ON THE EFFECTS OF PHASE ESTIMATION ERRORS ON COLLABORATIVE BEAMFORMING IN WIRELESS AD HOC NETWORKS

Hideki Ochiai[*][†], *Patrick Mitran*[†], *H. Vincent Poor*[‡], *and Vahid Tarokh*[†]

[†]Division of Engineering and Applied Sciences, Harvard University

[*]Division of Electrical and Computer Engineering, Yokohama National University

[‡]Department of Electrical Engineering, Princeton University

ABSTRACT

The performance of collaborative beamforming is studied using the theory of random arrays in the framework of wireless sensor networks. With the application to ad hoc networks in mind, two scenarios, one denoted closed-loop and the other open-loop, are considered. Associated with these scenarios, the effects of phase jitter and location estimation errors on the average beampattern are analyzed.

1. INTRODUCTION

Recent advances in the construction of low cost, low power, and mass produced micro sensors and Micro-Electro-Mechanical (MEM) systems have ushered in a new era in system design using distributed sensor networks [1, 2]. In ad hoc sensor networks, *collaborative beamforming* [3, 4] has a significant potential to improve bandwidth-efficient communications. If the sensor nodes in the cluster share the information *a priori* and synchronously transmit their data collaboratively, it may be possible to beamform when transmitting (or receiving) data in a distributed manner and only in the specified target direction. This enables Space-Division Multiple Access (SDMA), a technology that has the potential to increase significantly the capacity of multiple access channels.

The obvious question is whether one can form a nice beampattern with a narrow mainbeam. As the sensor nodes in ad hoc networks are by nature located randomly, it is natural to treat the beampattern with probabilistic arguments. In the antenna design literature, Lo [5] has developed a comprehensive theory of linear random arrays in the late 1960's, and it has been shown that randomly generated linear arrays with large numbers of nodes can in fact form a good beampattern with high probability. (The theory of random arrays has been discussed and developed almost exclusively in the antenna design community, e.g., in [5, 6].) In previous work [3], we have shown that with N sensor nodes, a sharp mainbeam with sidelobe as low as $1/N$ can be formed with high probability, under the assumption that all nodes estimate their initial phases perfectly and transmit with perfect synchronization.

The major difference between classical beamforming by antenna arrays and distributed beamforming is that whereas the geometry of the former is usually known *a priori*, the exact location of the sensor nodes in an ad hoc network is not, and it should be acquired dynamically. Even if their relative location is estimated by some adaptive algorithm, considering the low SNR operation of the sensor nodes, it is certain that the acquired geometric information has some inaccuracy. Also, since all nodes are operated with physically different local oscillators, each node may suffer from statistically independent phase offsets. In order to model and investigate the effect of these impairments, we consider the following two scenarios: *closed loop* and *open loop*.

1) Closed loop: In this scenario, each node independently synchronizes itself to the beacon sent from the destination node (such as a base station) and adjusts its initial phase to it. Thus, the beam will be formed in the direction of arrival of the beacon. This kind of system is often referred to as a *self-phasing* array in the literature, and may be effective for systems operating in Time-Division Duplex (TDD) mode. The residual phase jitter due to synchronization and phase offset estimation among sensor nodes is then often the dominant impairment.

2) Open loop: Here, we assume that all nodes within the cluster acquire their relative locations from the beacon of a nearby reference point or cluster head. The beam will then be steered toward an arbitrary direction. Thus, the destination need not transmit a beacon, but each node requires precise knowledge of its relative position from a predetermined reference point within the cluster. This case may occur in ad hoc sensor networks where sensor nodes do not have sufficient knowledge of the destination direction *a priori*. The location estimation ambiguity among sensors may also affect the beampattern in this case.

In this paper, we extend our study of [3] to the case where the nodes do not have perfect phase information. Specifically, we analyze the impact of phase jitter or location estimation errors on the resultant beampattern in conjunction with the above two scenarios.

