ON OPTIMAL TRANSMISSION ALGORITHMS FOR SLOTTED ALOHA SENSOR NETWORKS WITH MULTI-PACKET RECEPTION

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ABSTRACT

In this paper we utilize decentralized channel state information (CSI) for designing optimal transmission schemes for slotted ALOHA sensor networks that have multi-packet reception capability. We prove that under certain conditions the optimal transmit probability function is deterministic, i.e. it is optimal for sensors to either transmit or not transmit with certainty depending on their channel states. We present a provably convergent stochastic approximation optimization algorithm to estimate the optimal transmit policy. Numerical studies illustrate the performance of the algorithm and the degenerate, non-randomized structure of the optimal transmission policy.

1. INTRODUCTION

The simplicity and distributed nature of ALOHA make it particularly attractive for sensor networks. Traditionally, ALOHA is based on the collision model which assumes that all packets are lost when two or more users transmit at the same time. However, this is not the case in many communication systems such as CDMA systems. [2] introduced the Multi-Packet Reception (MPR) model that allows modelling systems where one or more packets can be received correctly with probabilities in the presence of simultaneous transmissions. Important results related to this MPR model include the possibility of a positive stable asymptotic throughput [3], [11],[4] and an algorithm to achieve the best asymptotic throughput [3] without CSI. The introduction of MPR model has great impact on the studies of ALOHA. However, its two limitations are the decoupling between the physical (PHY) layer and the MAC layer and the total symmetry between users. [1] proposed a Generalized MPR (G-MPR) model that interfaces the two layer: the reception of packets depends not only on the number of user transmit but also on the channel states of the transmitting users. Under this reception model it is natural to design transmission algorithms using a cross-layer approach, i.e. taking into account the channel states of users. [1] gives bounds on the maximal achievable asymptotic throughput under G-MPR. [1] also gives an expression for the maximum stable throughput for finite user slotted ALOHA systems deploying G-MPR. In this paper we follow the G-MPR model and use a cross-layer approach to design optimal transmission algorithms for sensor networks.

The contributions of this paper include:

1. We prove that under certain conditions the optimal transmission policy has a non-randomized structure: it is optimal for sensors to transmit with probabilities 0 or 1 depending on the channel states. This is a surprising result as probabilistic transmissions are necessary under the collision model or the MPR model without CSI [3].
2. We propose a provably convergent stochastic algorithm to estimate optimal transmission probabilities for a finite-sensor slotted ALOHA system deploying the G-MPR model in [1]. We illustrate the performance of the algorithm in numerical examples.

Utilizing decentralized CSI for optimal transmissions we obtain a variant of slotted ALOHA that is highly scalable and efficient. These properties are particularly useful for sensor networks where the number of nodes is large and energy is an important issue.

This paper is organized as follows: Section 2 briefly covers the system model and problem formulation. Section 3 gives a theorem on the structure of the optimal transmission policy. We propose a stochastic approximation algorithm for estimating the optimal policy and prove its convergence in Section 4. Section 5 presents numerical examples.

2. G-MPR MODEL AND PROBLEM FORMULATION

Let $i = 1, 2, \ldots, K$ ($K < \infty$) index the K sensors in a slotted ALOHA system with multipacket reception capability. Let the random variable γ_i denote the channel state

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of sensor i. We assume that every sensor knows its channel state at the beginning of each transmission slot. Define $\vec{\gamma}^{(k)} = (\gamma_1, \dots, \gamma_k)$ for any $k \le K$. We assume that the channel distributions of all sensors are i.i.d. and denote it by $F(.)$. This assumption is appropriate for large scale sensor networks where the sensors are approximately equi-distant from the base station. It is also an assumption in [1]. Let $p(.) : R_+ \to [0,1]$ be a function mapping channel states to transmit probabilities. Our objective in this paper is to find the optimal $p(.)$ that can be deployed by all sensors to maximize the maximum system stable throughput.

