A GAME THEORETICAL APPROACH FOR TRANSMISSION STRATEGIES IN SLOTTED ALOHA NETWORKS WITH MULTI-PACKET RECEPTION

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ABSTRACT

In this paper we consider finite-size slotted ALOHA sensor networks with multiple packet reception capability and selfish sensors. Each sensor wishes to maximize its individual expected reward. We exploit decentralized channel state information (CSI) to obtain transmission policies that are optimal for each sensor. The problem is formulated as a finite player,finite action, non-cooperative stochastic game where each sensor is a selfish but rational player. We prove for the first time that under the Signal to Interference Noise Ratio (SINR) threshold reception model the optimal transmission policy for each player belongs to the class of threshold policies. As a result, there exists a Nash-equilibrium at which all players adopt pure strategies. The optimality of threshold policies greatly simplifies the estimation of optimal transmission schemes. We present a provably convergent algorithm for finding the threshold for each sensor and illustrate its performance via numerical examples.

1. INTRODUCTION

In this paper we formulate the problem of selecting optimal transmission probabilities in slotted ALOHA system given decentralized CSI as a non-cooperative game of a finite number of players and actions where randomized strategies are allowed.

ALOHA is distributed by nature, it is traditionally based on the collision model which limits the throughput of any system to maximum 1 packet/slot assuming that all packets are lost when two or more users transmit. This collision model is pessimistic and does not hold in many cases for example when CDMA is deployed. The Multi-packet Reception (MPR) model has almost replaced the collision model since it was proposed in [3],[4]. The limitations of the MPR model are: an assumption that all users are indistinguishable at all times and totally ignorance of CSI. [1] proposed a Generalized MPR model (G-MPR) that takes into account the channel states of transmitting users and only assumes long term symmetry among users.

In [4],[11],[5], the system asymptotic stable throughput and stability of ALOHA are analysed under the MPR model using a non-game theoretic approach. [1] gives theoretical limits on the system asymptotic maximal stable throughput taking into account decentralized CSI under the G-MPR model also using a centralized design approach. Allen B. MacKenzie et.al. provides the first results on the stability regions of ALOHA systems in the presence of selfish users under the collision model [8],[7]. A more recent paper [9] considers stability of slotted ALOHA under the MPR model. It is shown that an equilibrium exists and rational users would not try to transmit at all times.

In this paper, we deploy the SINR threshold receiver model, which is an instance of the G-MPR model. We prove the optimality of threshold transmission strategies for players in the ALOHA stochastic game assuming that each player knows its channel perfectly prior to each transmission slot. We present a provably convergent algorithm for estimating the optimal threshold for each sensor.

The paper is organized as follows: Section 2 is a brief cover of the reception model and problem formulation. The theorem on the optimality of threshold policies is in Section 3. We present a stochastic algorithm for finding the threshold and prove its convergence in Section 4. Section 5 contains numerical examples.

2. ALOHA GAME MODEL AND PROBLEM FORMULATION

We consider a slotted ALOHA sensor network of K ($K < \infty$) sensors. We assume the uplink signal to noise ratios (SNRs) represent the channel states of a sensor as in [1]. Let the random variable γ_i denote the channel state of sensor i . We assume that at the begining of each time slot sensor i knows its channel state (SNR), i.e. the instantaneous realization of the random variable γ_i. Let $F_i(.)$ be the probability distribution function of γ_i. For simplicity, we assume that at each time slot all sensors have packets to

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transmit. This assumption is equivalent to the assumption that the number of sensors who wish to access the channel is known in [9]. During a time slot if sensor i does not transmit a waiting cost $c_w^{(i)}$ occurs, if it transmits and its packet is not received successfully a transmission cost $c_t^{(i)}$ occurs. Lastly, if sensor i transmits and its packet goes through a reward of $1 - c_t^{(i)}$ is obtained. A neccessary condition is $0 \leq c_w^{(i)} < c_t^{(i)} < 1$ for all sensor i. The waiting and transmission costs may vary among systems and even sensors depending on several factors such as battery constraints, performance requirements. It is worth noting that without a positive transmission cost, every sensor will always attempt to transmit regardless of its channel state. In comparison, the waiting costs can be adjusted to account for the long-term fairness issue.