Throughout the paper, the nodes and channel are assumed to be static over the communication period, and for simplicity the information rate is sufficiently low that Inter-Symbol Interference (ISI), due to residual timing offset, is negligible. It will also be assumed that all nodes share the same transmitting information *a priori*, as the main focus of the paper is on the beampattern, rather than the front-end communication performance.

2. SYSTEM MODEL AND AVERAGE BEAMPATTERN

In this section, we review the system model and properties of the average beampattern discussed in [3]. The geometrical configuration of the distributed nodes and destination (or target) is illustrated in Fig. 1 where, without loss of generality, all the collabora-

This material is based upon research supported in part by the National Science Foundation under the Alan T. Waterman Award, Grant No. CCR-0139398, in part by the Office of Naval Research under Grant No. N00014-03-1-0102, and in part by a Fellowship from the John Simon Guggenheim Memorial Foundation. Any opinions, findings, and conclusions or recommendations expressed in this publication are those of the authors and do not necessarily reflect the views of the National Science Foundation.

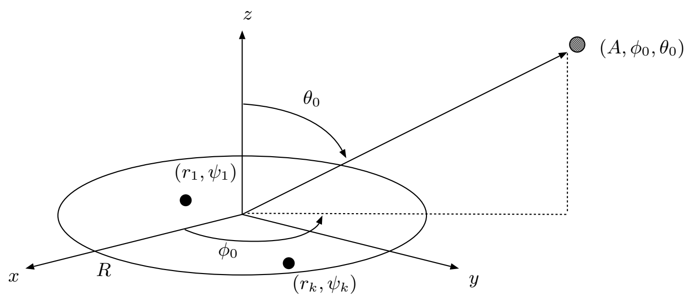

Fig. 1. Definitions of notation.

tive sensor nodes are assumed to be located on the x-y plane. The kth node location is thus denoted in polar coordinates by (r_k, ψ_k). The location of the destination is given in spherical coordinates by (A, ϕ_0, θ_0). Following the standard notation in antenna theory [7], the angle $\theta \in [0, \pi]$ denotes the elevation direction, whereas the angle $\phi \in [-\pi, \pi]$ represents the azimuth direction. In order to simplify the analysis, the following assumptions are made:

1. The location of each node is chosen randomly, following a uniform distribution within a disk of radius R.
2. Each node is equipped with a single ideal isotropic antenna.
3. All sensor nodes transmit identical energies, and the path losses of all nodes are also identical.
4. There is no multipath fading or shadowing.
5. Mutual coupling effects [7] among the antennas of different sensor nodes are negligible.

Of particular interest in practice is the case where $\theta_0 = \frac{\pi}{2}$, i.e., the destination node is in the same plane as the collaborative sensor nodes. Thus, for simplicity, we will assume that $\theta_0 = \frac{\pi}{2}$ for the rest of the paper. Let $d_k(\phi)$ denote the Euclidean distance between the kth node and the reference location $(A, \phi, \theta)|_{\theta=\pi/2}$, which is given by

$$d_k(\phi) = \sqrt{A^2 + r_k^2 - 2r_k A \cos(\phi - \psi_k)}. \tag{1}$$

In the closed-loop scenario, we assume that each node acquires accurate knowledge of the distance (relative to the wavelength of the radio frequency (RF) carrier λ) from the destination. By setting the initial phase of the node $k \in \{1, 2, \dots, N\}$ to

$$\Psi_k = -\frac{2\pi}{\lambda} d_k(\phi_0), \tag{2}$$

the corresponding array factor, given the realization of node locations $\boldsymbol{r} = [r_1, r_2, \dots, r_N] \in [0, R]^N$ and $\boldsymbol{\psi} = [\psi_1, \psi_2, \dots, \psi_N] \in [-\pi, \pi]^N$, is given by

$$F(\phi|\boldsymbol{r}, \boldsymbol{\psi}) = \frac{1}{N} \sum_{k=1}^{N} e^{j\Psi_k} e^{j\frac{2\pi}{\lambda} d_k(\phi)}, \tag{3}$$

where N is the number of sensor nodes.