We follow the G-MPR model proposed in [1]. The set of possible outcomes when only k ($k \le K$) sensors with channel states $\vec{\gamma}^{(k)}$ transmit is the set of binary k-tuples:

$$\Theta_k = \left(\theta_1^{(k)}, \dots, \theta_k^{(k)}\right)$$

$\theta_i^{(k)} = \{0,1\}; i = 1, \dots, k$. $\theta_i = 1$ indicates that the packet from sensor i is correctly received and $\theta_i = 0$ indicates otherwise. We are concerned with the expected total number of packets that are received correctly. This information is summarized in the following set of K functions:

$$\Psi^{(k)}(\vec{\gamma}^{(k)}) \triangleq \sum_{i=1}^{k} \mathbf{E}\left[\theta_i^{(k)} | k \text{ sensors transmit}, \vec{\gamma}^{(k)}\right] \quad (1)$$

Using the reception model (1) the maximum system stable throughput is given in [1]:

$$L_K(p(.)) = \sum_{k=1}^{K} \binom{K}{k} (1-\overline{p})^{K-k} \int_0^\infty \dots \int_0^\infty p(\gamma_1) \dots p(\gamma_k) \Psi^{(k)}(\vec{\gamma}^{(k)}) dF(\gamma_1) \dots dF(\gamma_k), \quad (2)$$

where $\overline{p} = \int_0^\infty p(\gamma) dF(\gamma)$ is the average transmit probability of each sensor. The above maximum stable throughput is the expected throughput when all sensors have data to transmit [1]. Our optimization is therefore equivalent to maximize the expected throughput when the system is fully loaded and can be formulated as :

$$\begin{array}{ll} \max\limits_{p(.)} & L(p(.)) \\ s.t & 0 \le p(.) \le 1 \end{array} \quad (3)$$

This optimization problem is infinite dimensional and cannot be solved analytically. Our approach in this paper is as follows: 1) We first prove that under certain conditions the optimal transmit probability function is either 1 or 0. 2) We then use the stochastic gradient ascent method to estimate the optimal transmit probability function.

3. OPTIMAL TRANSMISSION POLICY

Theorem 1. *Consider a slotted ALOHA system of K sensors as described in Section 2. Use the reception model (1). Assume that there exists uniquely an optimal transmission probability function $p^*(.) : R_+ \to [0,1]$, which maximizes the expected system throughput (2) in the sense that if one or more sensors deviate from $p^*(.)$ the expected system throughput decreases. Then $p^*(.)$ satisfies:*

$$p^*(.) \in \{0,1\} \quad (4)$$

Proof. Let $p^*(.)$ be the optimal transmit probability function. Let sensors $1, \dots, K-1$ use $p^*(.)$ and denote the throughput obtained by these $K-1$ sensors when sensor K does not transmit by $L^{(K-1)}(p^*(.))$. We aim to find a transmit probability function $p(.)$ for sensor K to maximize the system throughput. $p^*(.)$ must be the unique solution.

If sensor K has channel state realization $\overline{\gamma_K}$ and transmits with probability $p(\overline{\gamma_K})$, it causes a drift $D(\overline{\gamma_K})$ in the throughput:

$$\begin{aligned} D(\overline{\gamma_K}) &= p(\overline{\gamma_K}) \sum_{k=0}^{K-1} \binom{K-1}{k} (1-\overline{p}^*)^{K-1-k} \int_0^\infty \dots \\ &\int_0^\infty p^*(\gamma_1) \dots\ p^*(\gamma_k) \left(\Psi^{(k+1)}(\vec{\gamma}^{(k)}, \overline{\gamma_K}) - \Psi^{(k)}(\gamma^{(\vec{k})})\right) \\ &dF(\gamma_1) \dots dF(\gamma_k) \\ &= p(\overline{\gamma_K}) h(\overline{\gamma_K}) \end{aligned}$$

Maximizing the drift point-wise, i.e. for all $\overline{\gamma_K}$, will automatically maximize the throughput. Hence, selecting

$$p(\overline{\gamma_K}) = \begin{cases} 1 & \text{if } h(\overline{\gamma_K}) > 0 \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

for all $\overline{\gamma_K} \in [0, \infty)$ will optimize the system throughput. $p^*(.)$ is the unique solution and hence must be (5), which satisfies (4). □

Remarks: The assumption that the global optimum exists uniquely is not satisfied under the collision model but it can be satisfied under the G-MPR model, for example under the Signal to Interference Noise Ratio (SINR) threshold receiver model for CDMA systems with matched filters [8].