Even though our results are valid for a broad class of reception models, for simplicity and rigor of analytical proofs we specifically consider CDMA systems with matched filter receivers and a SINR (Signal to Interference Noise Ratio) threshold reception model: a packet from sensor i is considered successfully received if and only if [10]:

$$\frac{P_j}{\sigma^2 + \frac{1}{N}\sum_{i\neq j} P_i} = \frac{\gamma_j}{1 + \frac{1}{N}\sum_{i\neq j}\gamma_i} > \beta, \qquad (1)$$

where P_j is the received power of sensor i; $P_j = \gamma_j \sigma^2$; σ^2 is the noise power, γ_j as defined above is the uplink SNR and represents channel state of sensor j, N is the spreading gain, β is some constant and often referred to as QoS requirement.

Our non-cooperative stochastic game then can be set up as follows:

- The set of players I is the set of sensors indexed by $i = 1, 2, \ldots, K$.
- For any player i; $i = 1, 2, \ldots, K$, the set of actions $A_i = \{W, T\}$ where W means to wait, T means to transmit. A player can choose to transmit with some probability, i.e. randomized strategies are allowed.
- A strategy is a mapping from channel states to transmit probabilities and the strategy of sensor i will be represented by a function $p_i(.) : R_+ \to [0, 1]$.

Since we have a game of finite players, finite number of actions and allow mixed strategies, at least one Nash equilibrium exists [2]. A Nash equilibrium is a point at which no player can gain by individually deviating from its current policy. Our objective is to prove that there exists a Nash equilibrium at which every sensor deploys a threshold transmission strategy.

3. OPTIMAL STRATEGY

Without loss of generality let us consider player (sensor) 1. Recall the notation of a strategy for sensor 1: $p_1(.) : R_+ \to [0, 1]$. We drop the index for sensor 1 in this section, i.e. we write $p(.)$ instead of $p_1(.)$. We wish to prove that the best strategy for sensor 1 is a threshold policy or in other words the optimal function $p_1^*(.)$ is a step function.

We now give an expression for the expected reward of sensor 1 when it plays with transmit policy $p(.)$, which we denote by $L(p(.))$:

$$\begin{aligned} L(p(.)) &= \int_0^\infty (p(\overline{\gamma})\Psi(\overline{\gamma}) - (1 - p(\overline{\gamma}))c_w) f(\overline{\gamma}) d\overline{\gamma} \\ &= \int_0^\infty p(\overline{\gamma})(\Psi(\overline{\gamma}) + c_w) f(\overline{\gamma}) d\overline{\gamma} - c_w \qquad (2) \end{aligned}$$

$\Psi(\overline{\gamma})$ is the expected reward when the channel state of the sensor is $\overline{\gamma}$. For CDMA ALOHA system with matched filter receivers and SINR threshold reception model $\Psi(\overline{\gamma})$ is:

$$\begin{aligned} \Psi(\gamma) &= \int_0^\infty \!\!\ldots \int_0^\infty \mathbf{I}\Big(\frac{\gamma}{1 + \frac{1}{N}\sum_{j=2}^{j=K}(\gamma_j)} > \beta\Big) - c_t \\ &\quad dF_2(\gamma_2)\ldots dF_K(\gamma_K) \qquad (3) \end{aligned}$$

In (3) $\mathbf{I}(.)$ is the indicator function. Maximizing the individual expected reward of the player can be formulated as:

$$\max_{p(.)} \quad L(p(.)) \qquad (4)$$

Theorem 1. *Consider an ALOHA network of $K < \infty$ sensors and the non-cooperative stochastic game setting described in Section 2. There exists a Nash-equilibrium at which each sensor adopts a threshold transmission strategy:*

$$p(\gamma) = \boldsymbol{I}(\gamma > \theta) \qquad (5)$$

for some $\theta \in [0, \infty)$.