In this paper, we are interested in the radiation pattern in the far-field region, and we assume that the far-field condition $A \gg r_k$ holds. The far-field distance $d_k(\phi)$ in (1) can then be approximated as

$$d_k(\phi) \approx A - r_k \cos(\phi - \psi_k). \tag{4}$$

The far-field beam pattern is thus approximated by

$$\tilde{F}(\phi|\boldsymbol{r}, \boldsymbol{\psi}) \triangleq \frac{1}{N} \sum_{k=1}^{N} e^{j\frac{2\pi}{\lambda} r_k [\cos(\phi_0 - \psi_k) - \cos(\phi - \psi_k)]}. \tag{5}$$

Alternatively, in the open-loop scenario, instead of applying Ψ_k as in (2), each node chooses its initial phase as

$$\Psi_k^\dagger = \frac{2\pi}{\lambda} r_k \cos(\phi_0 - \psi_k). \tag{6}$$

Accurate knowledge of the node positions relative to some common reference (such as the origin in this example) may thus be required. The array factor in this case is given by

$$\tilde{F}^\dagger(\phi|\boldsymbol{r}, \boldsymbol{\psi}) \triangleq \frac{e^{j\frac{2\pi A}{\lambda}}}{N} \sum_{k=1}^{N} e^{j\frac{2\pi}{\lambda} r_k [\cos(\phi_0 - \psi_k) - \cos(\phi - \psi_k)]}. \tag{7}$$

Note that the only difference between $\tilde{F}(\phi|\boldsymbol{r}, \boldsymbol{\psi})$ in (5) and $\tilde{F}^\dagger(\phi|\boldsymbol{r}, \boldsymbol{\psi})$ in (7) is the existence of the initial phase offset of $\frac{2\pi A}{\lambda}$. The far-field beampattern is thus identical for both systems, and the received signal exhibits no difference as long as the base station compensates for this phase rotation.

From (5), we have

$$\tilde{F}(\phi|\boldsymbol{r}, \boldsymbol{\psi}) = \frac{1}{N} \sum_{k=1}^{N} e^{j4\pi \frac{R}{\lambda} \sin\left(\frac{\phi_0 - \phi}{2}\right) \tilde{r}_k \sin \tilde{\psi}_k}, \tag{8}$$

where $\tilde{r}_k \triangleq r_k / R$ and $\tilde{\psi}_k \triangleq \psi_k - \frac{\phi_0 + \phi}{2}$. By assumption, the node locations (r_k, ψ_k) follow a uniform distribution over the disk of radius R. The compound random variable

$$z_k \triangleq \tilde{r}_k \sin \tilde{\psi}_k, \tag{9}$$

has the following probability density function (pdf):

$$f_{z_k}(z) = \frac{2}{\pi} \sqrt{1 - z^2}, \quad -1 \le z \le 1. \tag{10}$$

Without loss of generality, we assume that $\phi_0 = 0$. The array factor of (8) can then be rewritten as

$$\tilde{F}(\phi|\boldsymbol{z}) = \frac{1}{N} \sum_{k=1}^{N} e^{-j4\pi \tilde{R} \sin\left(\frac{\phi}{2}\right) z_k}, \tag{11}$$

where $\tilde{R} \triangleq \frac{R}{\lambda}$ is the radius of the disk normalized by the wavelength. Finally, the far-field beampattern, which may be defined as $P(\phi|\boldsymbol{z}) \triangleq \left|\tilde{F}(\phi|\boldsymbol{z})\right|^2$, is thus expressed as

$$P(\phi|\boldsymbol{z}) = \frac{1}{N} + \frac{1}{N^2} \sum_{k=1}^{N} e^{-j\alpha(\phi) z_k} \sum_{\substack{l=1 \\ l \ne k}}^{N} e^{j\alpha(\phi) z_l}, \tag{12}$$

where $\alpha(\phi) \triangleq 4\pi \tilde{R} \sin \frac{\phi}{2}$.