4. ESTIMATE THE OPTIMAL POLICY

In this section we divide the channel state into a finite number of regions and estimate an optimal transmit probability for each region. Selecting the uplink Signal to Noise Ratios to represent channel states we approximate the transmit probability function as below:

$$p_{\vec{\theta}}(\gamma) = \sum_{i=1}^{M} \sin^2 \theta_i I_{\alpha_i \le \gamma < \alpha_{i+1}},$$

where I is the indicator function; M and $\alpha_i, i = 1, 2 \dots M$ ($\alpha_i < \alpha_j\ \forall i < j$) define the channel state (SNR) regions and can be chosen arbitrarily. Define $\vec{\theta} \triangleq (\theta_1, \dots, \theta_M)$.

The system stable throughput (2) can be rewritten as:

$$L(\vec{\theta}) = \sum_{k=1}^{K} \binom{K}{k} (1-\overline{p}_{\vec{\theta}})^{K-k} \mathbf{E}_{F^k} \left[p_{\vec{\theta}}(\gamma_1) \dots p_{\vec{\theta}}(\gamma_k) \Psi^{(k)}(\vec{\gamma}^{(k)}) \right], \quad (6)$$

$$\text{where} \quad \overline{p}_{\vec{\theta}} = \sum_{i=1}^{M} \sin^2 \theta_i (F(\alpha_{i+1}) - F(\alpha_i)) \quad (7)$$

Reformulate the optimization problem (3) as:

$$\max_{\vec{\theta}} \quad L(\vec{\theta}) \quad (8)$$

Let $X_k = p(\gamma_1) \dots p(\gamma_k) \Psi^{(k)}(\vec{\gamma})$. The gradient of (6) with respect to θ_i can be computed as:

$$\nabla_{\theta_i} L(\vec{\theta}) = \sum_{k=1}^{K} \binom{K}{k} \Big[(k-K)(1-\overline{p}_{\vec{\theta}})^{K-k-1} (F(\alpha_{i+1}) - F(\alpha_i)) \sin(2\theta_i) \mathbf{E}_{F^k}[X_k] + (1-\overline{p})^{K-k} \nabla_{\theta_i} \mathbf{E}_{F^k}[X_k] \Big] \quad (9)$$

Estimating $\nabla_{\theta_i} L(\vec{\theta})$ requires estimations of $\mathbf{E}[X_k]$ and its gradient. To reduce variances we estimate these values conditioned on the event that all k sensors transmit, which we denote as event Y_k. The aposteriori channel state distribution of a sensor given that it transmits is computed in [6], [1]:

$$G_{\vec{\theta}}(\gamma) = \frac{\int_0^{\gamma} p_{\vec{\theta}}(\overline{\gamma}) dF(\overline{\gamma})}{\int_0^{\infty} p_{\vec{\theta}}(\overline{\gamma}) dF(\overline{\gamma})} = \frac{\int_0^{\gamma} p_{\vec{\theta}}(\overline{\gamma}) dF(\overline{\gamma})}{\overline{p}} \quad (10)$$

The corresponding density function is $g_{\vec{\theta}}(\gamma) = \frac{p_{\vec{\theta}}(\gamma) f(\gamma)}{\overline{p}}$. $\mathbf{E}[X_k]$ can be estimated using (11) and its gradient is computed using the score function method [9] in (12) :

$$\begin{aligned} \mathbf{E}[X_k] &= \mathbf{E}[\mathbf{E}[X_k | Y_k]] = \Pr(Y) \mathbf{E}_{F^k | Y_k} \left[\Psi^{(k)}(\vec{\gamma}^{(k)}) \right] \\ &= \overline{p}^k \mathbf{E}_{G^k_{\vec{\theta}}} \left[\Psi^{(k)}(\vec{\gamma}^{(k)}) \right] \end{aligned} \quad (11)$$