Proof. For sensor 1 in the network rewite (3) as

$$\begin{aligned} \Psi(\gamma) &= \mathbf{E}_{F_2(\gamma_2),\ldots,F_K(\gamma_K)}\left[\mathbf{I}\left(\text{SINR} > \beta\right) \right.\\ &\qquad \left.|p_2(.), \ldots, p_K(.)\right] - c_t \\ &= \Pr\left((\text{SINR} > \beta)\,|p_2(.), \ldots, p_K(.)\right) - c_t \\ &= \Pr\left(\gamma > \beta * (\sigma^2 + P_I)|p_2(.), \ldots, p_K(.)\right) - c_t \qquad (6) \end{aligned}$$

In (6), P_I is the effective interference power. From (6) it is obvious that $\Psi(\gamma)$ is a non-decreasing function of γ. In addition, $\Psi(0) = -c_t < -c_w$, $\Psi(\infty) = 1 - c_t > 0$. As a result, there must exist a threshold θ so that $\Psi(\gamma) + c_w \leq 0$ for all $\gamma \leq \theta$ and $\Psi(\gamma) + c_w > 0$ for all $\gamma > \theta$. The value of θ may vary among sensors.

Recall that the objective is to maximize

$$L(p(.)) = \int_0^\infty p(\gamma)(\Psi(\gamma) + c_w)f(\gamma)d\gamma - c_w$$

It can easily be seen that it is optimal to select

$$p(\overline{\gamma}) = \begin{cases} 1 & \text{if } \Psi(\gamma) + c_w > 0 \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

In other words $p(\gamma) = \mathbf{I}(\gamma > \theta)$ is always an optimal policy. It follows that there exists a Nash equilibrium at which all players adopt (possibly different) threshold policies. □

Remark: The above theorem applies to any reception models that satisfy: 1) the expected probability that a packet is received correctly $\Psi(\gamma) + c_t$ is a non-decreasing function of γ, 2) $\Psi(0) + c_w < 0$, 3)$\Psi(\infty) + c_w > 0$.

4. ESTIMATE THE OPTIMAL STRATEGY

In light of therorem 1 we now restrict to the class of threshold policies: $p(\gamma) = \mathbf{I}(\gamma > \theta)$ and since the waiting cost c_w is only a constant we reformulate the optimization problem given by (2),(4) as:

$$\max_\theta \quad L(\theta) = \int_0^\infty \mathbf{I}(\gamma > \theta)\left(\Psi(\gamma) + c_w\right) f(\gamma)d\gamma \quad (8)$$

The gradient of the objective function is then:

$$\begin{aligned} \nabla_\theta L(\theta) &= \nabla_\theta \int_0^\infty \mathbf{I}(\gamma > \theta)\left(\Psi(\gamma) + c_w\right) f(\gamma)d\gamma \\ &= -(\Psi(\theta) + c_w)f(\theta) \quad (9) \end{aligned}$$

Since $f(\theta)$ can be absorbed into the step size, to utilize the gradient ascent method we only need to obtain a unbiased estimate of

$$\begin{aligned} \Psi(\gamma) &= \mathbf{E}_{F_2(\gamma_2),\ldots,F_K(\gamma_K)}\left[\mathbf{I}\left(\text{SINR} > \beta\right)\right. \\ &\quad \left.|p_2(.),\ldots,p_K(.)\right] - c_t \quad (10) \end{aligned}$$

In practice, this can be done by using a temporary power control strategy in the learning phase to make sure that the received SNR is equal to θ and counting the number of ACKs and NACKs that are sent from the base station.

We define a stationary point, which is also a local maximizer of the system throughput as

$$\theta^* = \{\theta : \nabla_\theta L(\theta) = 0, \nabla^2_\theta L(\theta) < 0\} \quad (11)$$

Algorithm 1. *An algorithm for Optimal Strategy Selection*

- Initialization:

  $l = 1$

  $\theta^{(l)} = \theta$

- Sampling, evaluation of gradient and update loop

  while $|\Psi(\theta^{(l)}) + c_w| > 0$ **do**

  Estimate: $\widehat{\Psi^{(l)}(\theta^{(l)})}$ using (10)

  Update Equation: $\theta^{(l+1)} = \theta^{(l)} - \varepsilon_l \left(\widehat{\Psi^{(l)}(\theta^{(l)})} + c_w\right)$ (12)

  $l = l + 1$

  end while

- Conditions: $\sum_{l=1}^{l=\infty} \varepsilon_l = \infty; \sum_{l=1}^{l=\infty} \varepsilon_l^2 < \infty$ (C1)

Theorem 2. *Convergence of Algorithm 2*

The sequence θ generated by Algorithm 1 converges to the threshold level corresponding to a local optimizer θ^*, defined in (11), of (8) and therefore of the sensor's individual expected reward.