The average beampattern of the random array resulting from the distributed sensor network model is defined as

$$P_{\text{av}}(\phi) \triangleq E_{\boldsymbol{z}} \left\{ P(\phi|\boldsymbol{z}) \right\}. \tag{13}$$

From (12) and (10), it can be readily shown that

$$P_{\text{av}}(\phi) = \frac{1}{N} + \left(1 - \frac{1}{N}\right) \left| 2 \cdot \frac{J_1(\alpha(\phi))}{\alpha(\phi)} \right|^2, \tag{14}$$

where $J_n(x)$ is the nth order Bessel function of the first kind. In (14), the first term represents the average power level of the sidelobe, which does not depend on the node location, whereas the second term is the contribution of the mainlobe factor. Since, conditioned on ϕ, the array factor of the form (11) is an average of bounded independent and identically distributed (i.i.d.) complex random variables, by the law of large numbers the beampattern (12) converges to the ensemble average (14) in probability as $N \to \infty$.

3. PERFORMANCE OF DISTRIBUTED BEAMFORMING WITH IMPERFECT PHASE

In this section, we analyze the effect of the phase ambiguities in the closed-loop scenario as well as location estimation errors in the open-loop scenario. For each of the two scenarios, we derive the average beampattern and calculate the amount of degradation.

3.1. Closed-loop case

In the closed-loop case, the effects of imperfect phase may be easily derived, following the approach developed by Steinberg [8]. The initial phase of node k in (2) will now be given by

$$\hat{\Psi}_k = -\frac{2\pi}{\lambda} d_k(\phi_0) + \varphi_k, \tag{15}$$

where φ_k corresponds to the phase offset due to the phase ambiguity caused by carrier phase jitter or offset between the transmitter and receiver nodes. In the following, the phase offset φ_k's are assumed to be i.i.d. random variables. Then, from (3), (4), (5), and (11), the far-field array factor will be given by

$$\tilde{F}(\phi|\boldsymbol{z}, \boldsymbol{\varphi}) = \frac{1}{N} \sum_{k=1}^{N} e^{-j z_k 4\pi \tilde{R} \sin\frac{\phi}{2}} e^{j\varphi_k}. \tag{16}$$

The average beampattern of (13) will be replaced by

$$P_{\text{av}}(\phi) \triangleq E_{\boldsymbol{z},\boldsymbol{\varphi}} \left\{ P(\phi|\boldsymbol{z}, \boldsymbol{\varphi}) \right\}. \tag{17}$$

Similar to (14), direct calculation of (17) results in

$$P_{\text{av}}(\phi) = \frac{1}{N} + \left(1 - \frac{1}{N}\right) \left| 2\frac{J_1(\alpha(\phi))}{\alpha(\phi)} \right|^2 |A_\varphi|^2, \tag{18}$$

where

$$A_\varphi \triangleq E_{\varphi_k} \left\{ e^{j\varphi_k} \right\}. \tag{19}$$

Thus, as $N \to \infty$, the average beampattern will simply become a version of the original scaled by a factor of $|A_\varphi|^2$.

Let us now assume that the phase offset follows a Tikhonov distribution, a typical phase jitter model for Phase-Locked Loop (PLL) circuits, given by

$$f_\varphi(x) = \frac{1}{2\pi I_0\left(1/\sigma_\varphi^2\right)} \exp\left(\cos(x)/\sigma_\varphi^2\right), \qquad |x| \le \pi \tag{20}$$

where σ_φ^2 is the variance of the phase noise and I_n is the nth order modified Bessel function of the first kind. The corresponding attenuation factor is given by

$$A_\varphi = \frac{I_1(1/\sigma_\varphi^2)}{I_0(1/\sigma_\varphi^2)}. \tag{21}$$