$$\begin{aligned} &\nabla_{\vec{\theta}} \mathbf{E}[X_k] = \overline{p}^k \nabla_{\vec{\theta}} \mathbf{E}_{G^k_{\vec{\theta}}} [\Psi^{(k)}(\vec{\gamma}^{(k)})] \\ &= \overline{p}^k \mathbf{E}_{G^k_{\vec{\theta}}} \left[\Psi^{(k)}(\vec{\gamma}^{(k)}) \left(\frac{\nabla_{\vec{\theta}} \left(g_{\vec{\theta}}(\gamma_1) \dots g_{\vec{\theta}}(\gamma_k) \right)}{g_{\vec{\theta}}(\gamma_1) \dots g_{\vec{\theta}}(\gamma_k)} \right) \right] \\ &= \overline{p}^k \mathbf{E}_{G^k_{\vec{\theta}}} \left[\Psi^{(k)}(\vec{\gamma}^{(k)}) \left(\frac{\nabla_{\vec{\theta}} g_{\vec{\theta}}(\gamma_1)}{g_{\vec{\theta}}(\gamma_k)} + \dots + \frac{\nabla_{\vec{\theta}} g_{\vec{\theta}}(\gamma_k)}{g_{\vec{\theta}}(\gamma_k)} \right) \right] \end{aligned} \quad (12)$$

We now define a stationary point, which is also a local maximizer of the system throughput as

$$\vec{\theta}^* = \{ \vec{\theta} : \nabla_{\vec{\theta}} L(\vec{\theta}) = 0, \nabla^2_{\vec{\theta}} L(\vec{\theta}) < 0 \} \quad (13)$$

Algorithm 1. *Optimal Transmit Policy Estimation*

- Initialization: $l = 1$; $\vec{\theta}^{(1)} = \theta_1, \dots, \theta_M$
- Sampling, evaluation of gradient and update loop
  while $|\nabla_{\vec{\theta}} L^{(l)}(\vec{\theta})| > 0$ **do**
  for $k = 1, \dots, K$ **do**
  Use Monte-Carlo simulations to estimate:
  $\mathbf{E}_{G_{\vec{\theta}^{(l)}}} \left[\Psi_l^{(k)}(\vec{\gamma}^{(k)}) \right]$
  $\mathbf{E}_{G_{\vec{\theta}^{(l)}}} \left[\Psi_l^{(k)}(\vec{\gamma}^{(k)}) \left(\frac{\nabla_{\vec{\theta}} g_{\vec{\theta}}(\gamma_1)}{g_{\vec{\theta}}(\gamma_1)} + \dots + \frac{\nabla_{\vec{\theta}} g_{\vec{\theta}}(\gamma_k)}{g_{\vec{\theta}}(\gamma_k)} \right) \right]$
  end for
  Compute $\nabla_{\vec{\theta}} L^{(l)}(\vec{\theta})|_{\vec{\theta}=\vec{\theta}^{(l)}}$ using (9),(7),(11),(12)
  $$\vec{\theta}^{(l+1)} = \vec{\theta}^{(l)} + \varepsilon_l * \nabla_{\vec{\theta}} L^{(l)}(\vec{\theta}) \quad (14)$$
  $l = l + 1$
  end while
- Conditions: $\sum_{l=1}^{l=\infty} \varepsilon_l = \infty$; $\sum_{l=1}^{l=\infty} \varepsilon_l^2 < \infty$ (C1)

Theorem 2. *The estimates $\vec{\theta}^{(l)}$ generated by Algorithm 1 converge w.p.1 to a local maximizer $\vec{\theta}^*$, defined in (13), of the system stable throughput (6).*

Proof. For a fixed initial $\vec{\theta}$, since the sequence $\nabla_{\vec{\theta}} L^{(l)}(\vec{\theta})$: $l = 1, 2 \dots$ are i.i.d, the above equation (14) is an instance of the well known Robbins Munro algorithm. The convergence of this algorithm (and much more general algorithms) is proved in [5] under the condition (C1) and uniform integrability of $\nabla_{\vec{\theta}} L^{(l)}(\vec{\theta})$. A sufficient condition for uniform integrability is that the channel distribution has finite variance. □