Proof. For a fixed initial θ the sequence $\nabla_\theta L^{(l)}(\theta) : l = 1, 2\ldots$ are i.i.d. Therefore equation (12) is an instance of the well known Robbins Munro algorithm. The convergence of this algorithm is proved in [6] under the condition (C1) and uniform integrability of $\nabla_\theta L^{(l)}(\theta)$. A sufficient condition for uniform integrability is that the channel distribution has finite variance. □

Remark: The above algorithm can be implemented in real time and can adapt to the changes in the statistics of the network. For adaptivity of the algorithm, a constant, reasonably small step size need to be used. In such cases, the constant step size acts as a forgetting factor.

5. NUMERICAL STUDIES

We present numerical results for slotted ALOHA CDMA networks with matched filter receivers and random signature sequences in Rayleigh Fading channels. Assume that the uplink SNR represents the channel state. We use the heuristic SINR threshold reception model (1).

In the first experiment, we let all sensors have the same channel distribution, fix the waiting cost at $c_w = 0.02$, vary the number of sensors in the network and the transmission cost c_t. We use algorithm 1 to find the transmission threshold at a Nash-equilibrium for each sensor in a completely decentralized manner. It can be seen from figure 1 that the threshold increases when the network size or the transmission cost increase. This implies that as the size of the network grows, each sensor transmit with a lower probability and it is possible to use the transmission cost to control the average transmission probability of network users.

In the second experiment, we consider a CDMA slotted ALOHA network of 20 sensors with matched filter receivers. We use a fixed transmission cost $c_t = 0.2$. We also let all sensors have same channel distribution. Figure

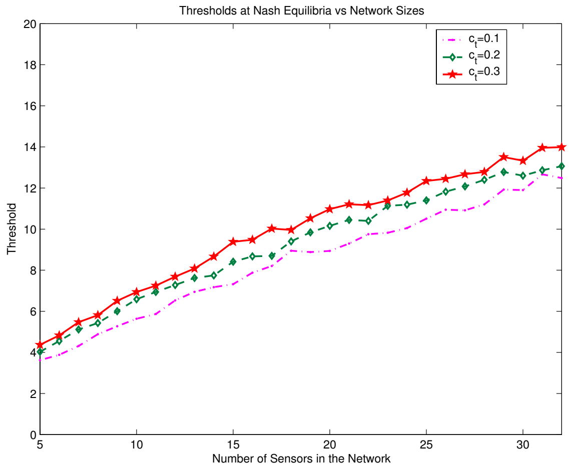

Fig. 1. Thresholds versus Network Sizes

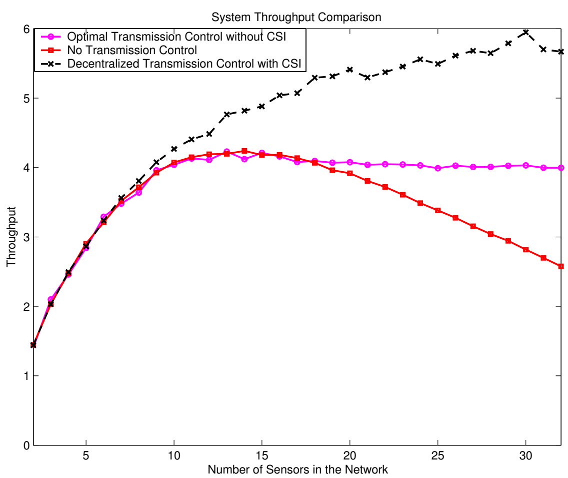

Fig. 2. Throughput Comparison between Transmission Control Schemes

2 compares the system throughput obtained by using algorithm 1 with two other cases: 1)without transmission control (which means a sensor always transmit with certainty), 2) Use the algorithm in [4] for optimal transmission control without CSI. It can also be seen from figure 2 that the system throughput obtained by the decentralized algorithm in this paper is superior to the other two cases.

6. CONCLUSION

In this paper we prove that there exists a Nash equilibrium at which every rational sensor deploys a threshold transmission strategy. The optimal thresholds may be different for different sensors. We present a convergent algorithm for estimating the optimal threshold for each sensor in a decentralized manner. The algorithm is totally distributed, inexpensive, highly scalable, and can adapt to changes in the statistical properties of the system and therefore particularly useful for large scale sensor networks.

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