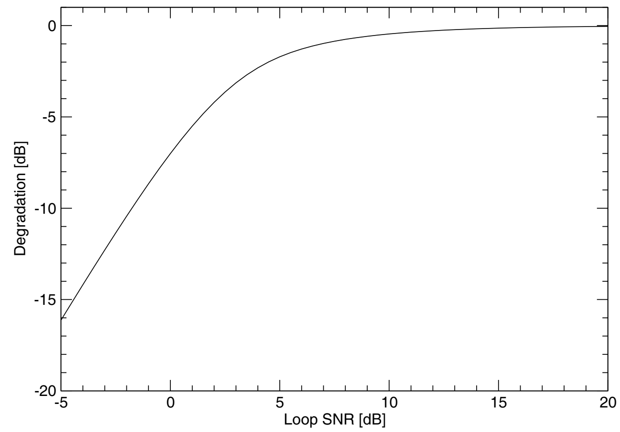

Fig. 2. Mainbeam degradation due to the phase noise in the closed-loop scenario.

The variance of the phase noise σ_φ^2 is related to the loop SNR of the PLL by

$$\rho_\varphi = 1/\sigma_\varphi^2. \tag{22}$$

Fig. 2 shows the degradation factor $|A_\varphi|^2$ with respect to the loop SNR. As observed from the figure, a loop SNR of 3 dB may be necessary for each node in order to reduce the overall beampattern degradation to less than 3 dB.

3.2. Open-loop case

In the open-loop case, our model of the initial phase is given in (6), and if there are estimation errors in the location parameters r_k and ψ_k, the initial phase will be replaced by

$$\hat{\Psi}_k^\dagger = \frac{2\pi}{\lambda}(r_k + \delta r_k)\cos(\phi_0 - (\psi_k + \delta\psi_k)), \tag{23}$$

where δr_k and $\delta\psi_k$ are the corresponding error random variables, each set assumed to be i.i.d. and also independent of r_k and ψ_k for simplicity. With the far-field approximation, we have [4]

$$\frac{2\pi}{\lambda} d_k(\phi) + \hat{\Psi}_k^\dagger \approx \frac{2\pi}{\lambda} A - \frac{4\pi}{\lambda} r_k \sin\tilde{\psi}_k \sin\left(\frac{\phi - \phi_0 - \delta\psi_k}{2}\right) + \frac{2\pi}{\lambda} \delta r_k \cos\left(\tilde{\psi}_k + \frac{\phi - \phi_0 + \delta\psi_k}{2}\right),$$

where $\tilde{\psi}_k \triangleq \psi_k - \frac{\phi + \phi_0 - \delta\psi_k}{2}$.

The resulting beampattern is expressed as

$$P(\phi|\boldsymbol{z}, \boldsymbol{v}, \boldsymbol{\delta\psi}) = \frac{1}{N} + \frac{1}{N^2} \sum_{k=1}^{N} \sum_{\substack{l=1 \\ l \ne k}}^{N} e^{j\frac{2\pi}{\lambda}(v_k - v_l)} \times e^{-j4\pi\tilde{R}\left\{ z_k \sin\left(\frac{\phi - \phi_0 - \delta\psi_k}{2}\right) - z_l \sin\left(\frac{\phi - \phi_0 - \delta\psi_l}{2}\right) \right\}},$$

where

$$z_k \triangleq \frac{r_k}{R} \sin\tilde{\psi}_k = \tilde{r}_k \sin\left(\psi_k + \frac{\delta\psi_k}{2} - \frac{\phi + \phi_0}{2}\right)$$

$$v_k \triangleq \delta r_k \cos\left(\tilde{\psi}_k + \frac{\phi + \delta\psi_k}{2}\right) = \delta r_k \cos\left(\psi_k + \delta\psi_k - \phi_0\right).$$

