean $\bar{v}(\mu_a^{EQ}, \sigma_a^{2,EQ})$. Let now $\bar{\mathbf{R}}_{\mathbf{ss}}$ denote matrix $\mathbf{R_{ss}}$ when replacing v by $\bar{v}(\mu_a^{EQ}, \sigma_a^{2,EQ})$, $\bar{\mathbf{w}}$ denote vector $\mathbf{w}$ when replacing by $\mathbf{R_{ss}}$ by $\bar{\mathbf{R}}_{\mathbf{ss}}$, $\bar{\mu}$ and $\bar{\sigma}_\nu^2$ respectively denote μ and σ_ν^2 when replacing $\mathbf{R_{ss}}$ and $\mathbf{w}$ by $\bar{\mathbf{R}}_{\mathbf{ss}}$ and $\bar{\mathbf{w}}$. Given $\hat{s}_i = \mu\, s_i + \nu_i$, equation (10) and the previous definitions which consider μ and σ_ν^2 as constants $\bar{\mu}$ and $\bar{\sigma}_\nu^2$, the mean and variance of the extrinsic LLRs output by the equalizer are given by

$$\mu_e^{EQ} \triangleq E\{L_e^{EQ}(x_i)|x_i = 1\} = \frac{2\,\bar{\mu}^2}{\bar{\sigma}_\nu^2} \tag{13}$$

$$\sigma_e^{2,EQ} \triangleq \text{var}\{L_e^{EQ}(x_i)\} = \frac{4\,\bar{\mu}^2}{\bar{\sigma}_\nu^2} = 2\,\mu_e^{EQ}. \tag{14}$$

They are both functions of μ_a^{EQ} and $\sigma_a^{2,EQ}$ via $\bar{v}(\mu_a^{EQ}, \sigma_a^{2,EQ})$ and of σ_n^2 via $\bar{\mathbf{w}}$. The equalizer behavior may thus totally be predicted by the preceding calculations since we have expressed the mean and the variance of its output LLRs as functions of the mean and variance of its input LLRs and of noise variance. The computational complexity of these calculations is very low since the sizes of vectors and matrixes involved in calculations of μ_e^{EQ} and $\sigma_e^{2,EQ}$ are small and independent of the frame length.

3.2. Predicting the decoder behavior

Unlike the equalizer, the decoder behavior may only be predicted by simulations. However, these simulations are not too time demanding since they let vary only one parameter at the decoder input : the mean of its a priori LLRs which is assumed to be Gaussian distributed. Indeed, the a priori decoder input LLRs are the de-interleaved extrinsic LLRs output by the equalizer. As we may see from (13) and (14), the mean and the variance of these LLRs are not independent since $\sigma_e^{2,EQ} = 2\,\mu_e^{EQ}$. Simulations of the decoder behavior may thus be performed by letting vary with a certain step the mean of its input LLRs from 0 up to a value regarded as great enough. For each of these simulated values, the mean and the variance of the extrinsic LLRs output by the decoder are stored in a look-up table. As we are interested in the system performance, the output BER corresponding to each simulated value is also retained.

3.3. Conclusion

As each constituent stage may now be characterized by its input/output relations, the overall system performance and convergence behavior may be easily predicted at any iteration. Starting from null a priori equalizer LLRs at the first iteration, the method proceeds then step by step by calculating (for the equalizer) or picking (for the decoder) the output parameters as functions of the inputs. Once the decoder has been simulated for some fixed parameters of the code, performance of the equalized system may be predicted given that decoder without further simulations for any frequency selective channels. System performance with the AWGN channel may also be obtained (case when $h_0 = 1$ and $h_j = 0\ \forall j \neq 0$) as well as performance with perfect a priori information at the equalizer input. For this latter case, var$\{s_i\}$ and $\bar{v}(\mu_a^{EQ}, \sigma_a^{2,EQ})$ just need to be replaced by 0 everywhere in the previous equations since $\bar{s}_i^2$ is then equal to 1 $\forall i$. Actually it may be easily shown that with the chosen equalizer those two cases are equivalent. The efficiency of this prediction method will be illustrated in next section.

4. RESULTS

We consider a rate-$1/2$ convolutional encoder with constraint length $K = 3$ and generator polynomials $[5_8, 7_8]$. Decoding is performed with well-known BCJR algorithm. In the sequel we consider length-11 Proakis A channel [0.04,-0.05,0.07,-0.21,-0.5,0.72,0.36,0.0,0.21,0.03,0.07] and length-5 Porat channel [2-0.4j,1.5+1.8j,1,1.2-1.3j,0.8+1.6j]. Both of them have to be normalized. Simulations have been run for frames of 1024 BPSK coded symbols and 6 turbo iterations. Fig. 1 shows for iterations 1 and 2 the BER versus E_b/N_0 obtained with Proakis A channel, $N_1 = 0$ and $N_2 = 10$. Figure 2 gives for iterations 1, 2 and 6 the performance obtained with Porat channel, $N_1 = 3$ and $N_2 = 7$. In each figure the solid curves represent the results obtained with plain simulations of the turbo equalization chain whereas the dashed curves are for results obtained with the proposed prediction method. The dashed bold curves with circles show the AWGN channel case or equivalently the perfect equalizer a priori information case. In this AWGN case, the curve obtained with pure simulations turned out to be indiscernible from that resulting from the application of the proposed method. In both figures solid and dashed curves are very close to each other and even tend to be indistinguishable which proves the efficiency of the prediction method.

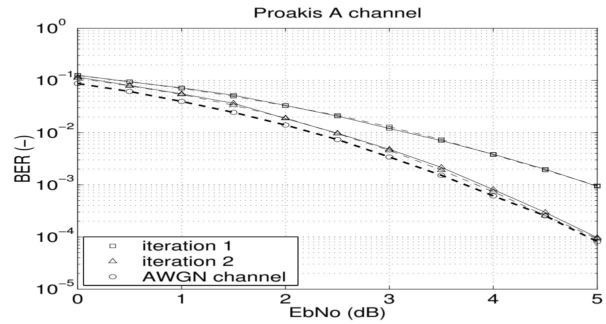

Fig. 1. Proakis A channel

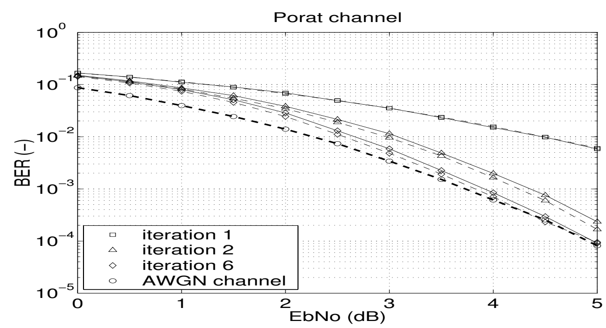

Fig. 2. Porat channel

5. CONCLUSION

This paper proposes a simple and time non-consuming method for predicting at any iteration the performance and convergence behavior of a turbo-equalization scheme. Efficiency of the method has been proved in section 4. An extension of this method to multiuser [2] and MIMO [5] turbo detection using the same kind of MMSE/IC equalizer is straightforward as the main equations of the equalizer keep the same form (only the size and entries of the channel matrix need to be changed) in those cases.

6. REFERENCES

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