5. NUMERICAL STUDIES

We study slotted ALOHA CDMA networks with matched filter receivers and random signature sequences in Rayleigh Fading channels. We assume the uplink SNRs represent the channel states. We use a heuristic SINR threshold model as in [10]: the packet from sensor i is successfully received only if

$$\frac{P_j}{\sigma^2 + \frac{1}{N} \sum_{i \neq j} P_i} = \frac{\gamma_j}{1 + \frac{1}{N} \sum_{i \neq j} \gamma_i} > \beta,$$

where P_j is the received power of sensor i; $P_j = \gamma_j \sigma^2$; σ^2 is the noise variance.

Figure 1 compares the system throughputs of 3 schemes: exploiting decentralized CSI via algorithm 1, using the algorithm in [3] which does not exploit CSI, always letting sensors transmit. The throughput is improved with the size of the network only in scheme 1. In addition, the successful rate remains high (0.76) in scheme 1. This means that fewer transmissions are required to achieve the best system throughput. This is important when battery life is an issue which is often the case in sensor networks.

Figure 2 shows the convergence property of algorithm 1. For a network of 20 sensors, spreading gain S = 32, only 100 samples are drawn at each iteration, algorithm 1 converges to a good neighborhood of the optimal point after about 10 iterations. Due to the quantization of the channel state, the transmit probabilities are not purely 0 and 1 as suggested by theorem 2, but figure 2 illustrates theorem 2 very closely.

6. CONCLUSION

In this paper we discover that under certain conditions, the optimal transmit probability function has a degenerate structure. We present a convergent algorithm to design optimal transmit probabilities for slotted ALOHA sensor networks under the G-MPR model. Throughout the paper we assume decentralized channel state knowledge and long term symmetry between sensors. The latter can be relaxed when we use a game theoretic approach to solve the problem [7].

7. REFERENCES

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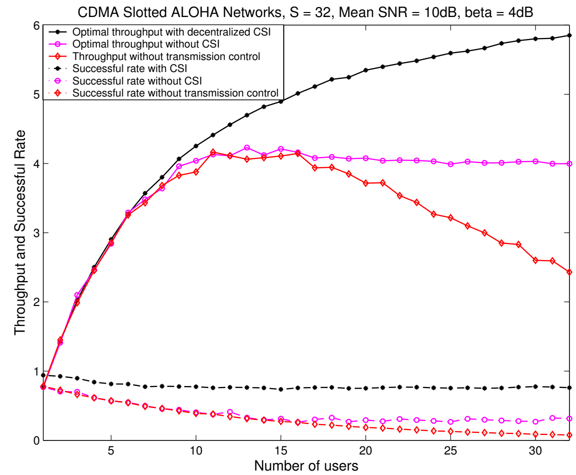

Fig. 1. Comparison between Transmission Algorithms

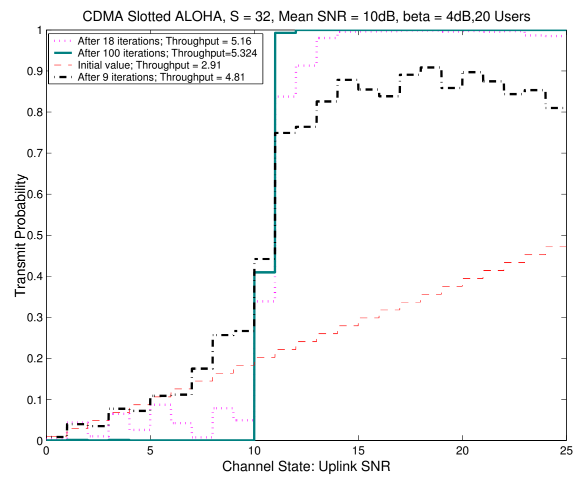

Fig. 2. Convergence of Algorithm 1

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