Conditioned on ϕ, ϕ_0 and $\delta\phi_k$, the angle $\tilde{\psi}_k$ can be seen as a uniformly distributed random variable, and thus the pdf of z_k is given by (10). Considering the fact that r_k and δr_k are assumed to be statistically independent, we further assume for analytical purposes that z_k and v_k are statistically independent. Then, again, the beampattern does not depend on the particular choice of ϕ_0. Furthermore, on modeling δr_k as being uniformly distributed over $[-r_{\text{max}}, r_{\text{max}}]$ and assuming the phase term of v_k to be uniformly distributed over $[0, 2\pi]$, the probability density function of v_k will be given, for $|v| \le r_{\text{max}}$, by

$$f_{v_k}(v) = \frac{1}{\pi r_{\text{max}}} \left[\ln \left(1 + \sqrt{1 - \left(\frac{v}{r_{\text{max}}} \right)^2} \right) - \ln \frac{|v|}{r_{\text{max}}} \right].$$

Consequently, the average beampattern can be written as

$$P_{\text{av}}(\phi) = \frac{1}{N} + \left(1 - \frac{1}{N} \right) |A_\psi(\phi)|^2 \, |A_r|^2, \tag{24}$$

where

$$A_r \triangleq E_{v_k} \left\{ e^{j \frac{2\pi}{\lambda} v_k} \right\} = {}_1F_2 \left(\frac{1}{2}\,;\, 1, \frac{3}{2}\,;\, - \left(\pi \frac{r_{\text{max}}}{\lambda} \right)^2 \right) \tag{25}$$

$$\begin{aligned} A_\psi(\phi) &\triangleq E_{z_k, \delta\psi_k} \left\{ e^{j 4\pi \tilde{R} z_k \sin\left(\frac{\phi_0 + \delta\psi_k - \phi}{2} \right)} \right\} \\ &= E_{\delta\psi_k} \left\{ \frac{J_1 \left(4\pi \tilde{R} \sin \frac{\phi - \delta\psi_k}{2} \right)}{2\pi \tilde{R} \sin \frac{\phi - \delta\psi_k}{2}} \right\}, \end{aligned} \tag{26}$$

and without loss of generality $\phi_0 = 0$ was assumed. In (25), ${}_1F_2\left(\frac{1}{2}\,;\,1, \frac{3}{2}\,;\,-x^2\right)$ denotes a generalized hypergeometric function which has an oscillatory tail but converges to zero as x increases.

Also, assuming that the $\delta\psi_k$ are uniformly distributed over $[-\psi_{\text{max}}, \psi_{\text{max}}]$ and using the approximation $\sin(\phi + \delta\psi_k) \approx \phi + \delta\psi_k$ which is valid for the beampattern around the mainbeam, we obtain

$$\begin{aligned} A_\psi(\phi) \approx& \frac{1}{2} \left(1 - \frac{\phi}{\psi_{\text{max}}} \right) {}_1F_2 \left(\frac{1}{2}\,;\, \frac{3}{2}, 2\,;\, -(\pi \tilde{R}(\phi + \psi_{\text{max}}))^2 \right) \\ &+ \frac{1}{2} \left(1 + \frac{\phi}{\psi_{\text{max}}} \right) {}_1F_2 \left(\frac{1}{2}\,;\, \frac{3}{2}, 2\,;\, -(\pi \tilde{R}(\phi - \psi_{\text{max}}))^2 \right). \end{aligned} \tag{27}$$

Since the hypergeometric function ${}_1F_2\left(\frac{1}{2}\,;\,\frac{3}{2}, 2\,;\,-x^2\right)$ has a maximum peak value of 1 at $x = 0$, the above expression indicates that regardless of the value of $\tilde{R}$, there may be two symmetric peaks around the mainbeam at $\phi = \pm\psi_{\text{max}}$ resulting in a *pointing error*. Therefore, the mainbeam may spread over by a factor of ψ_{max}. At the center of the mainbeam, we have

$$A_\psi(0) = {}_1F_2 \left(\frac{1}{2}\,;\, \frac{3}{2}, 2\,;\, - \left(\pi \frac{R\psi_{\text{max}}}{\lambda} \right)^2 \right). \tag{28}$$

Fig. 3 shows the degradation factor $|A_r|^2$ and $|A_\psi(0)|^2$ for a given $\frac{r_{\text{max}}}{\lambda}$ and $\frac{R\psi_{\text{max}}}{\lambda}$. As observed from the figure and discussion above, the angle estimation error has two effects, i.e., pointing error and mainbeam degradation. In particular, if we wish to suppress the degradation below 3 dB, from the figure, we should choose $R\psi_{\text{max}}/\lambda \le 1/2$. This means that the maximum angle estimation error should satisfy

$$\psi_{\text{max}} \le \frac{\lambda}{2R} = \frac{1}{2\tilde{R}}, \tag{29}$$

and as $\tilde{R}$ becomes large, the requirement of minimum angle ambiguity from (29) becomes severe.

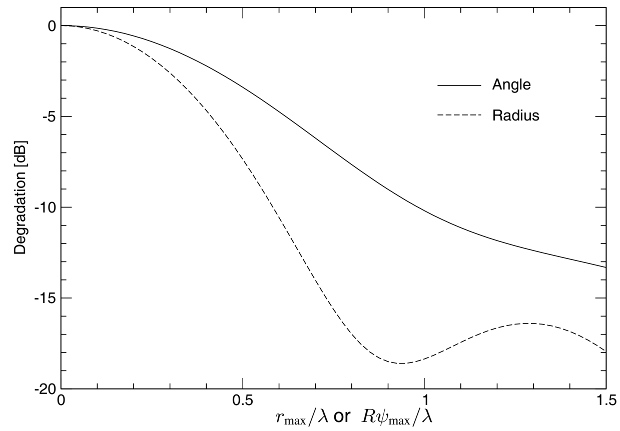

Fig. 3. Mainbeam degradation due to location estimation errors in open-loop scenario.

4. CONCLUSION

We have analyzed the stochastic performance of random arrays for distributed collaborative beamforming, in the framework of wireless ad hoc sensor networks. We have considered two scenarios of distributed beamforming and investigated the effects of phase ambiguity and location estimation error upon the resultant average beampatterns quantitatively. In the closed-loop scenario, for example, it has been shown that the PLL of each sensor node should be operated with a loop SNR higher than 3 dB in order to reduce the overall beampattern degradation to less than 3 dB. The results may thus serve as a design factor that collaborative sensors should satisfy for distributed beamforming under phase ambiguity.

5. REFERENCES

[1] K. Yao, R. E. Hudson, C. W. Reed, D. Chen, and F. Lorenzelli, "Blind beamforming on a randomly distributed sensor array system," *IEEE J. Select. Areas Commun.*, vol. 16, no. 8, pp. 1555–1567, Oct. 1998.

[2] J. C. Chen, L. Yip, J. Elson, H. Wang, D. Maniezzo, R. E. Hudson, K. Yao, and D. Estrin, "Coherent acoustic array processing and localization on wireless sensor networks," *Proc. IEEE*, vol. 91, no. 8, pp. 1154–1162, Aug. 2003.

[3] H. Ochiai, P. Mitran, H. V. Poor, and V. Tarokh, "Collaborative beamforming in ad hoc networks," in *Proc. IEEE Inform. Theory Workshop*, Oct. 2004.

[4] H. Ochiai, P. Mitran, H. V. Poor, and V. Tarokh, "Collaborative beamforming for distributed wireless ad hoc sensor networks," to appear in *IEEE Trans. Signal Processing*.

[5] Y. T. Lo, "A mathematical theory of antenna arrays with randomly spaced elements," *IRE Trans. Antennas Propagat.*, vol. 12, pp. 257–268, May 1964.

[6] B. D. Steinberg, "The peak sidelobe of the phased array having randomly located elements," *IEEE Trans. Antennas Propagat.*, vol. 20, no. 2, pp. 129–136, Mar. 1972.

[7] C. A. Balanis, *Antenna Theory: Analysis and Design*, Wiley, New York, 1997.

[8] B. D. Steinberg, *Principles of Aperture & Array System Design*, Wiley, New York, 1976.